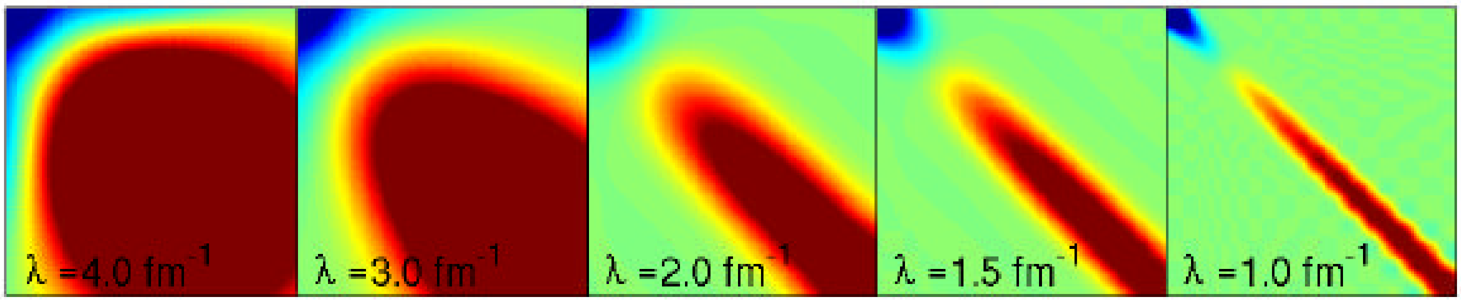
The Similarity Renormalization Group – In Pictures



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THE OHIO STATE UNIVERSITY

In collaboration with: S.K. Bogner, R.J. Furnstahl, E. Jurgenson, & R.J. Perry

The Big Picture

Nuclear DFT

DME ⇑

Nuclear Matter

PT+ ⇑

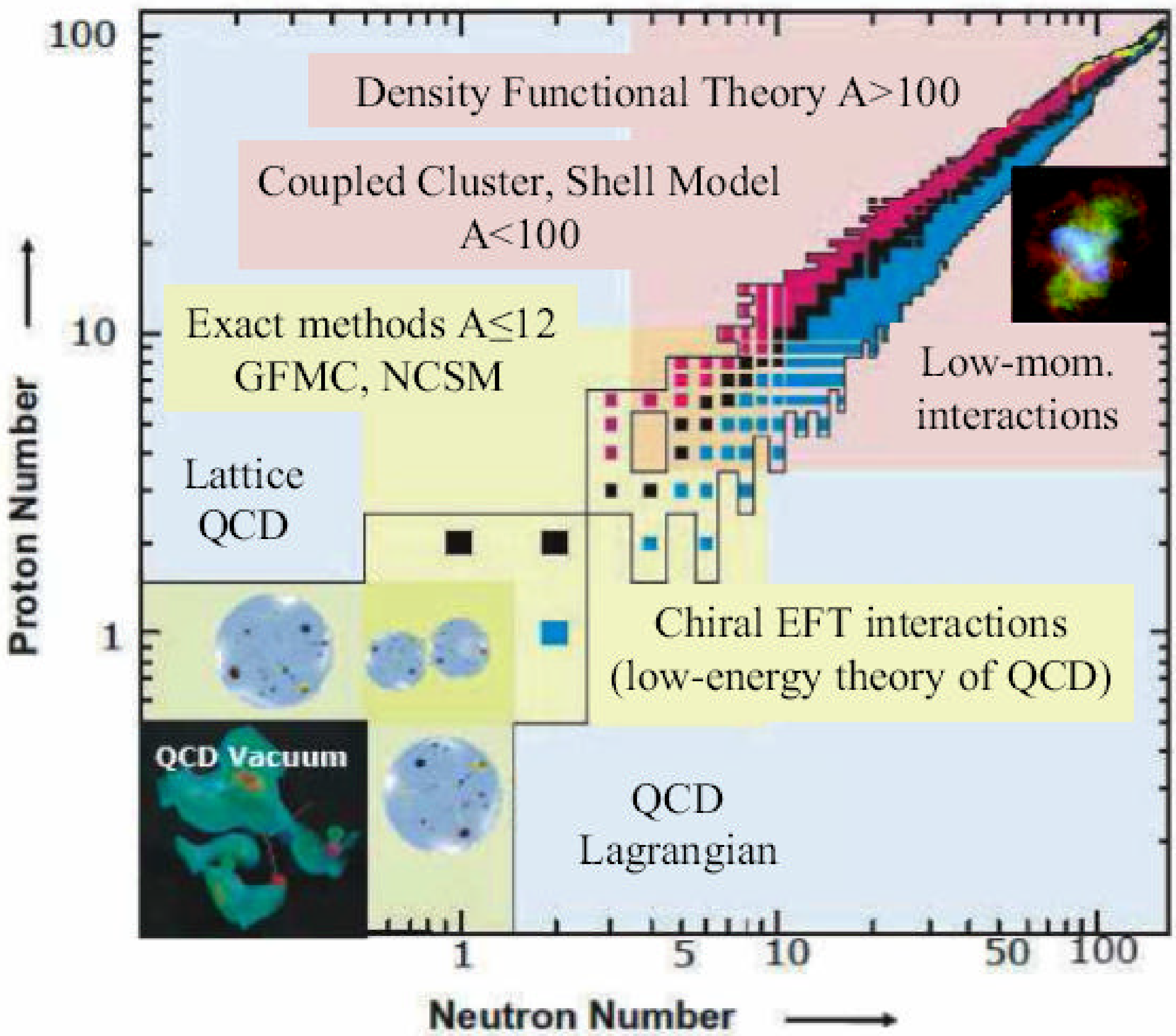
$V_{\mathrm{low}\,k}$
NN$\cdots$N

RG ⇑

Chiral EFT
NN$\cdots$N

LEC's ⇑

Lattice QCD



(adapted from A. Richter @ INPC2004)

The Big Picture

Nuclear DFT

DME ⇑

Nuclear Matter

PT+ ⇑

$V_{\mathrm{low}\,k}$

NN$\cdots$N

RG ⇑

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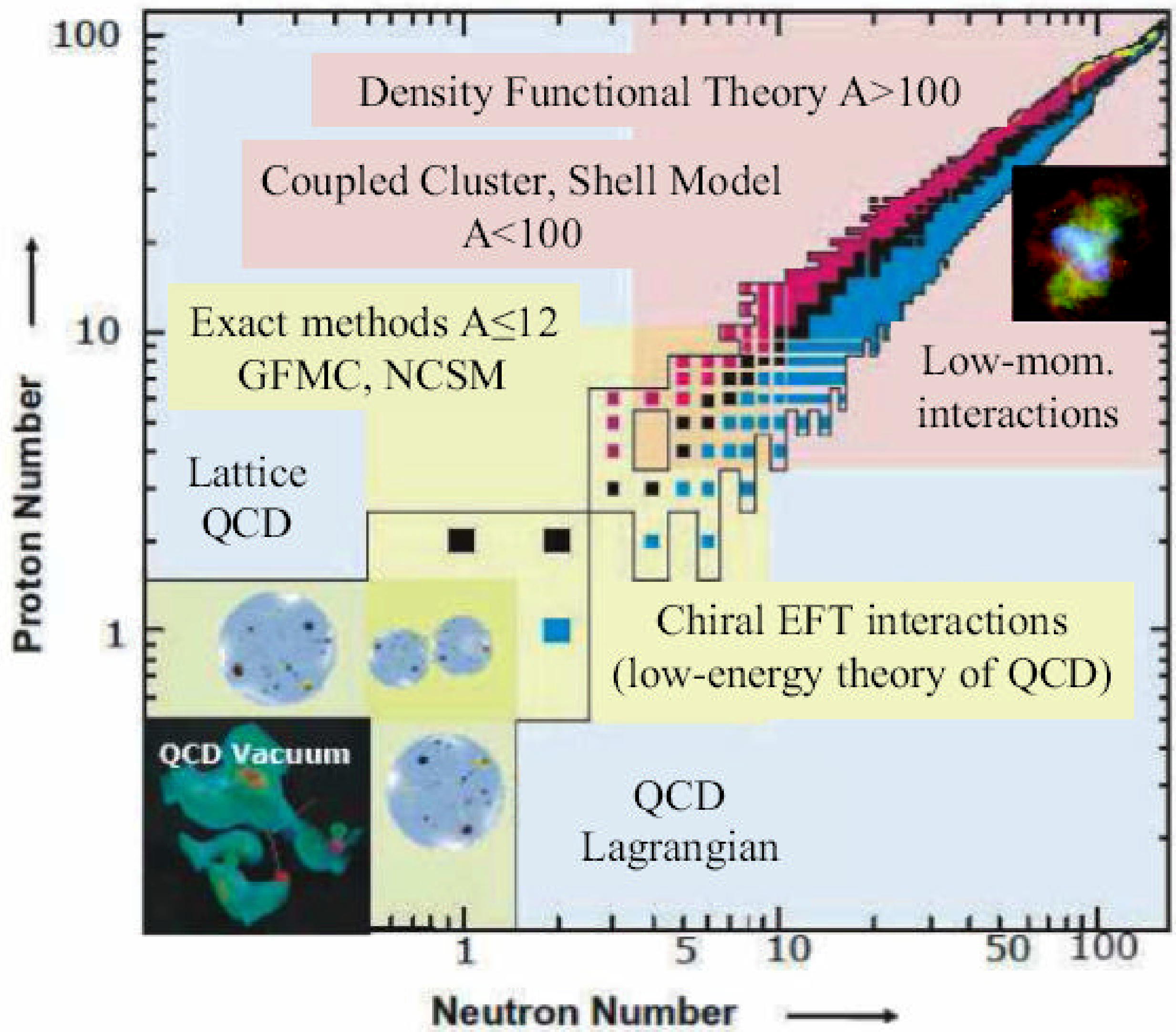
Lattice QCD



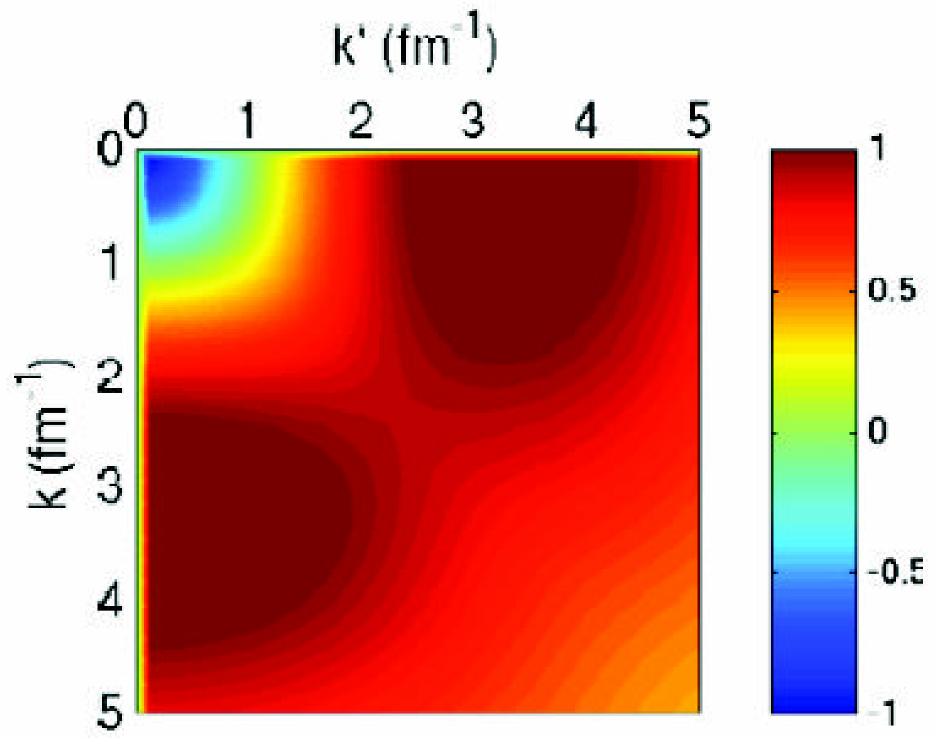
(adapted from A. Richter @ INPC2004)

SRG Fundamentals

- We need a method to simplify the EFT potentials
- Ideally, low energy physics will be insensitive to the short distance details of a Hamiltonian
- SRG provides a means to systematically evolve computationally difficult Hamiltonians toward diagonalized form
- Based on unitary transformations!
- Get a lower acceptable cutoff to simplify short distance physics

Representation of AV18 Potential

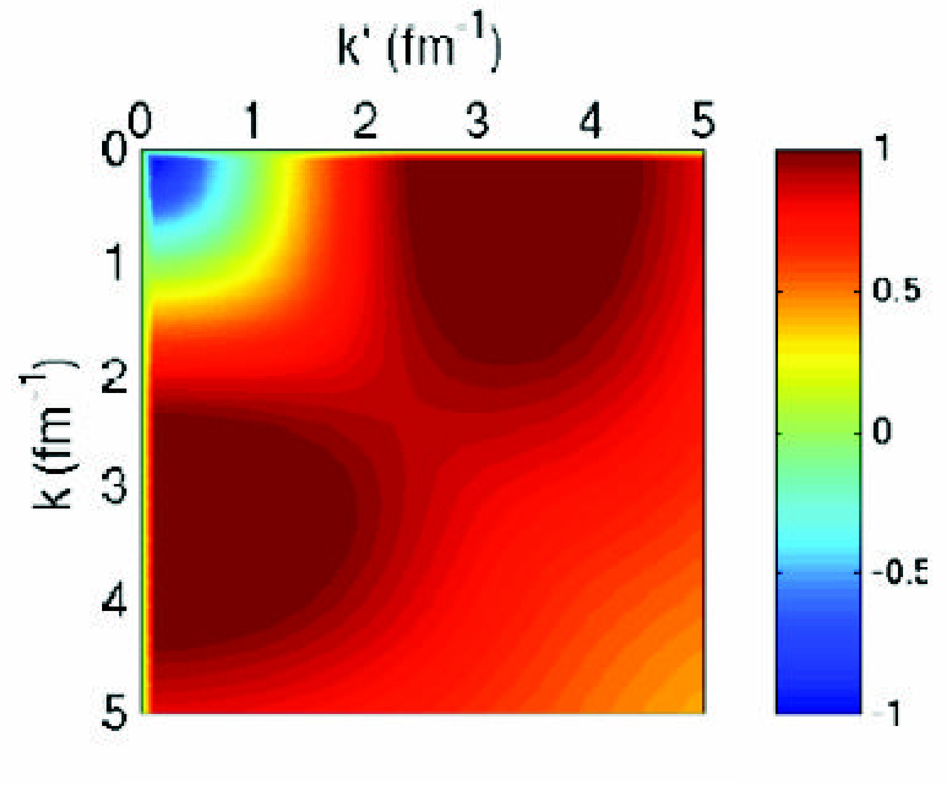
- Contains strong off diagonal elements
- => Coupling of high and low momentum values



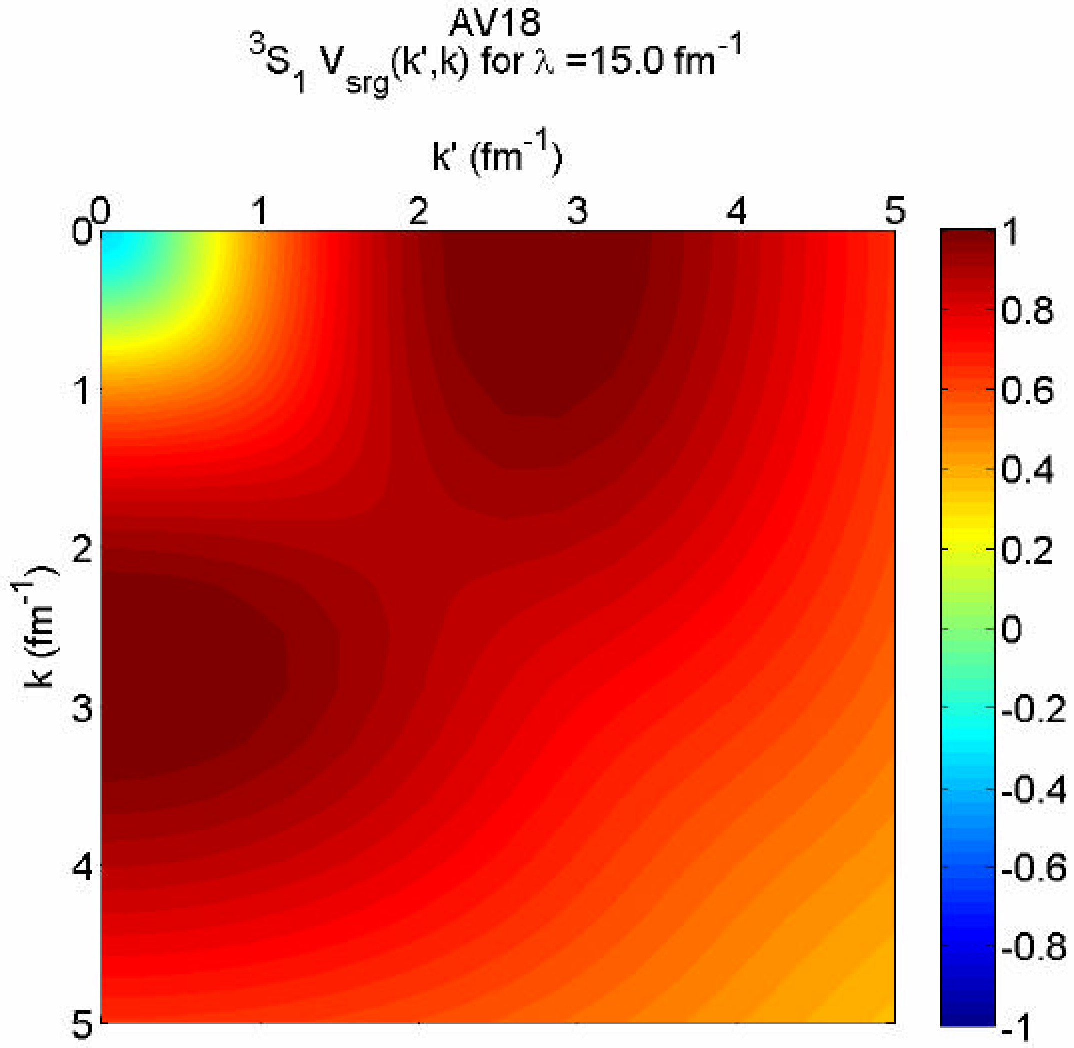
$$V(k, k') \sim < k|V|k' >$$

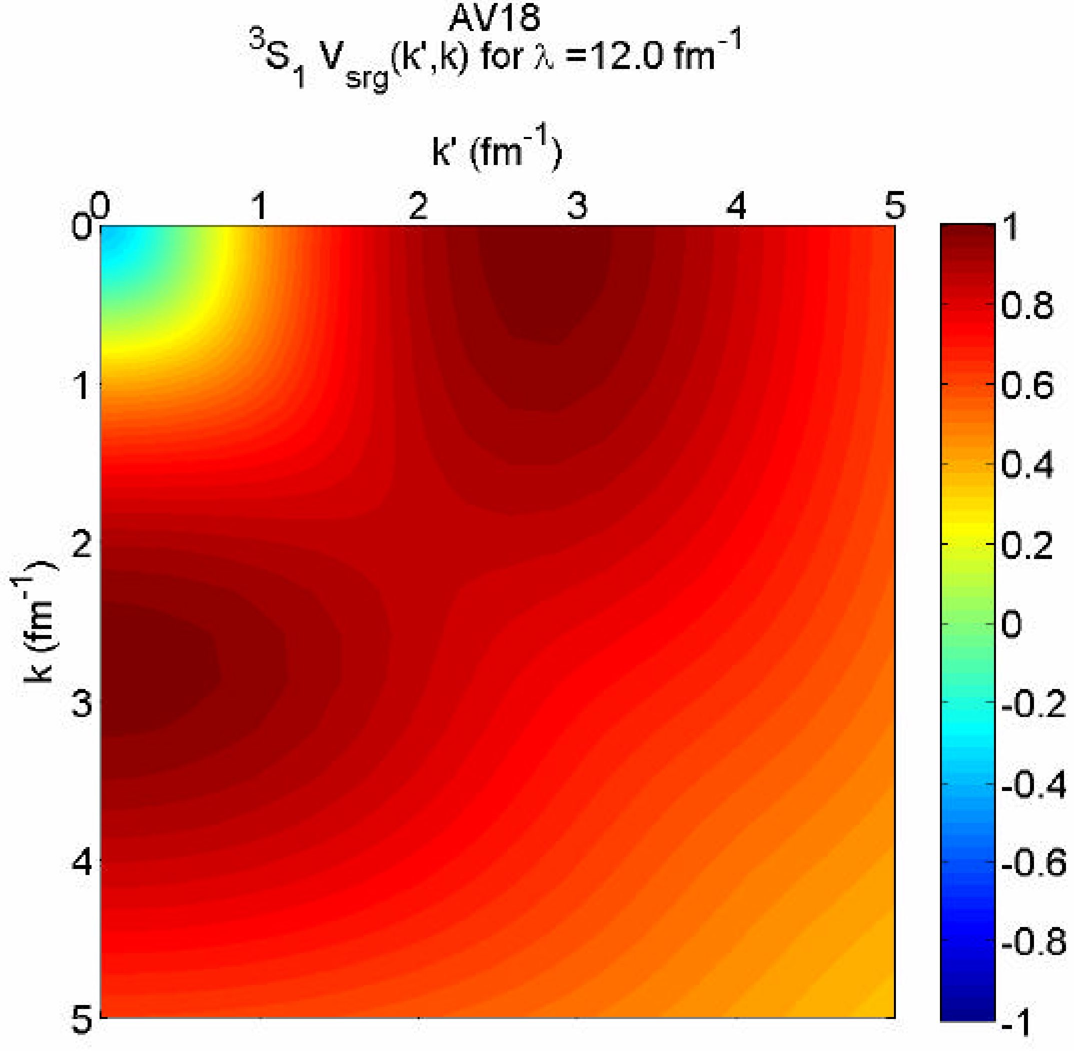
Representation of AV18 Potential

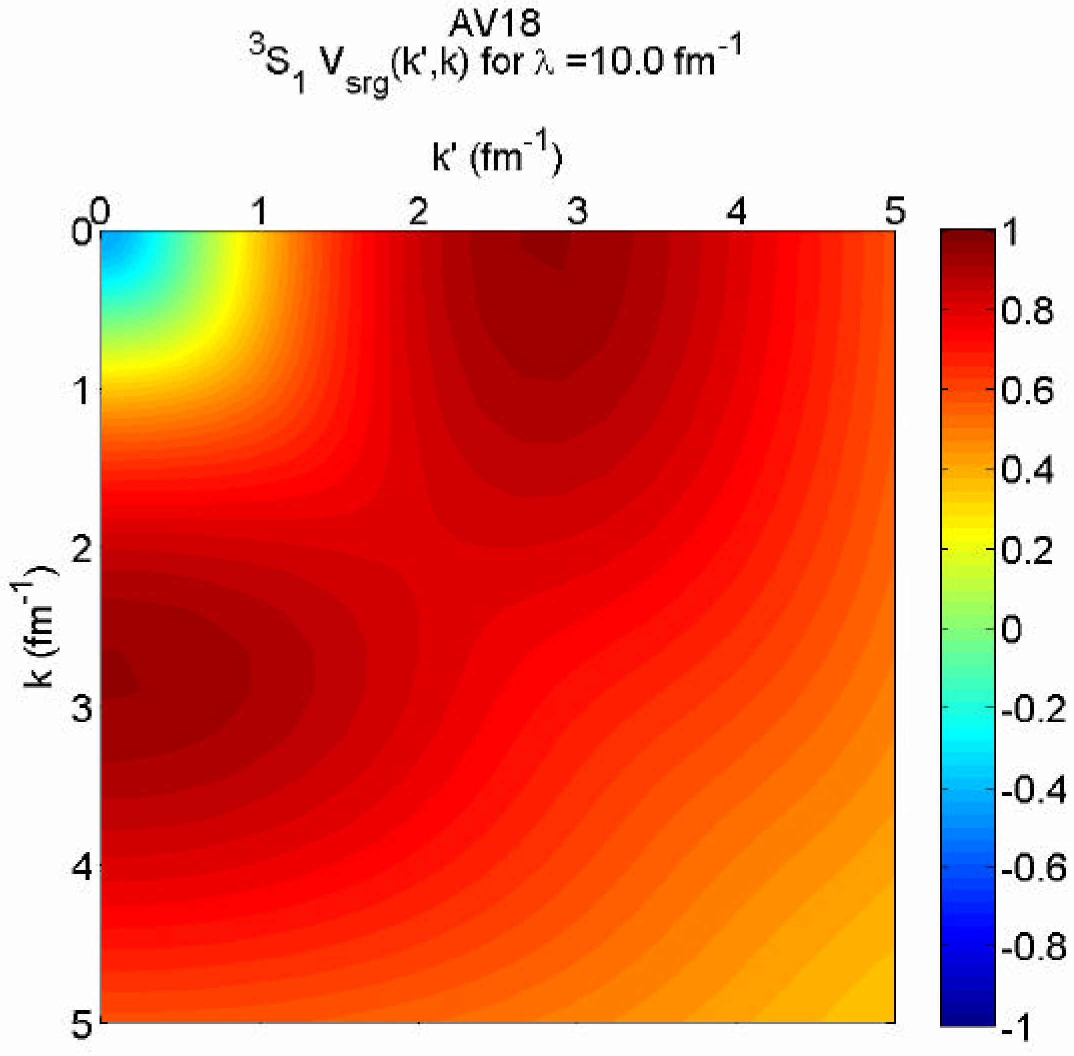
- Contains strong off diagonal elements
- => Coupling of high and low momentum values
- Apply SRG . . .

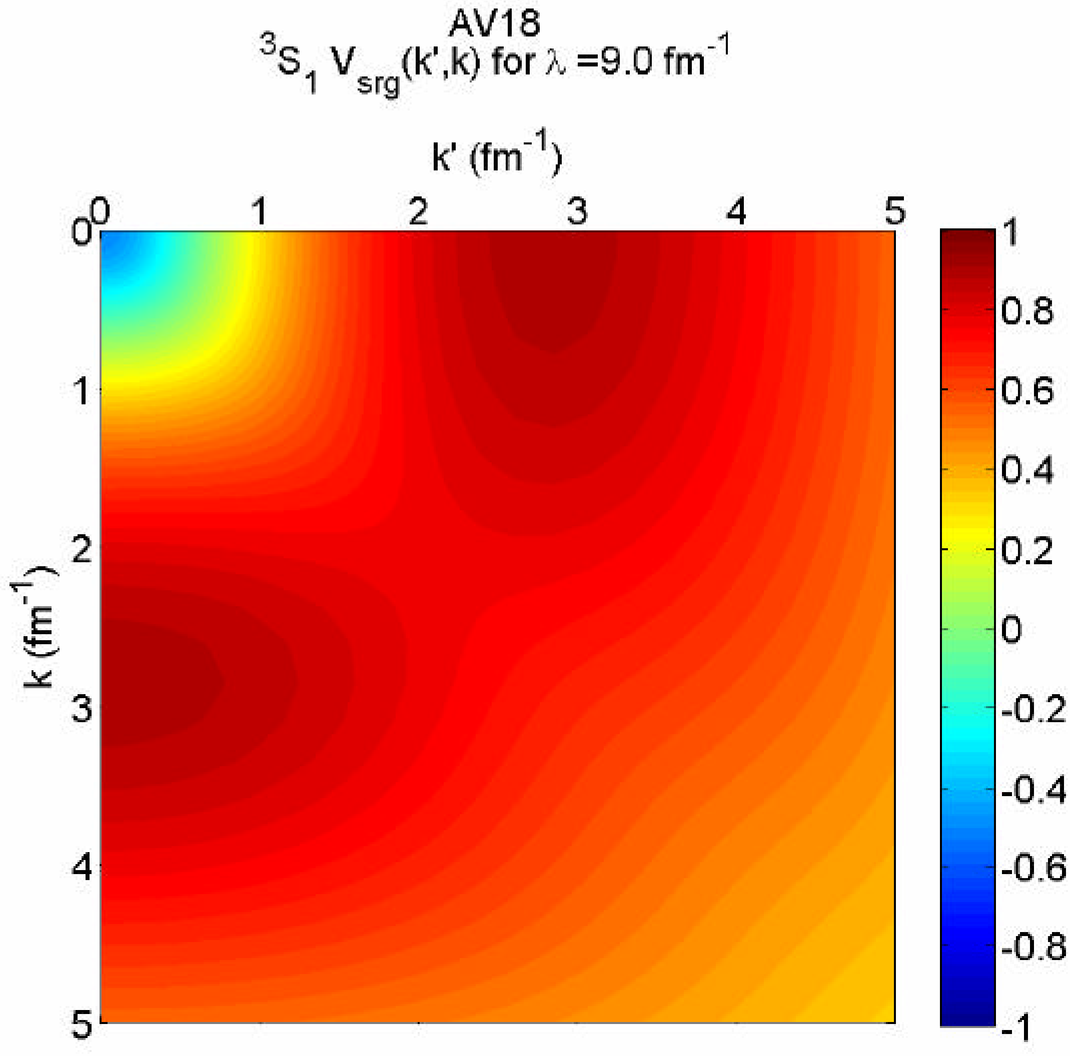


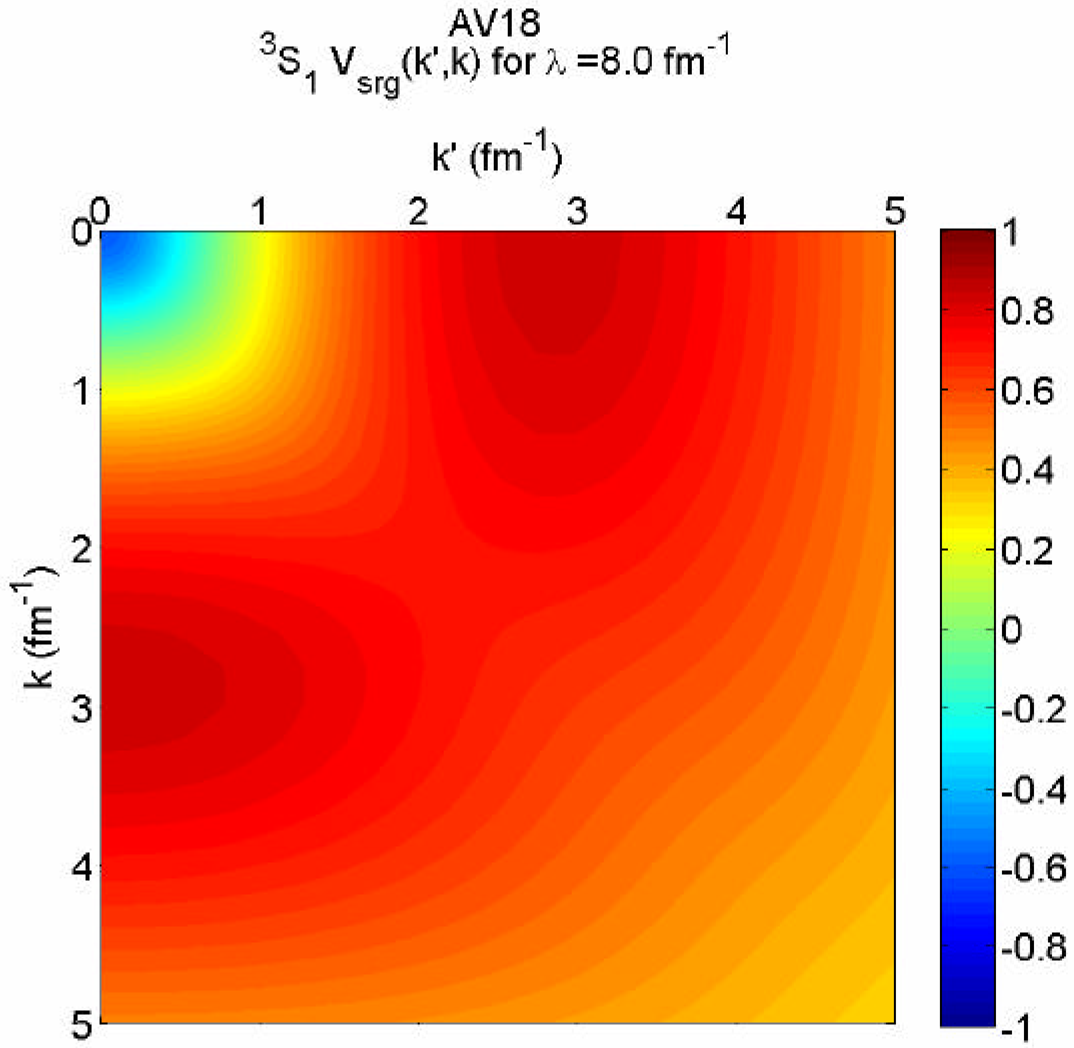
$$V(k, k') \sim \langle k|V|k' \rangle$$

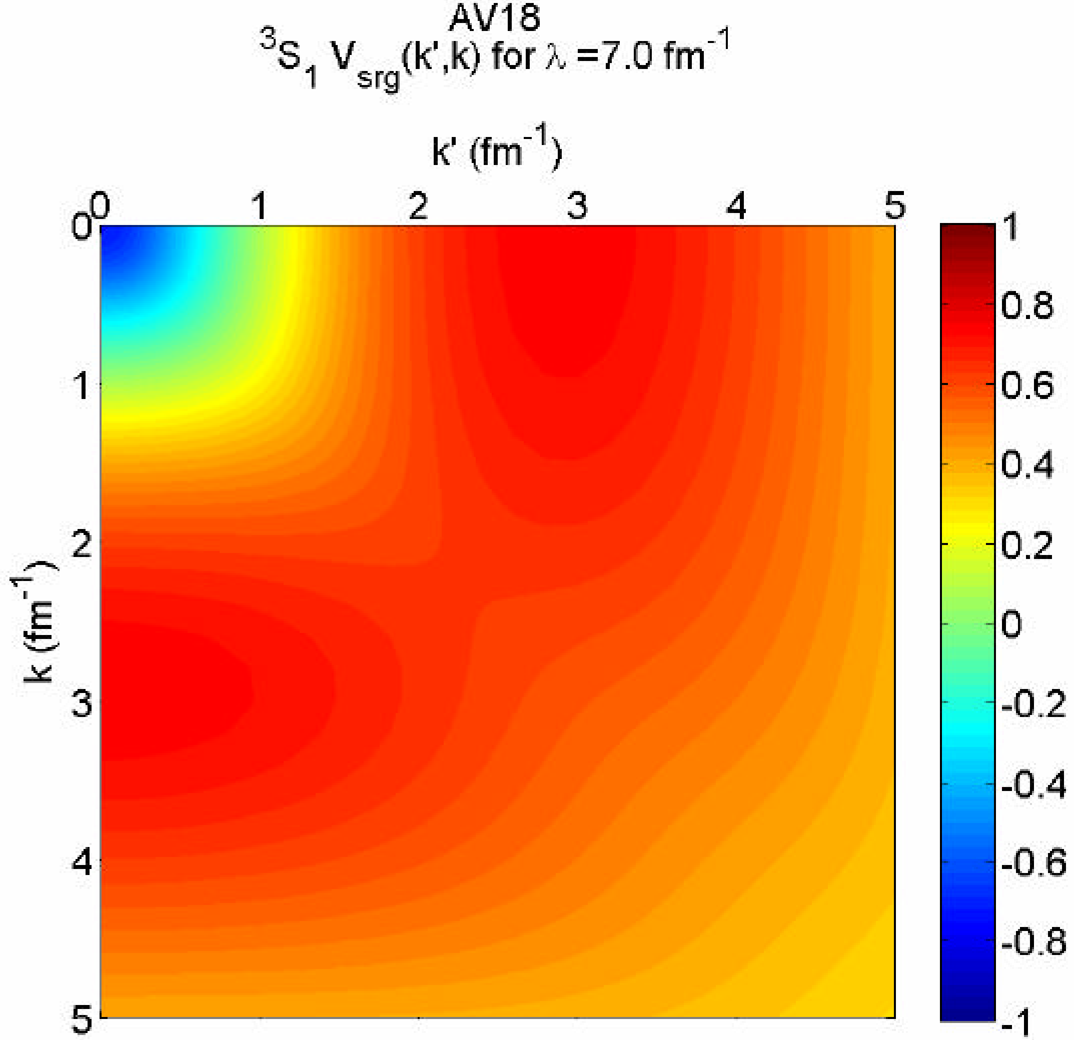

AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =15.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

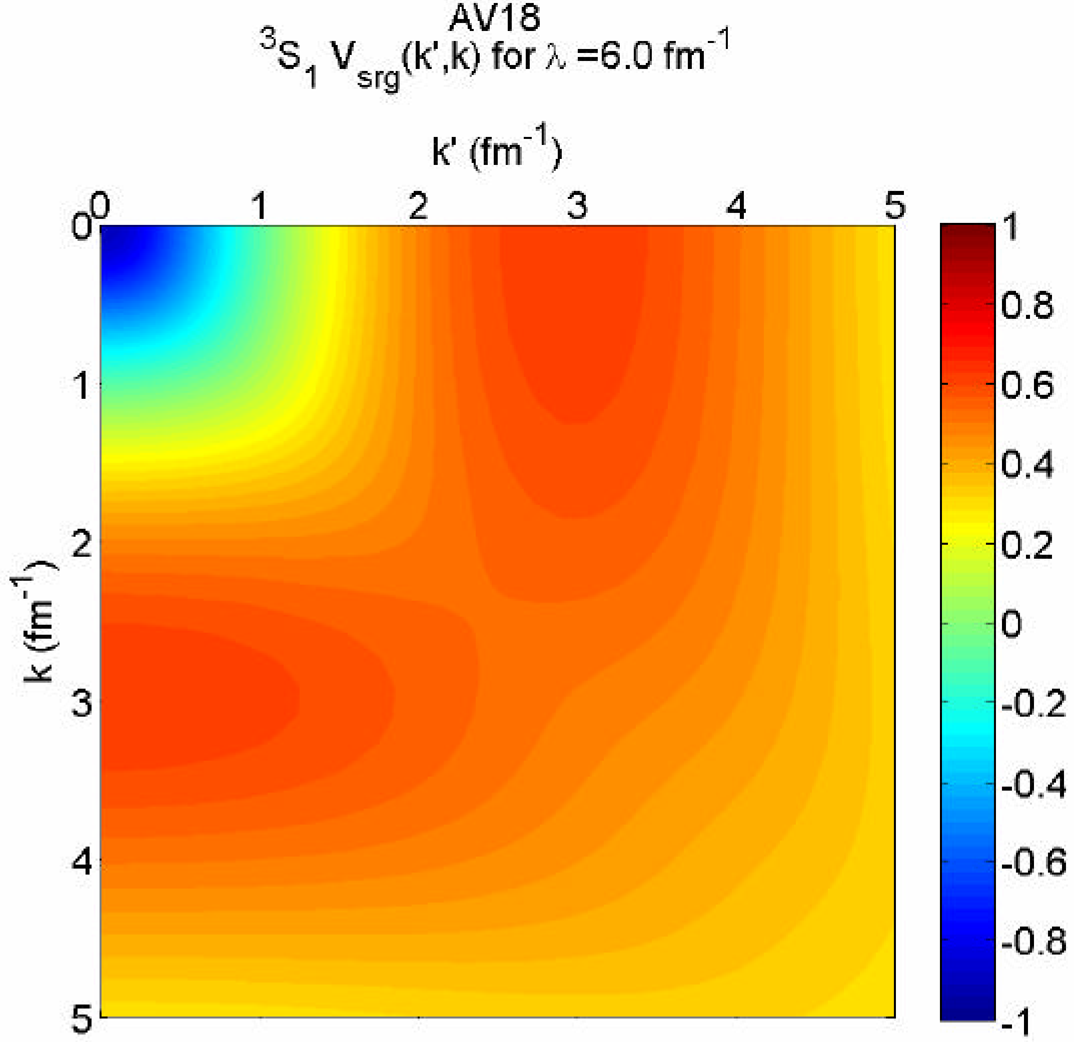

AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =12.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =10.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

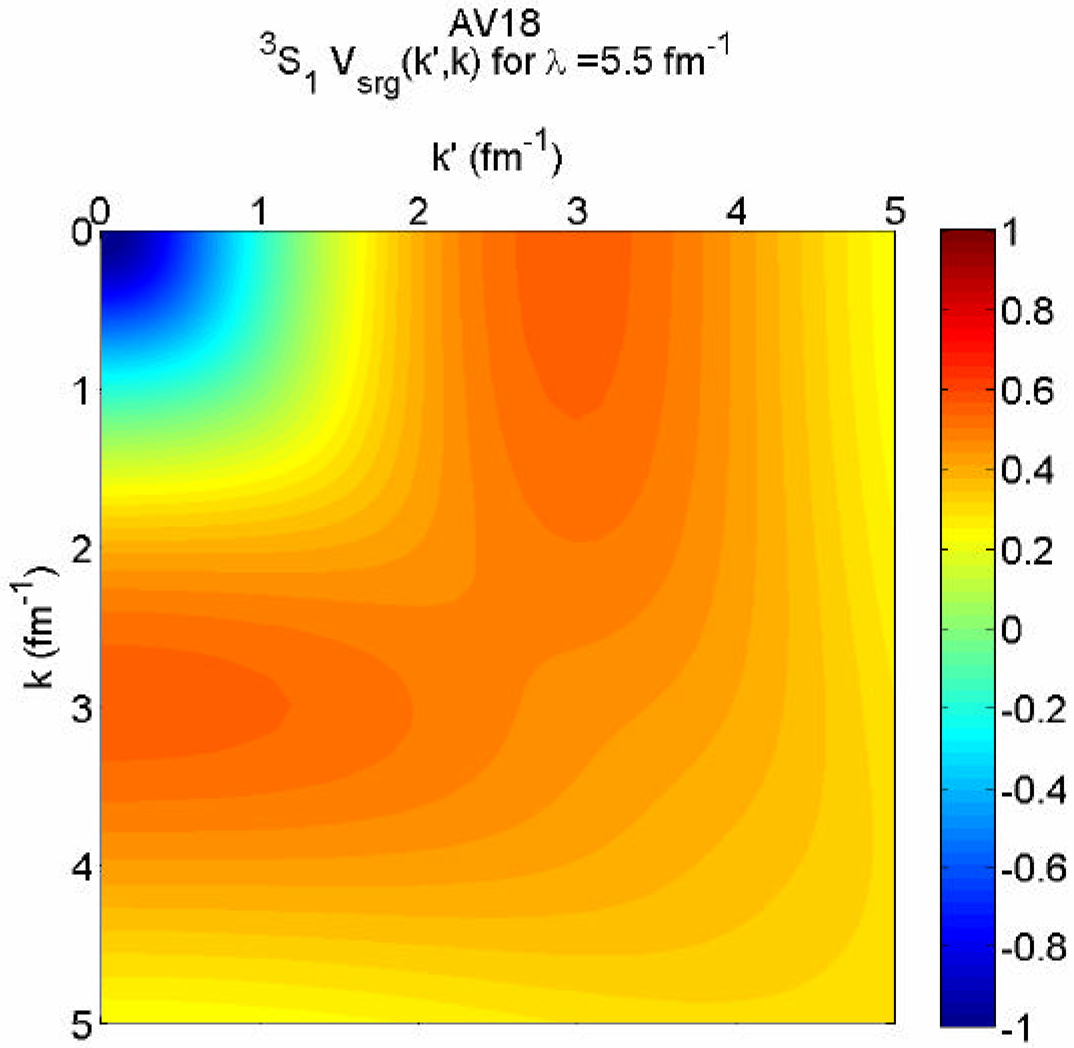

AV18
3S1 Vsrg(k',k) for λ =9.0 fm-1
k' (fm-1)
k (fm-1)
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

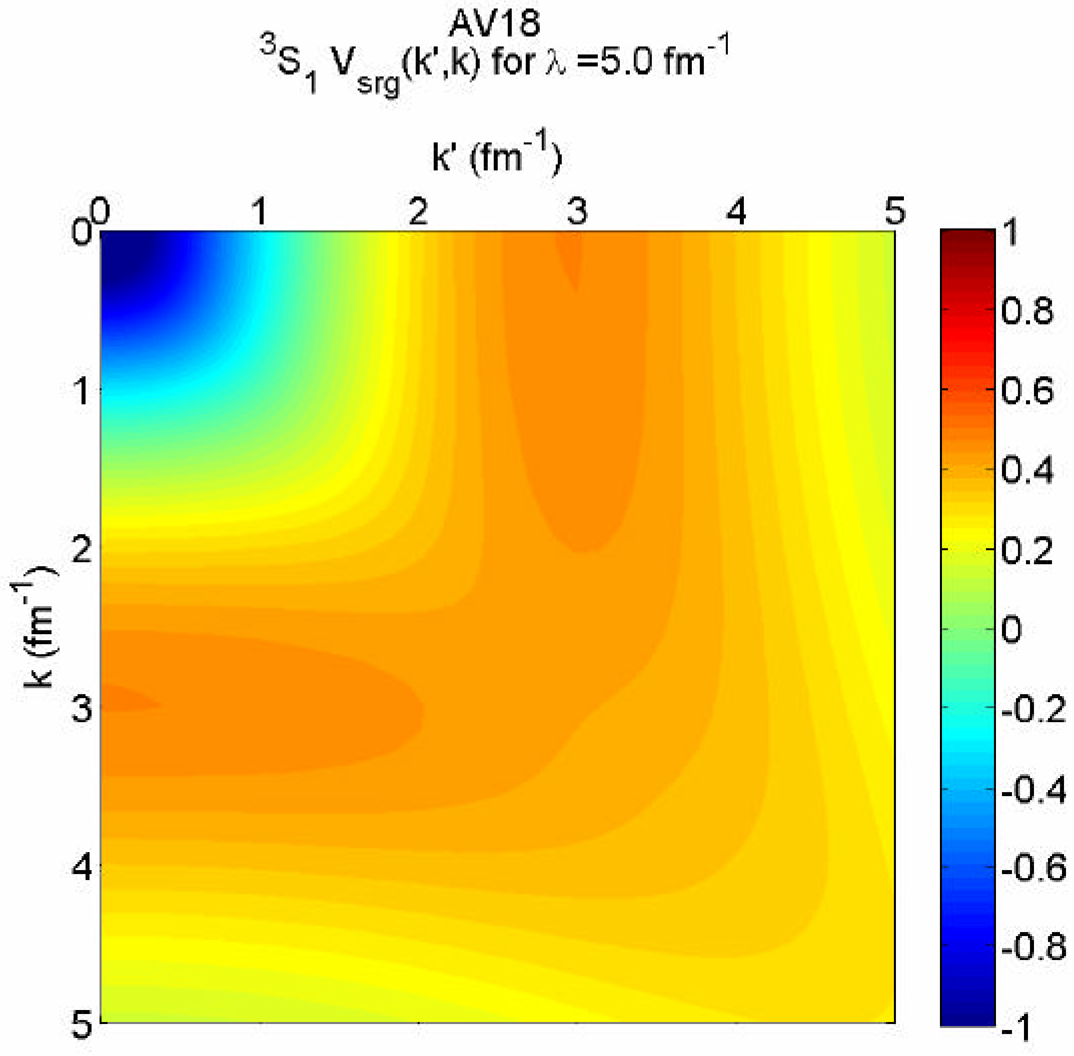

AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =8.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
3S1 Vsrg(k',k) for λ =7.0 fm-1
k' (fm-1)
k (fm-1)
0 1 2 3 4 5
0 1 2 3 4 5
1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1



AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =6.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =5.5 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =5.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
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5
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0.8
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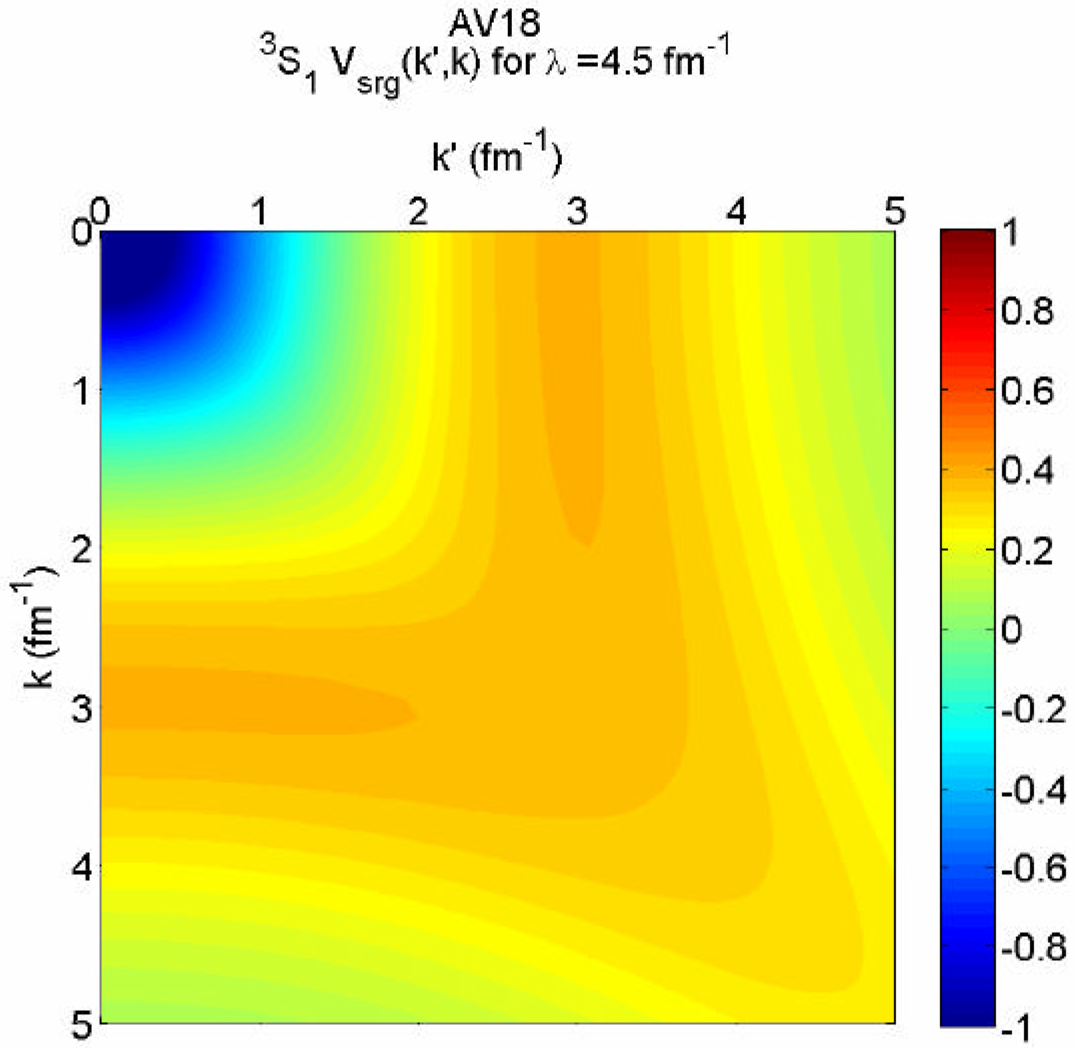
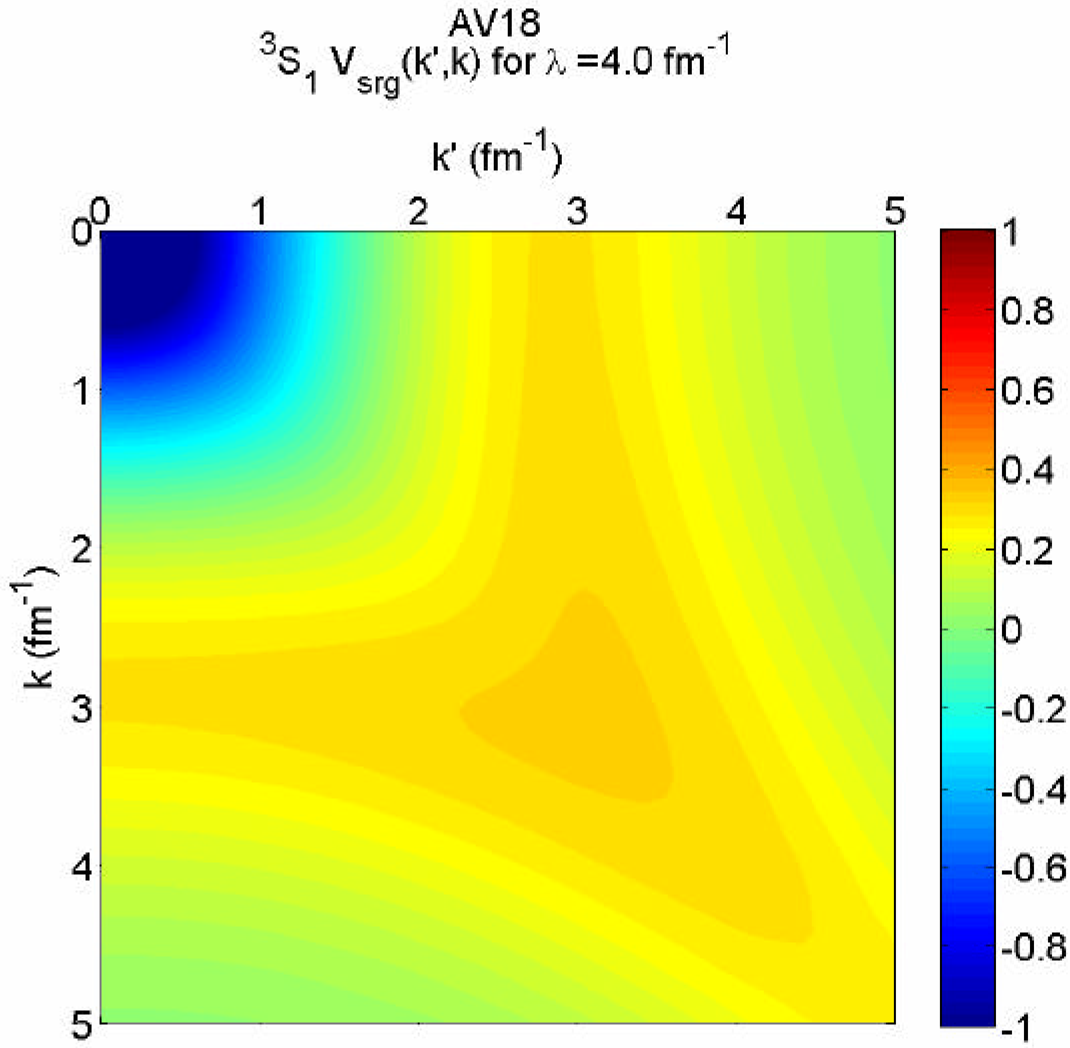
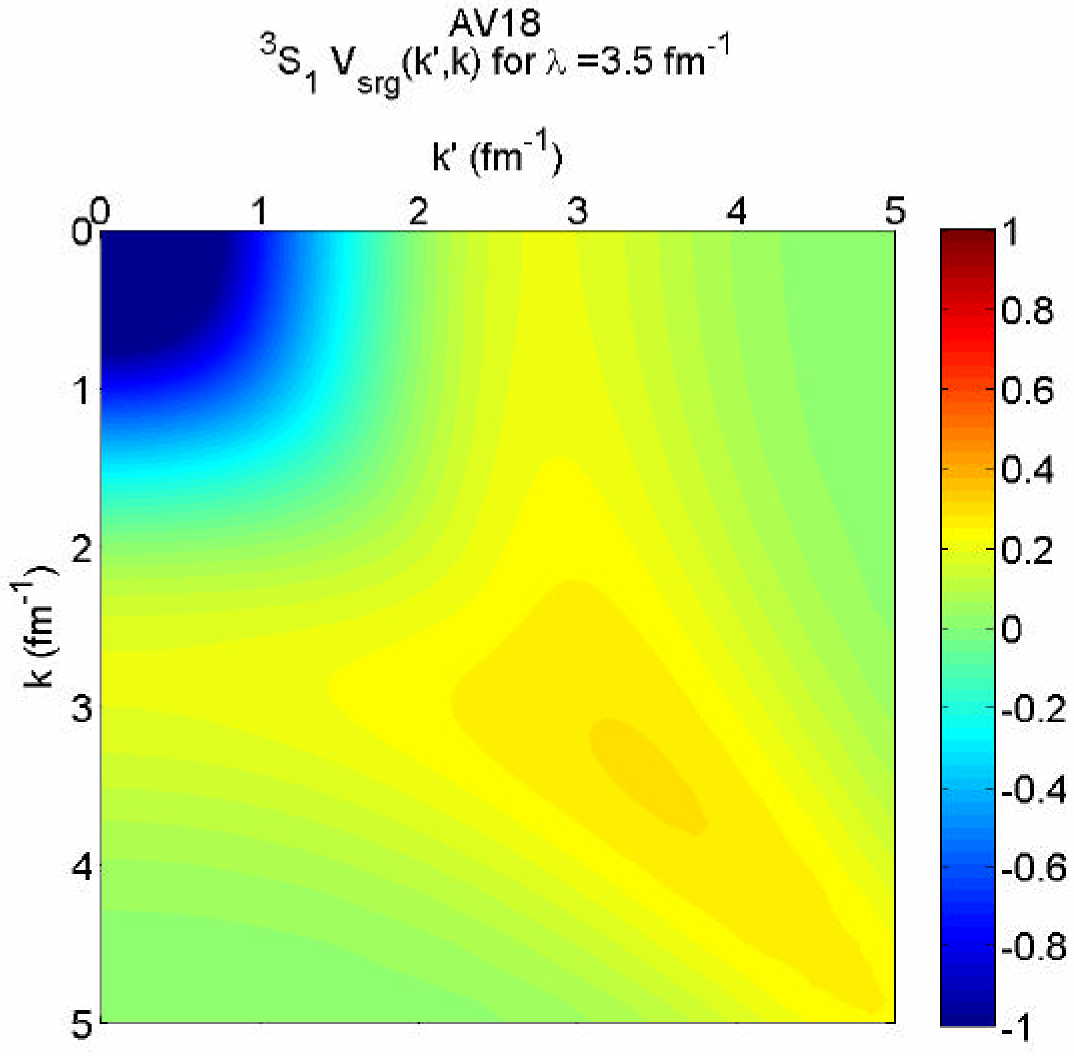
-0.2
-0.4
-0.6
-0.8
-1

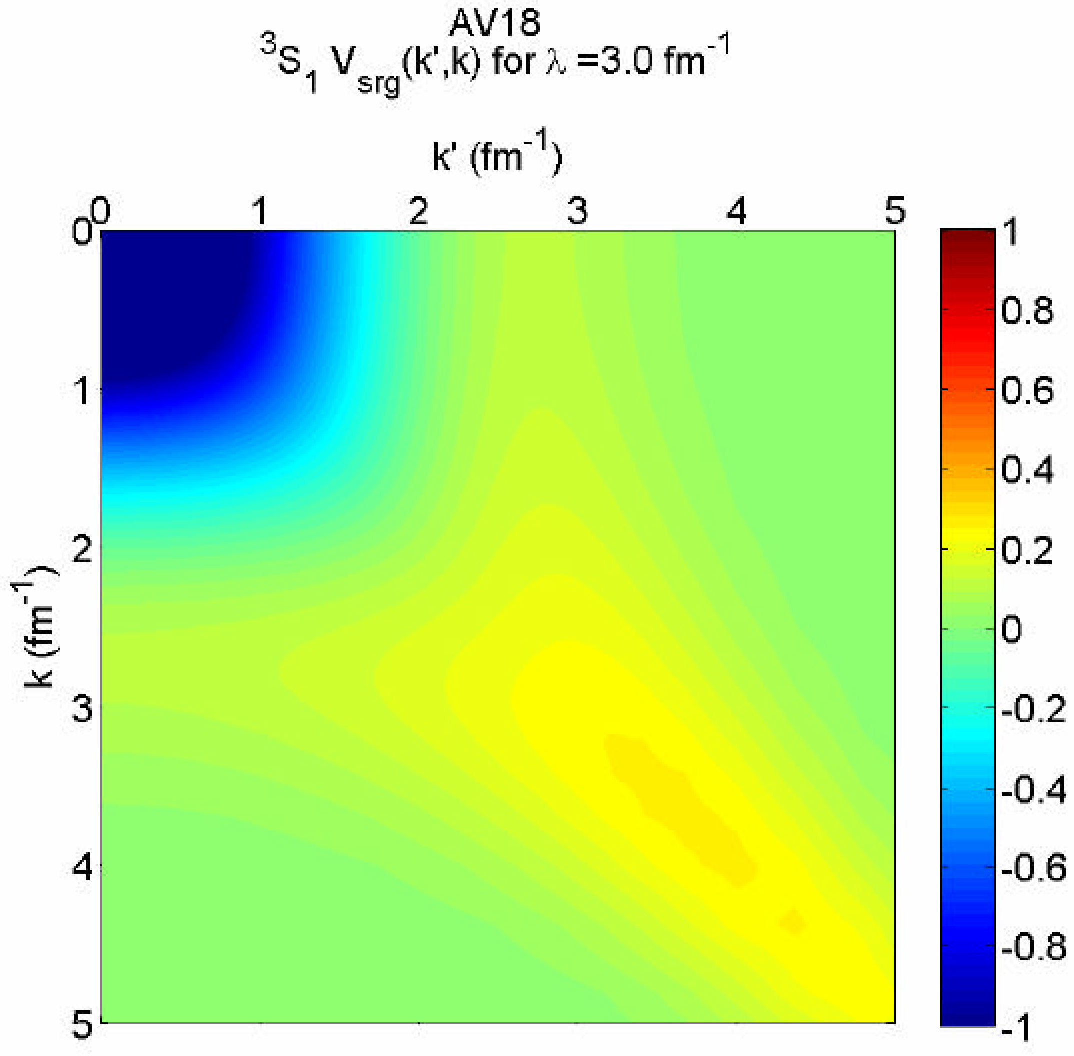


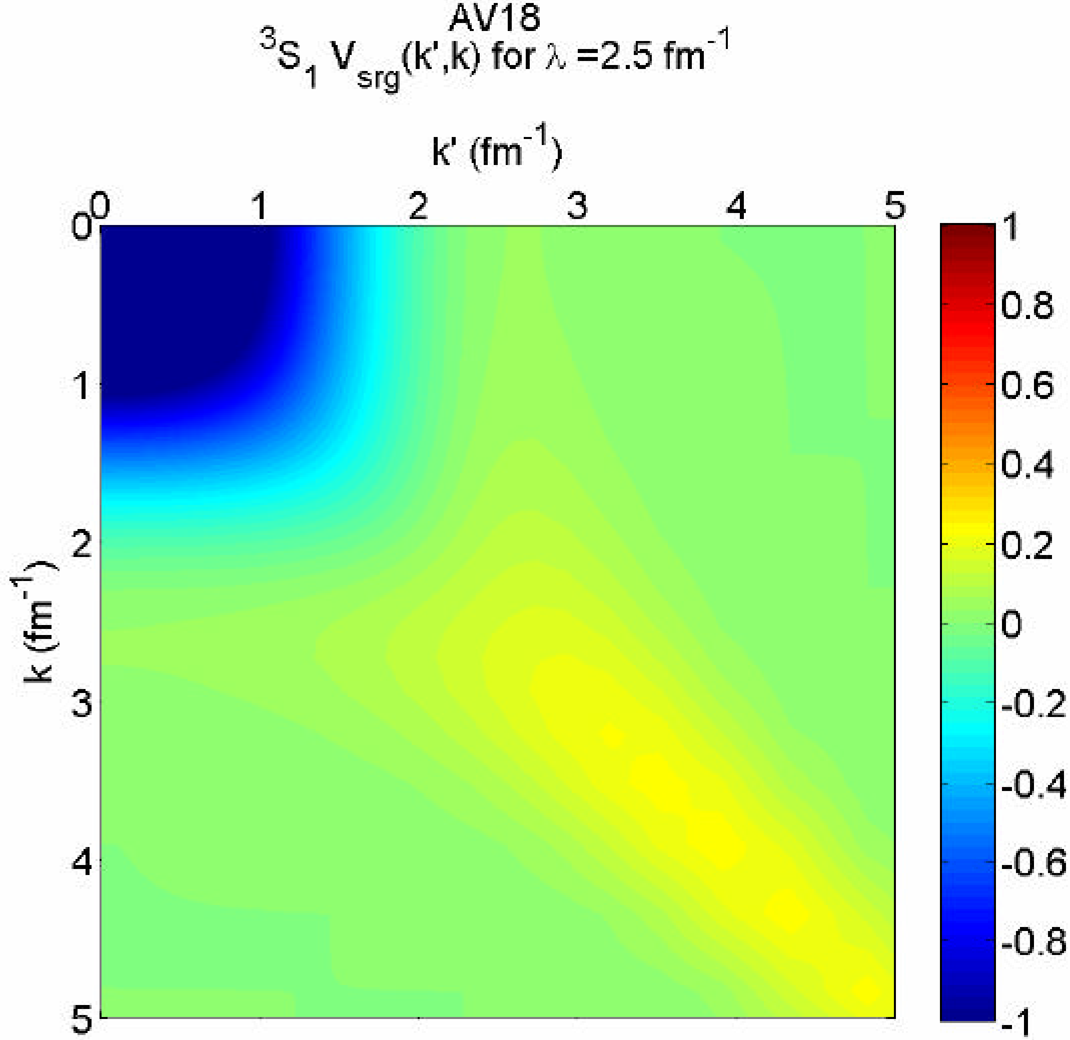
AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =4.5 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



AV18
3S1 Vsrg(k',k) for λ =4.0 fm-1
k' (fm-1)
0
1
2
3
4
5
k (fm-1)
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =3.5 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =3.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =2.5 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
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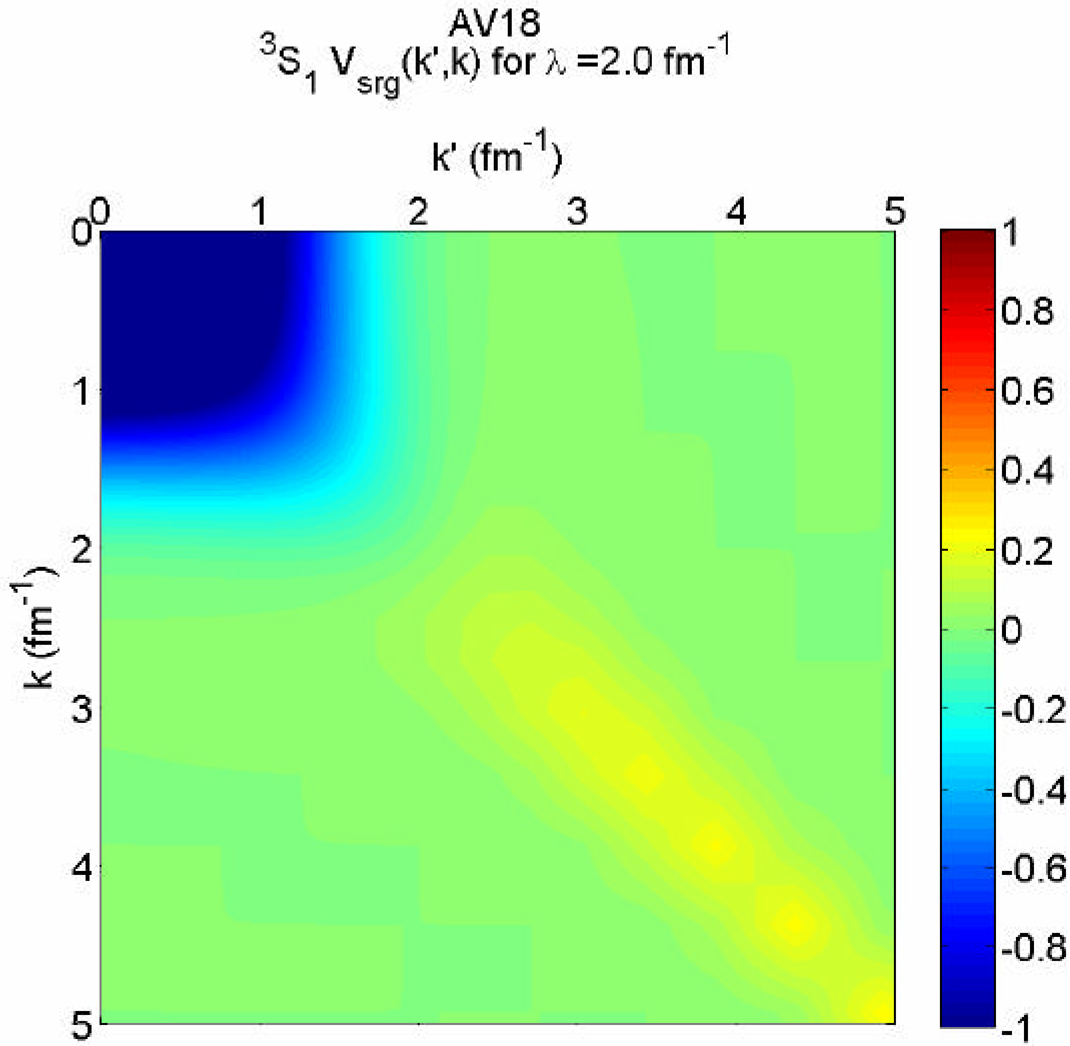
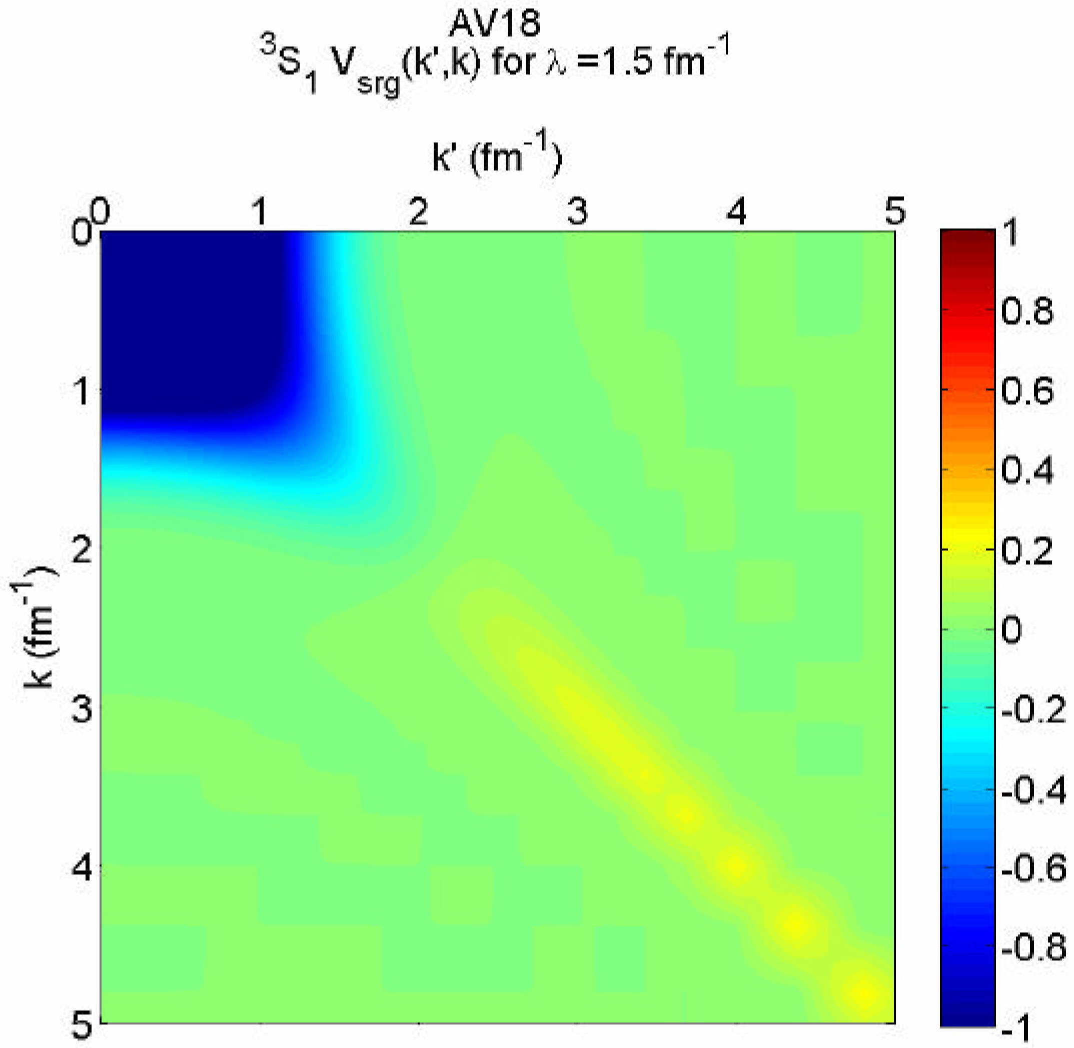
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



AV18
$^{3}S_{1}$ $V_{srg}(k',k)$ for λ =2.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


AV18
$^3S_1$ $V_{srg}(k',k)$ for λ =1.5 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

General formulation

- Transform an initial hamiltonian: $H_0 = T_{rel} + V$

$$H_S = U(s)\, H\, U^\dagger(s) = T_{rel} + V_s$$

where s is the flow parameter. Then, differentiating wrt to s [fm^4]:

$$\frac{dH_s}{ds} = [\eta(s), H_s]$$ with $$\eta(s) \equiv \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

- Choosing $\eta(s)$ specifies the transformation . . . for example, using the relative kinetic energy :

$$\eta(s) = [T_{rel}, H_s]$$

gives the flow equation,

$$\frac{\mathrm{d}H_s}{\mathrm{d}s} = [[T_{rel}, H_s], H_s]$$

- For an NN potential, project onto partial-wave momentum basis $|k>$ using

$$1 = \frac{2}{\pi}\int_0^{\infty} |q> q^2\, \mathrm{d}q < q|$$

with $\frac{\hbar^2}{M} = 1$

$$\frac{\mathrm{d}H_s}{\mathrm{d}s} = [[T_{rel}, H_s], H_s] \Rightarrow [[T_{rel}, V_s], H_s]$$

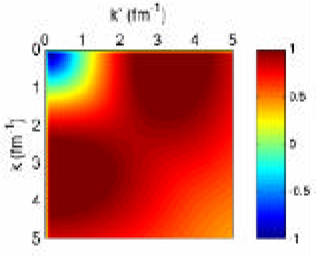
$$\Rightarrow \frac{\mathrm{d}V_s(k,k')}{\mathrm{d}s} = -\left(k^2 - k'^2\right)^2 V_s(k,k')$$

$$+ \frac{2}{\pi}\int_0^{\infty} q^2\, \mathrm{d}q \left(k^2 + k'^2 - 2q^2\right) V_s(k,q) V_s(q,k')$$

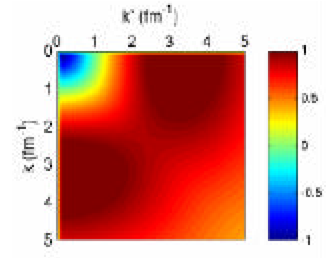
- The evolution of s of any operator O is given by the same unitary transformation, $O_S = U(s)\, O\, U^{\dagger}(s)$ which means that O evolves according to

$$\frac{dO_s}{ds} = \left[\eta(s), O(s)\right] = \left[\left[T_{rel}, V_s\right], O_s\right] \Rightarrow$$

$$\frac{dO_s(k,k')}{ds} = \frac{2}{\pi}\int_0^{\infty} q^2\, dq \left[\left(k^2 - q^2\right) V_s(k,q) O_s(q,k') + \left(k'^2 - q^2\right) O_s(k,q) V_s(q,k') \right]$$



Potential Evolution

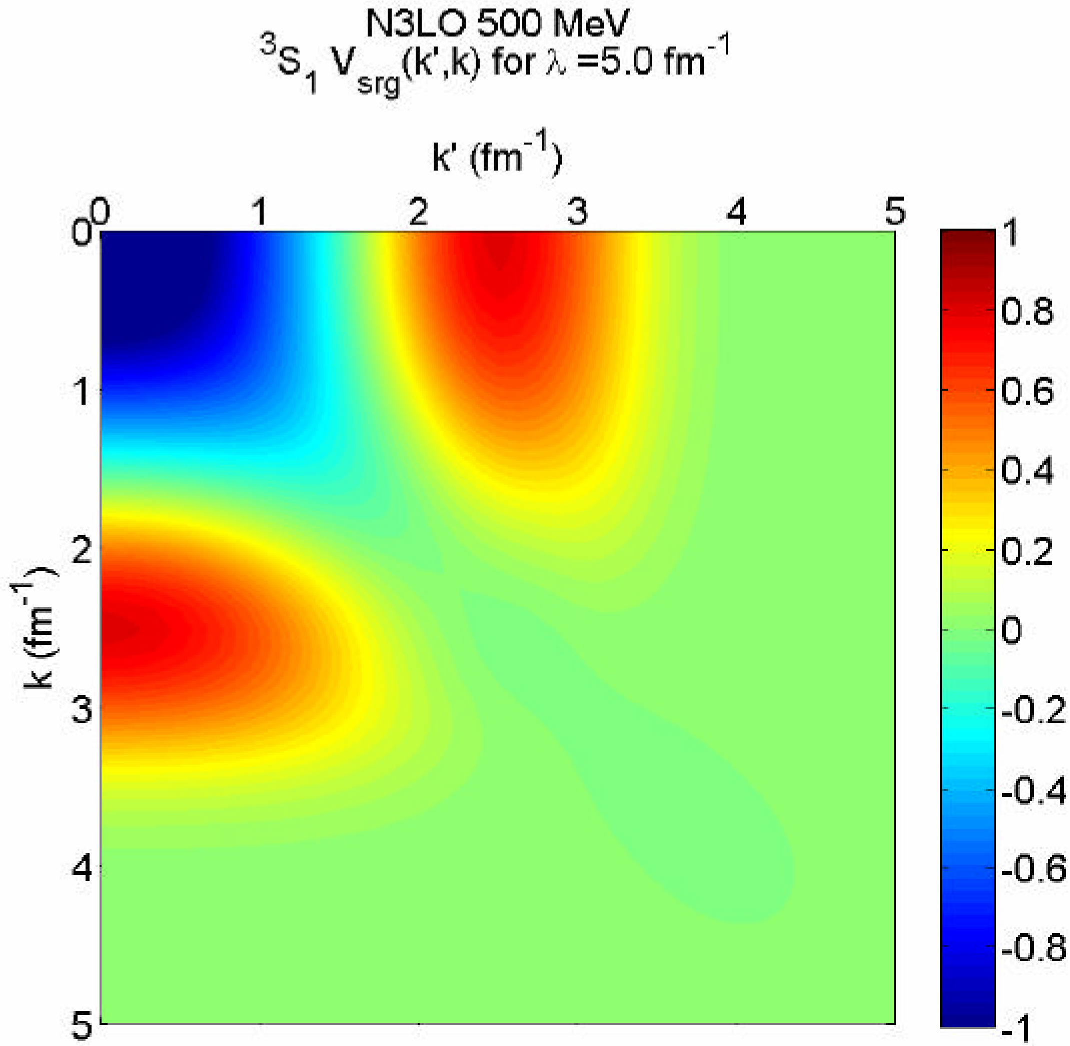


$$\frac{\mathrm{d}H_s}{\mathrm{d}s} = \left[\left[T_{rel}, H_s\right], H_s\right] = \left[\left[T_{rel}, V_s\right], H_s\right]$$

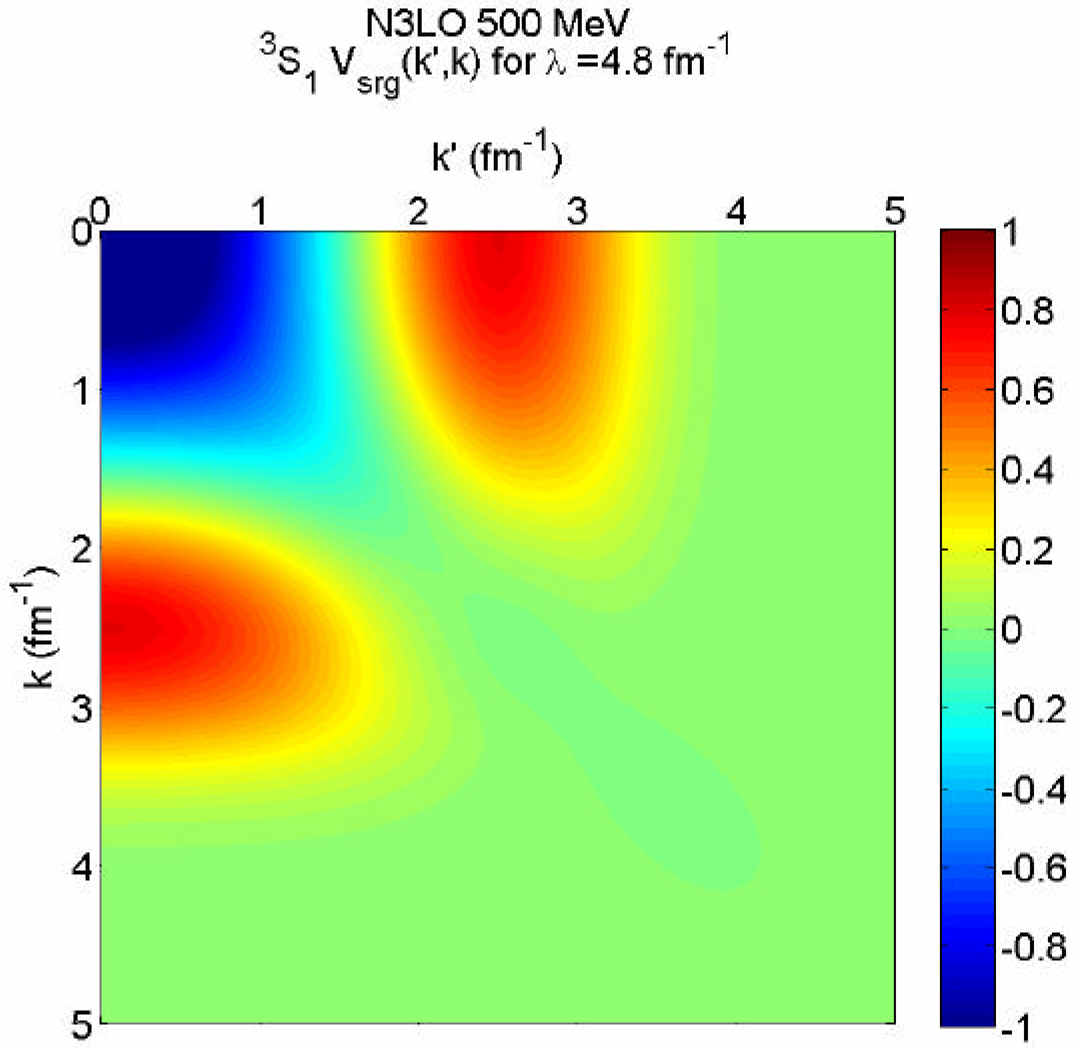
$$\frac{\mathrm{d}V_s(k,k')}{\mathrm{d}s} = -\left(k^2 - k'^2\right)^2 V_s(k,k')$$

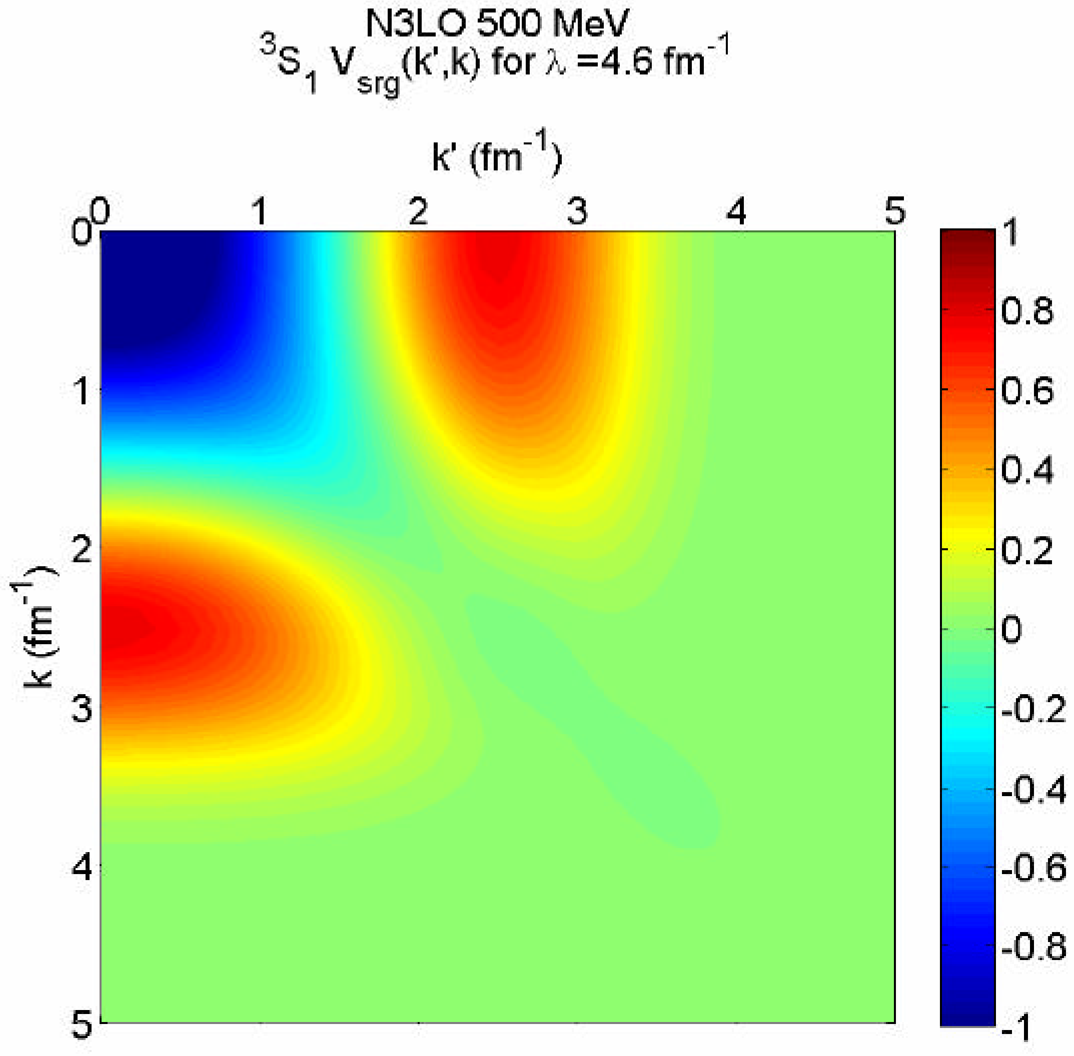
$$+ \frac{2}{\pi}\int_0^\infty q^2\, \mathrm{d}q \left(k^2 + k'^2 - 2q^2\right) V_s(k,q) V_s(q,k')$$

$$\boxed{\lambda = \frac{1}{s^{1/4}} \quad \left[fm^{-1}\right]}$$



N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =5.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =4.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0 1 2 3 4 5
0 1 2 3 4 5
1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1



N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =4.6 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
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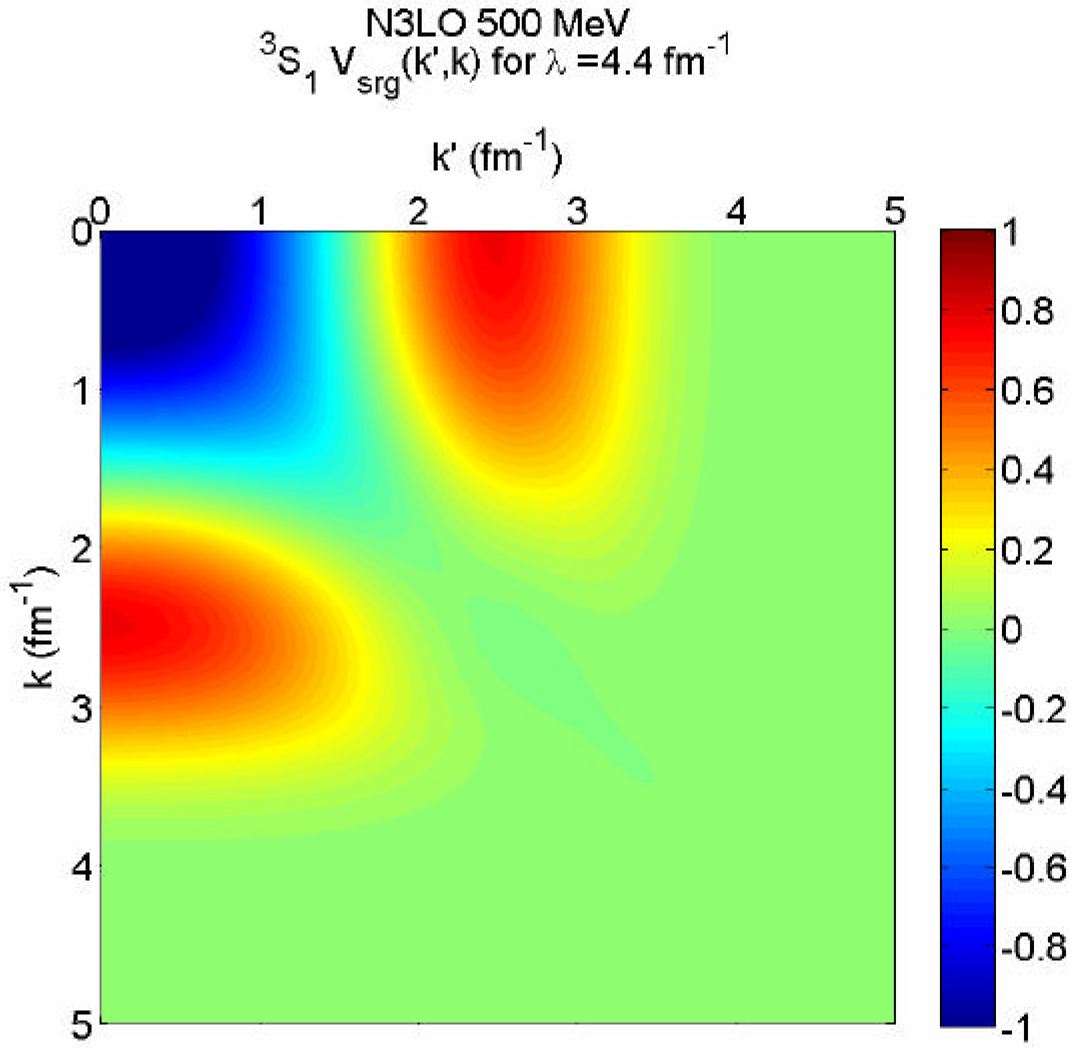
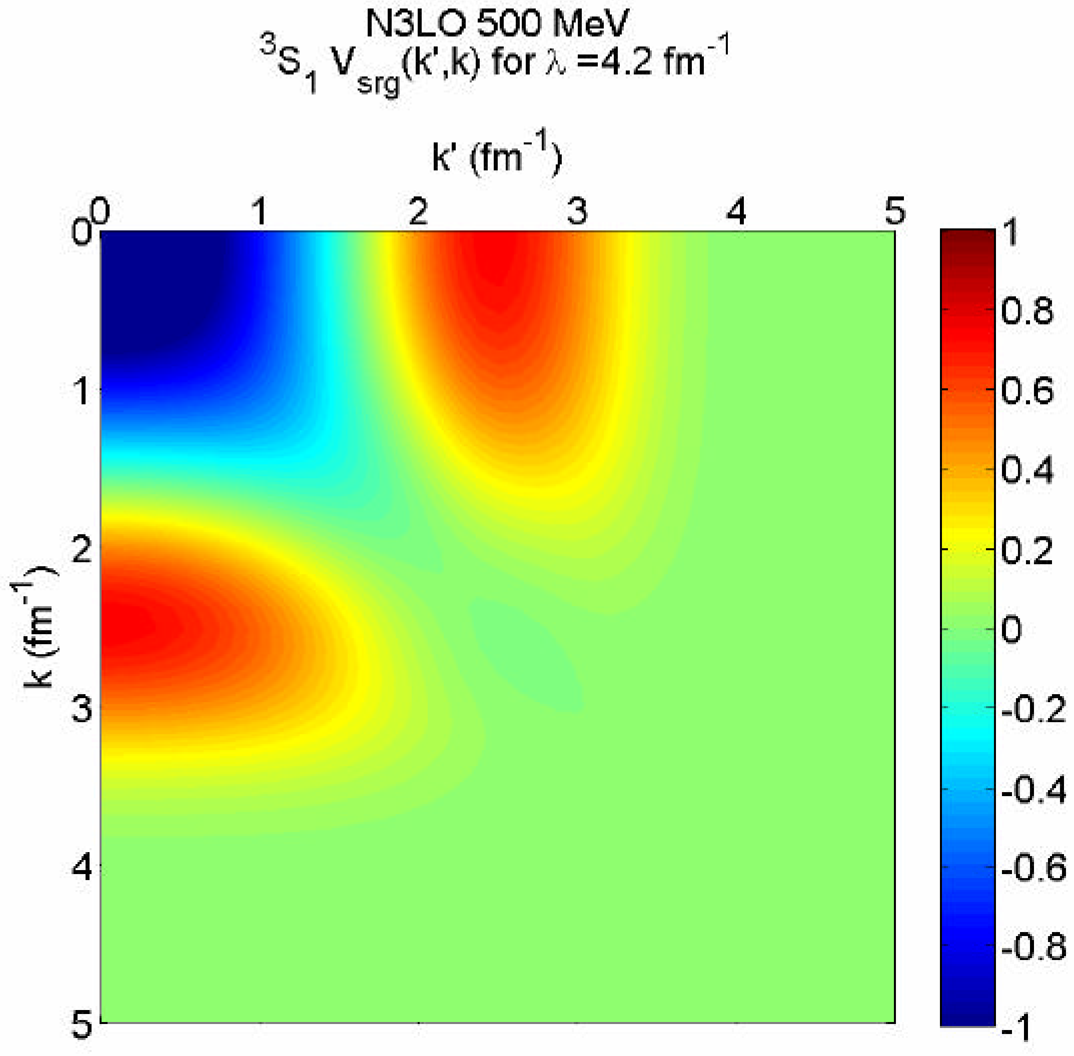
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =4.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =4.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
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0.8
0.6
0.4
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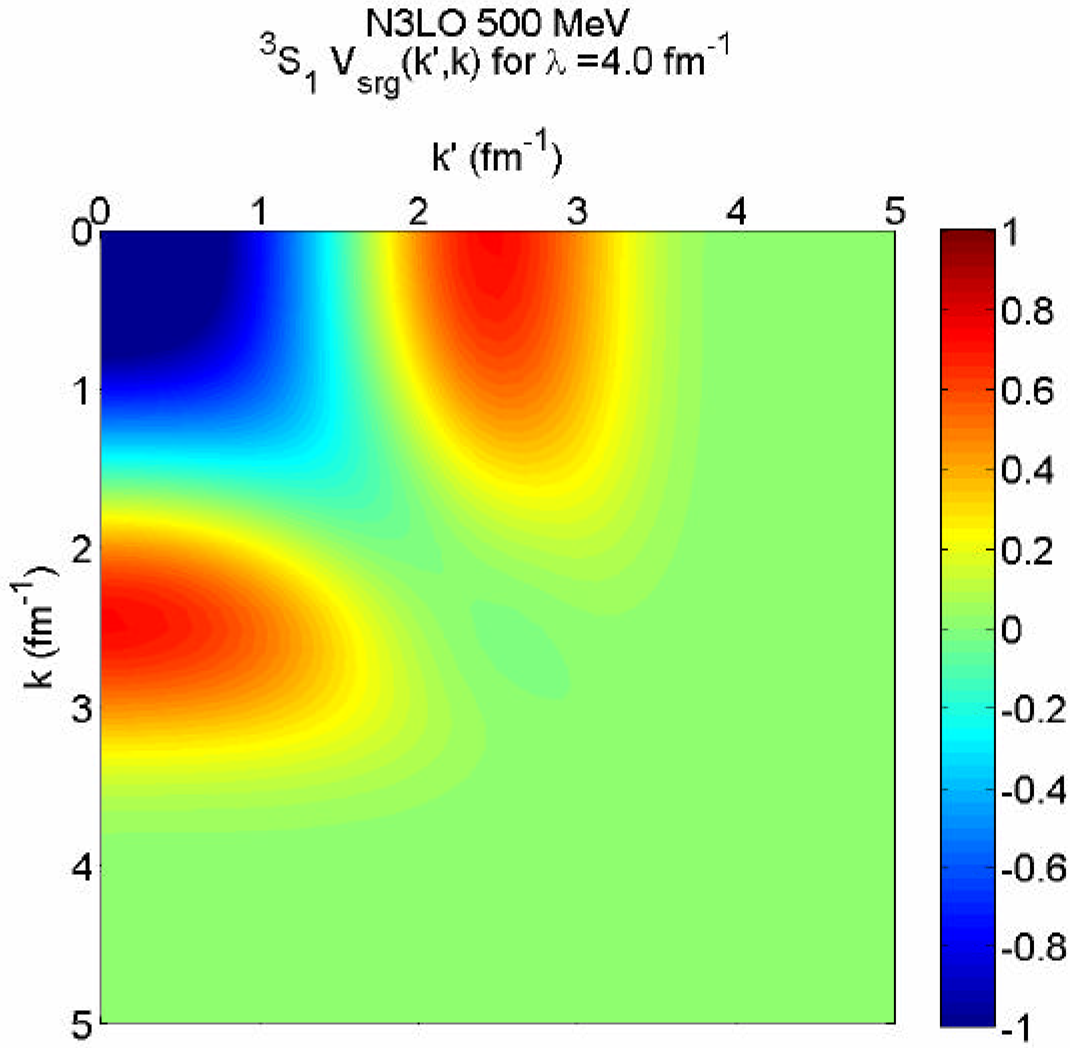
0
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =4.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
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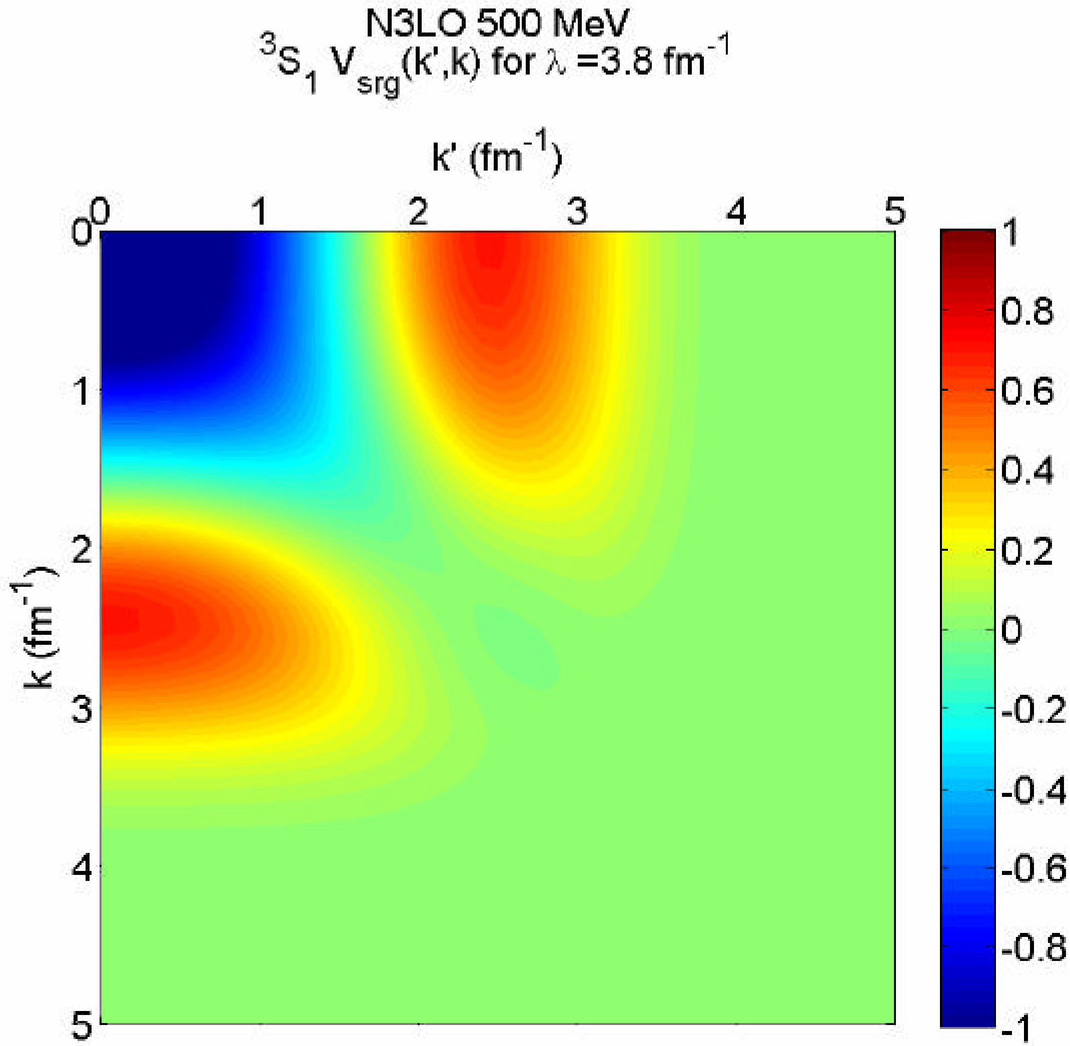
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =3.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
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0.6
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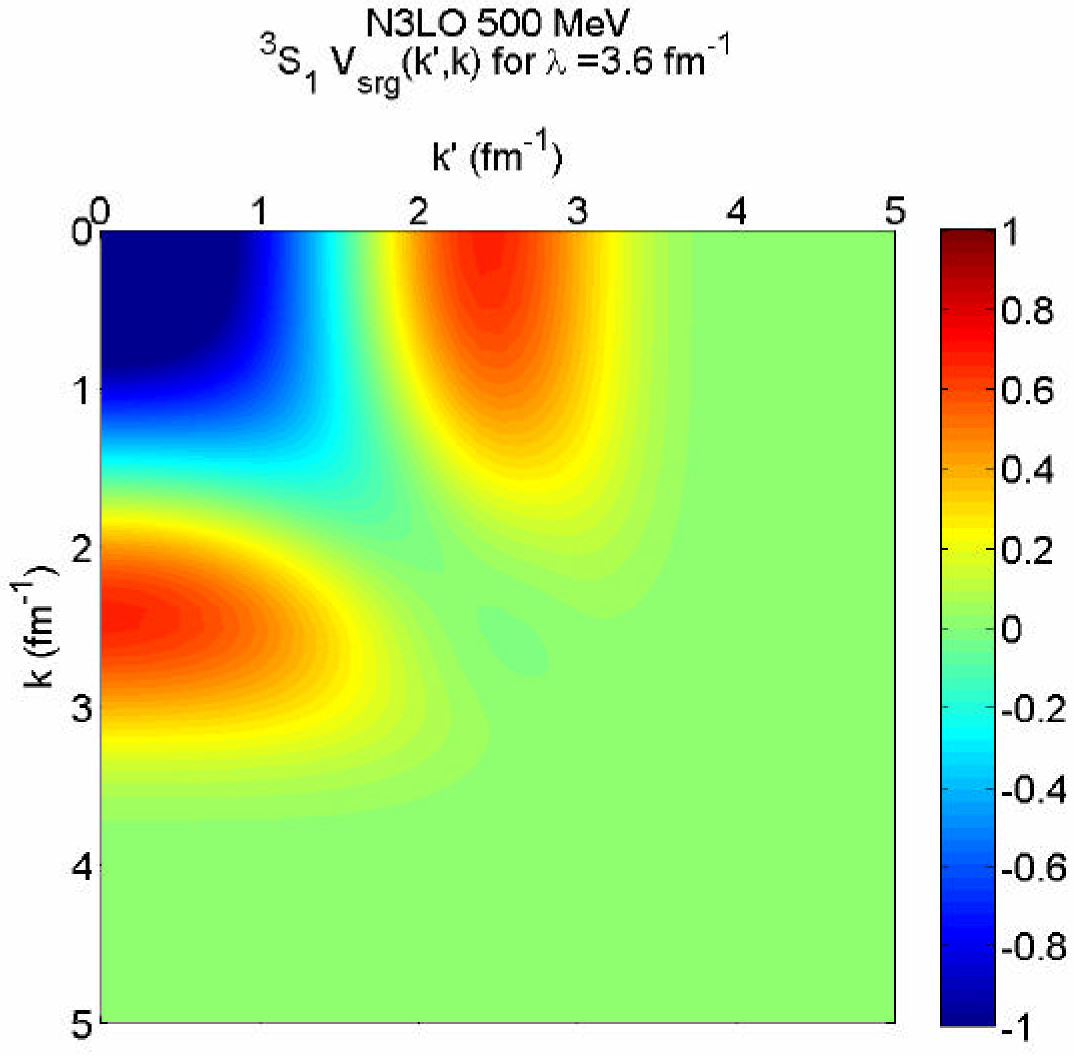
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =3.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
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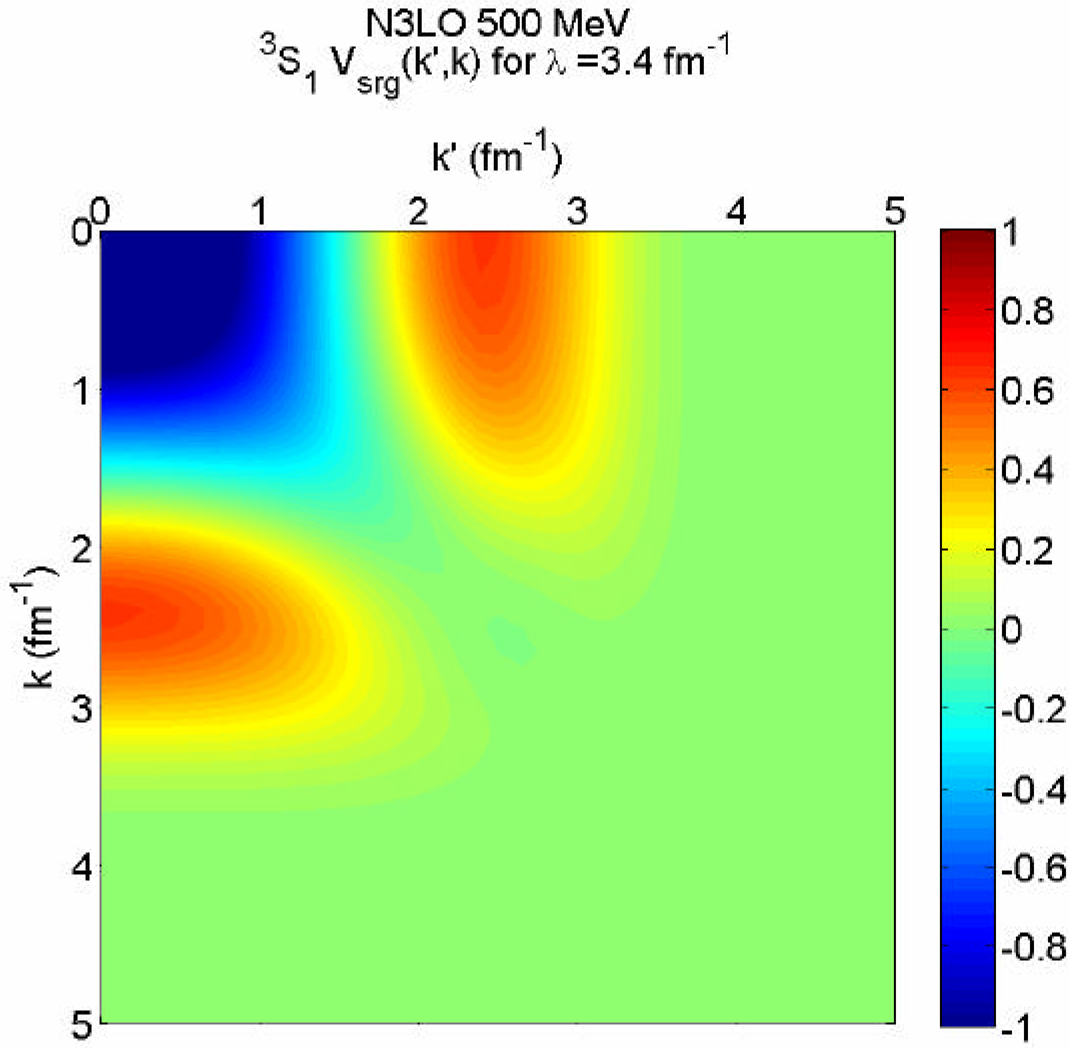
N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =3.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
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0.6
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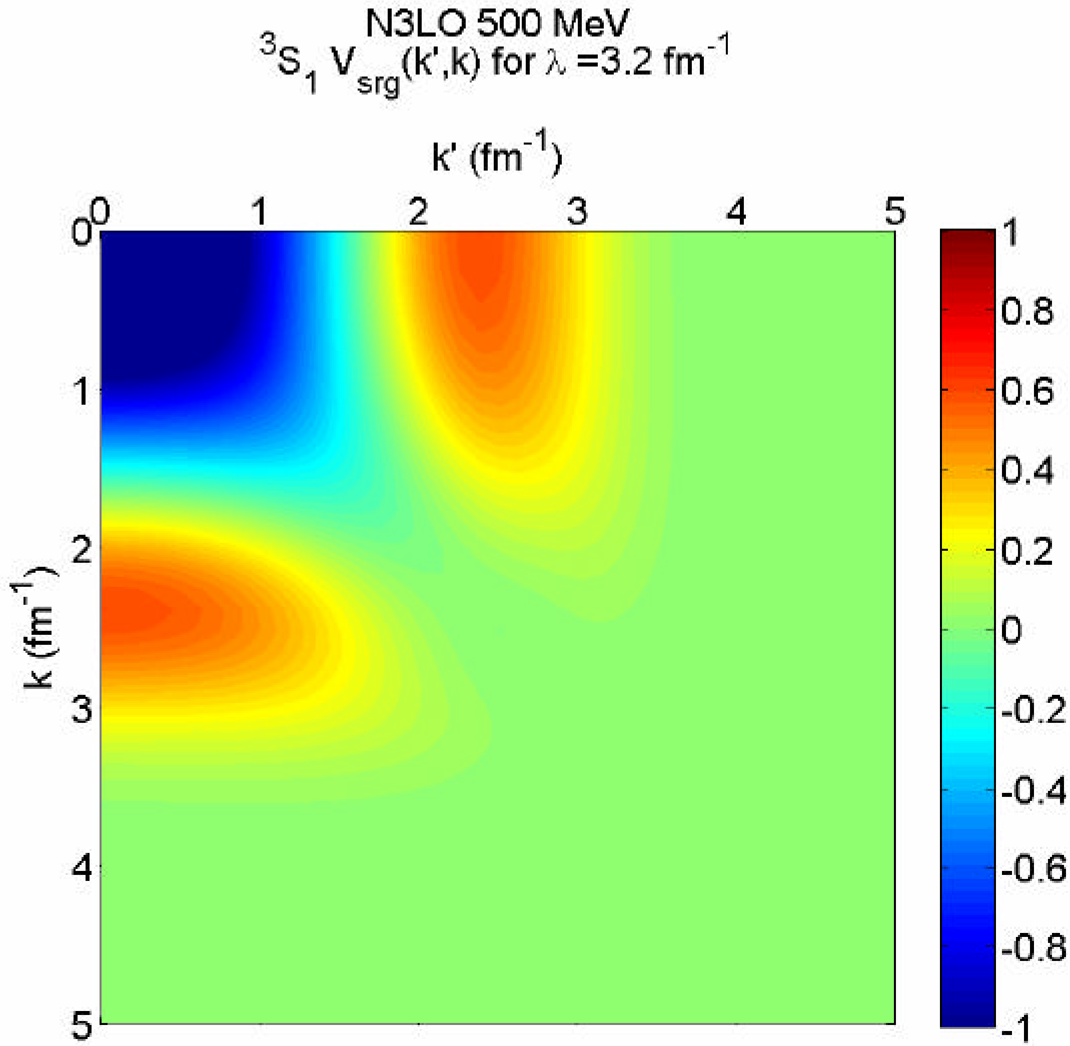
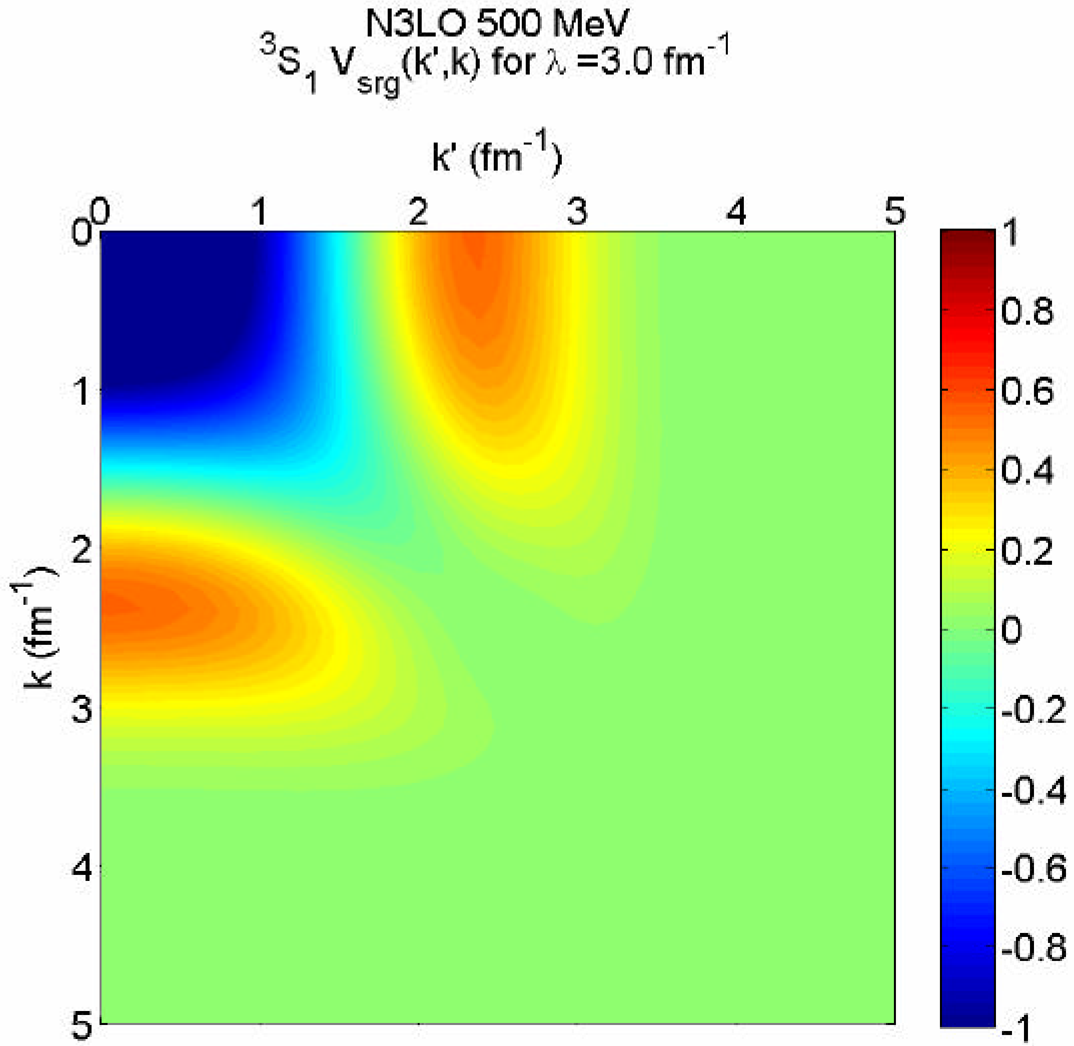
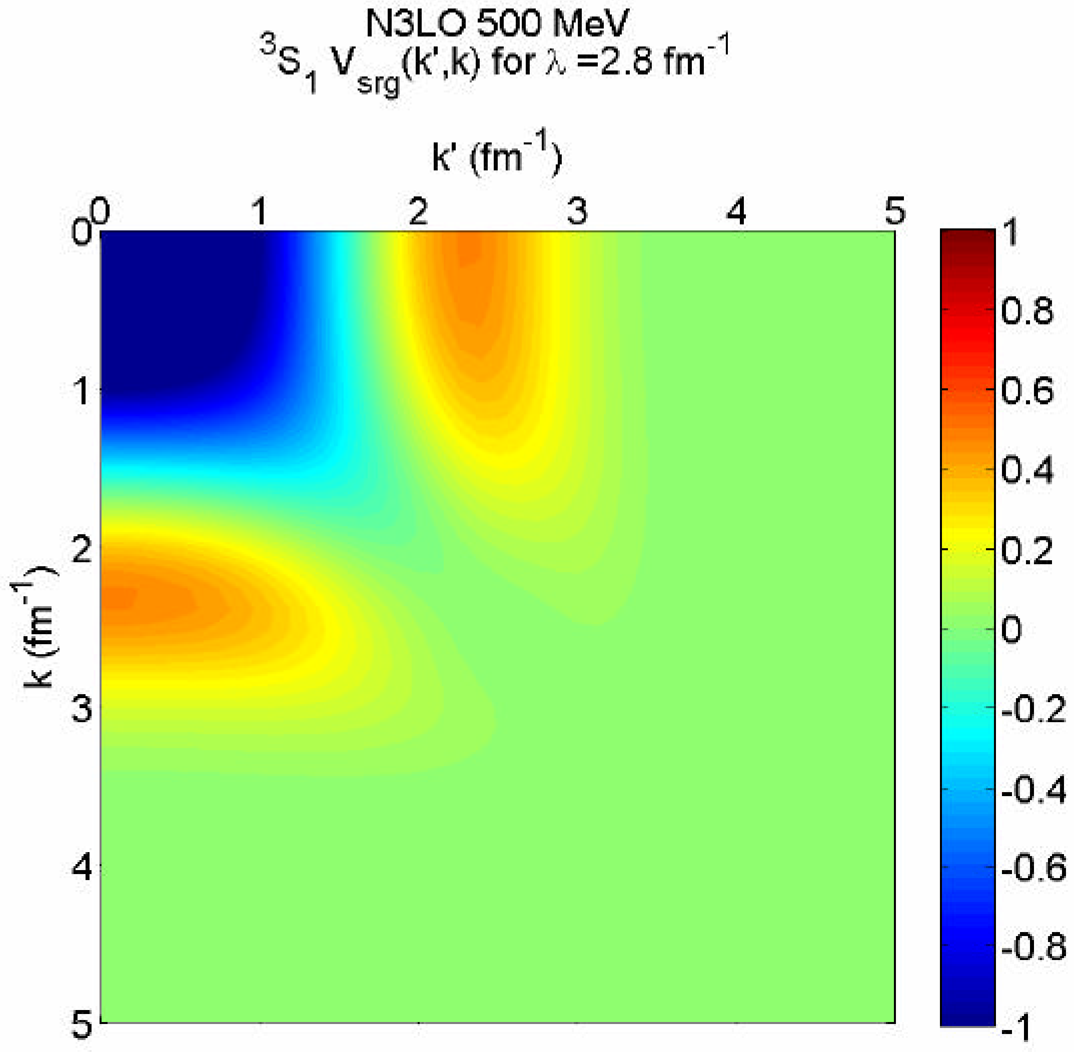
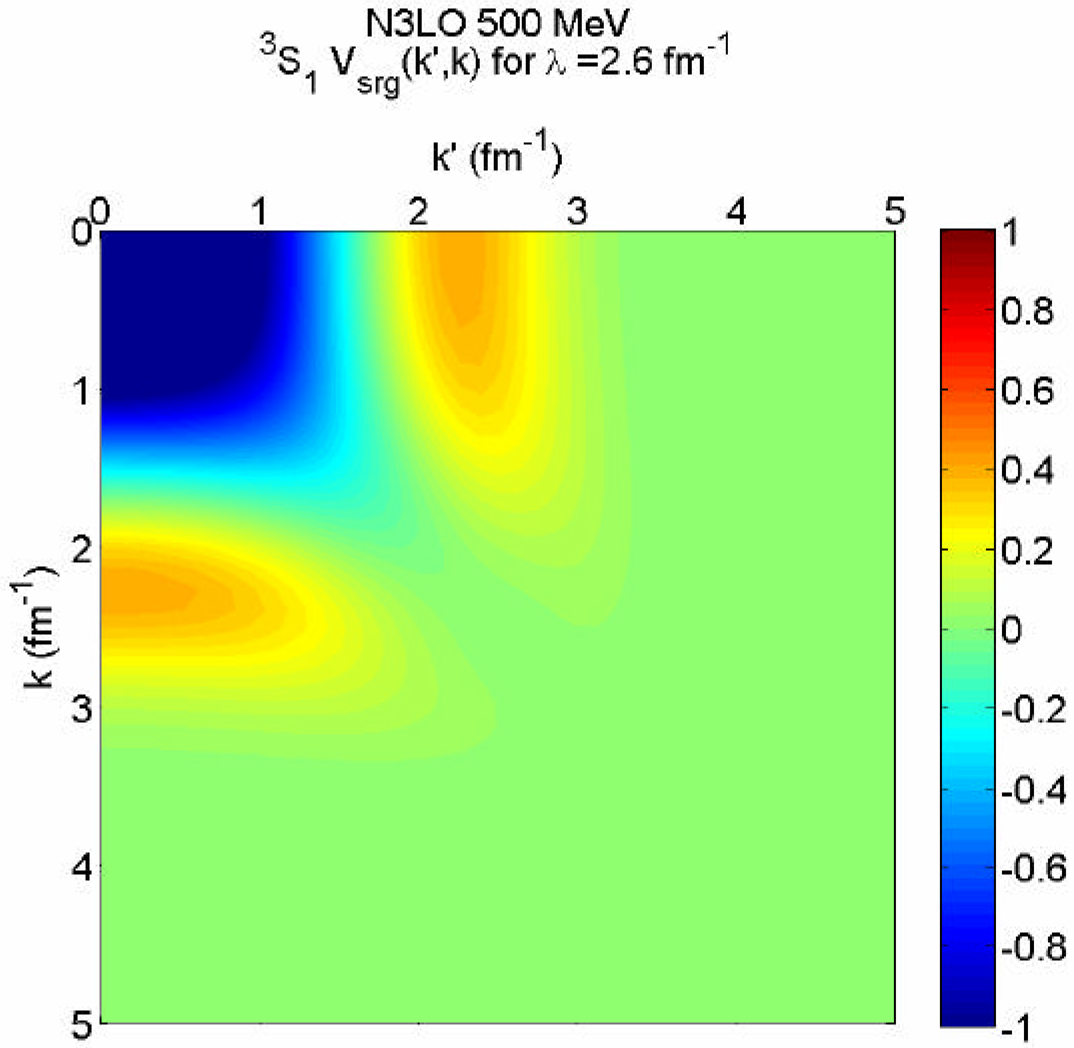
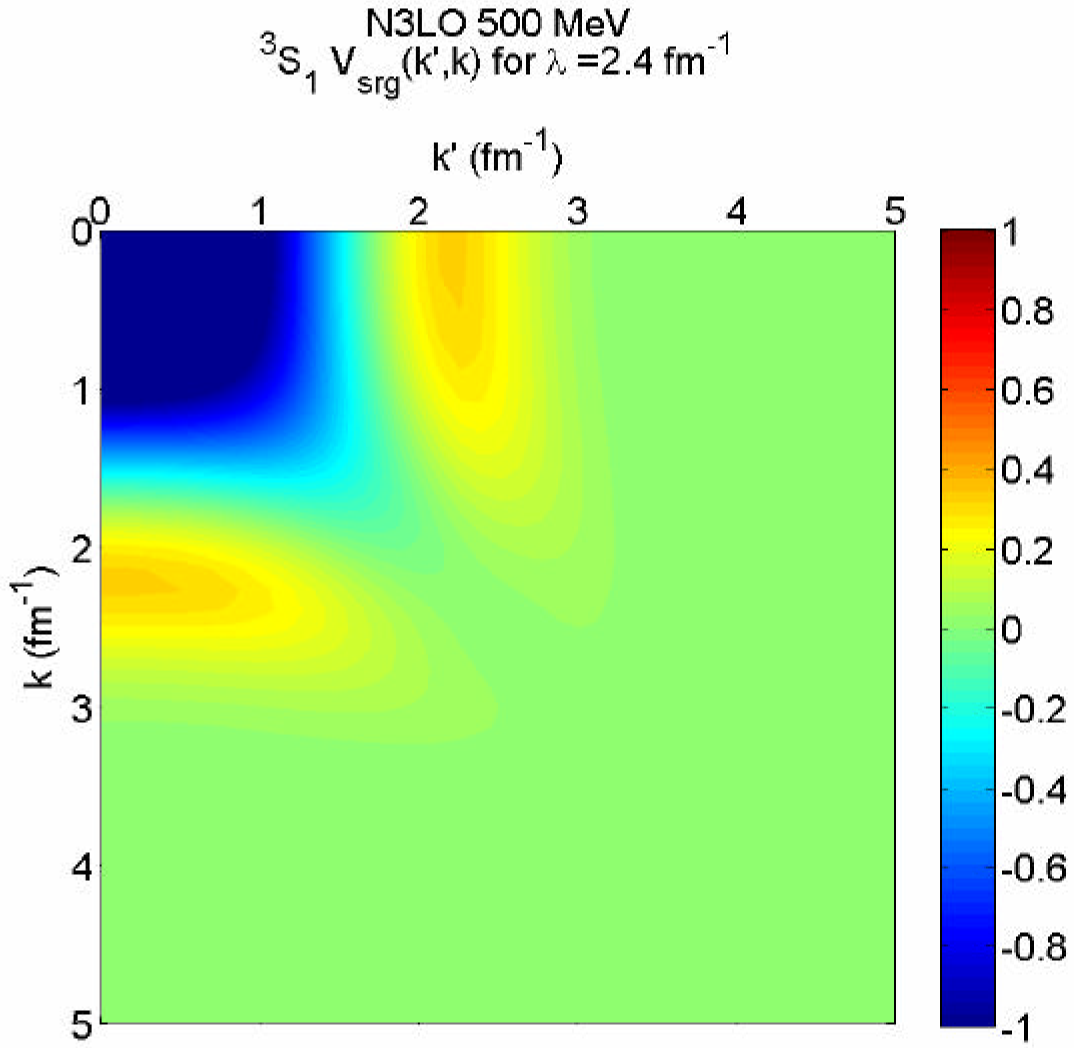

N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =3.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =3.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =2.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

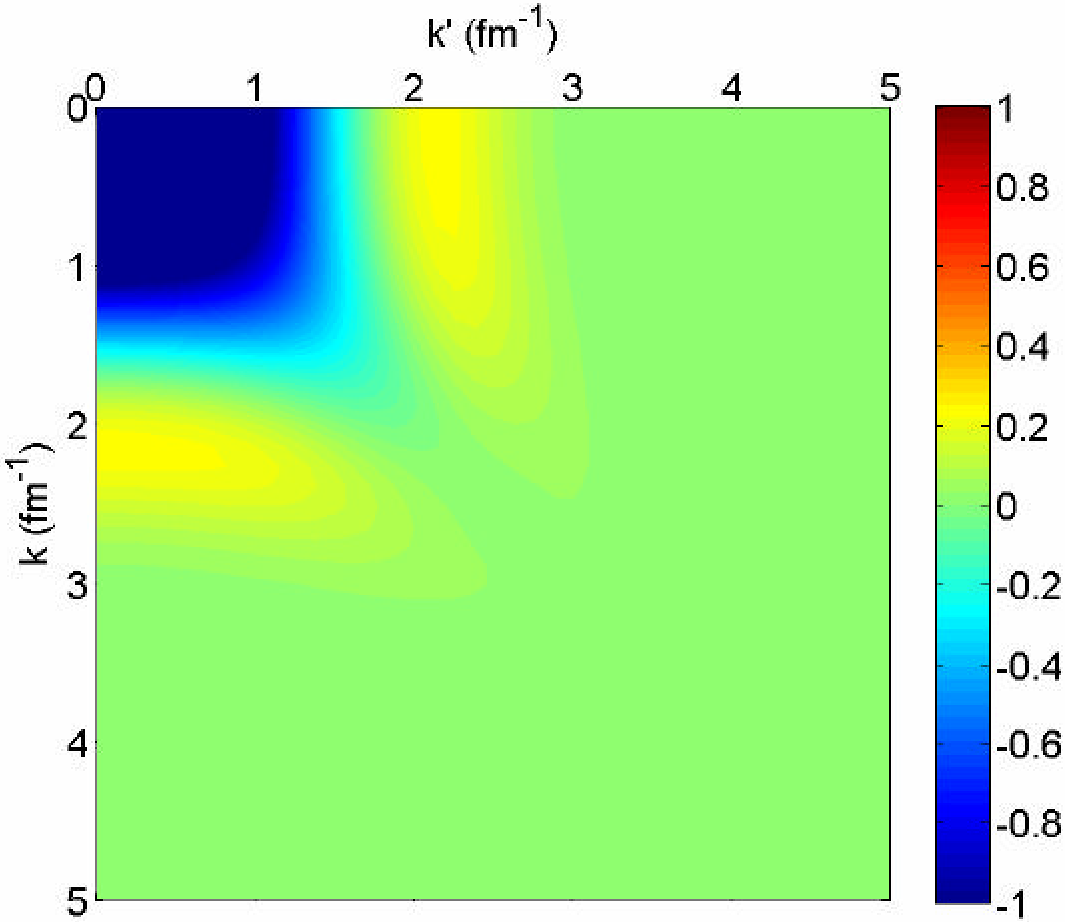

N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =2.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

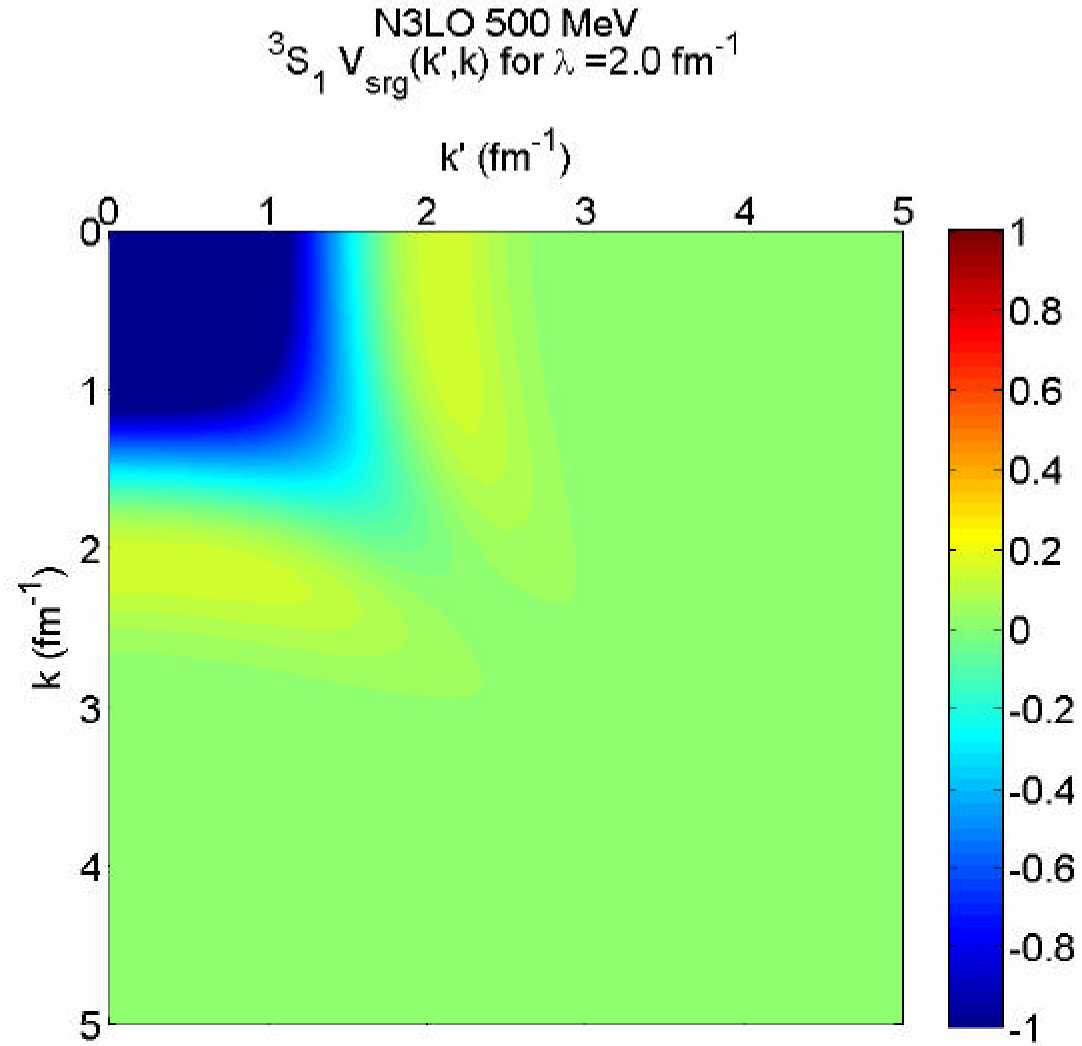


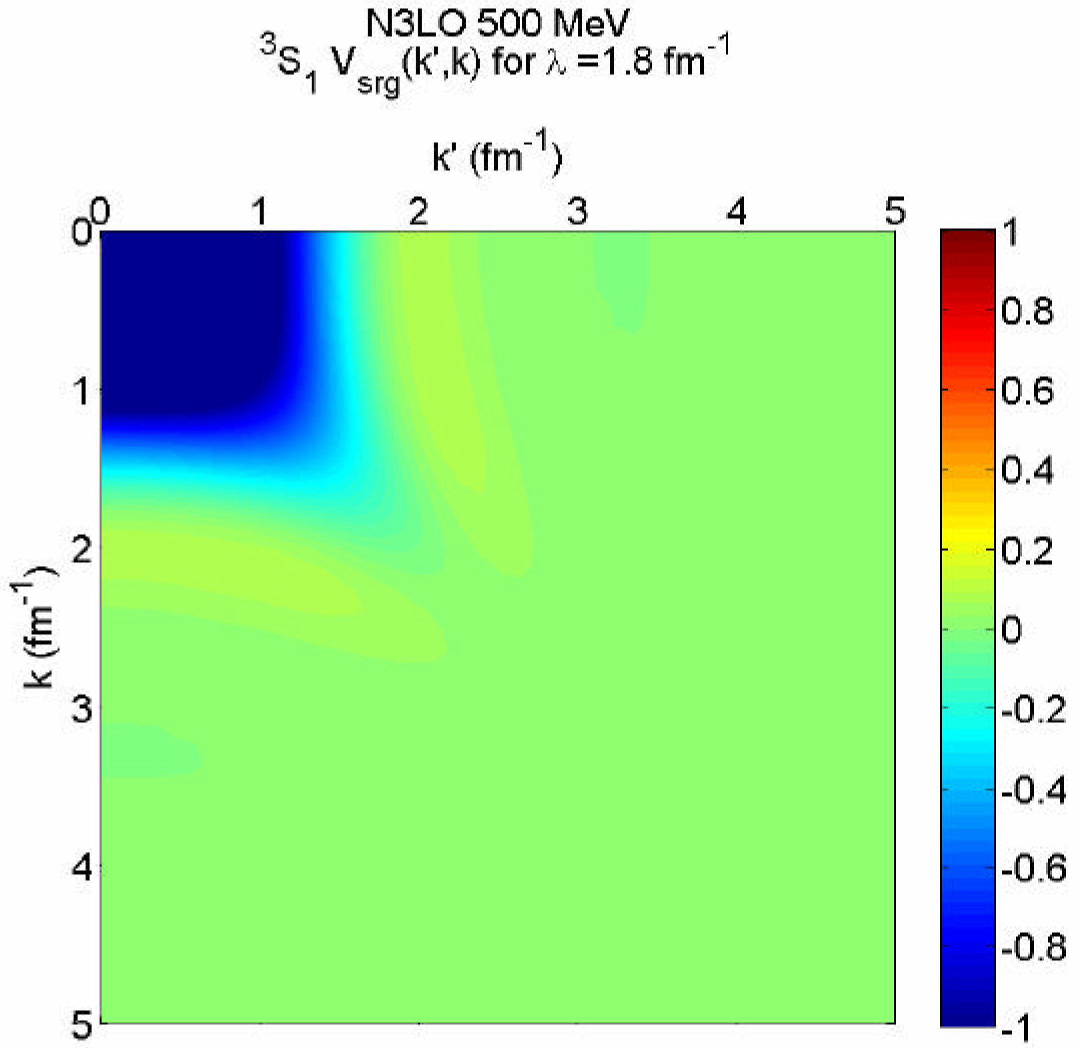
N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =2.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

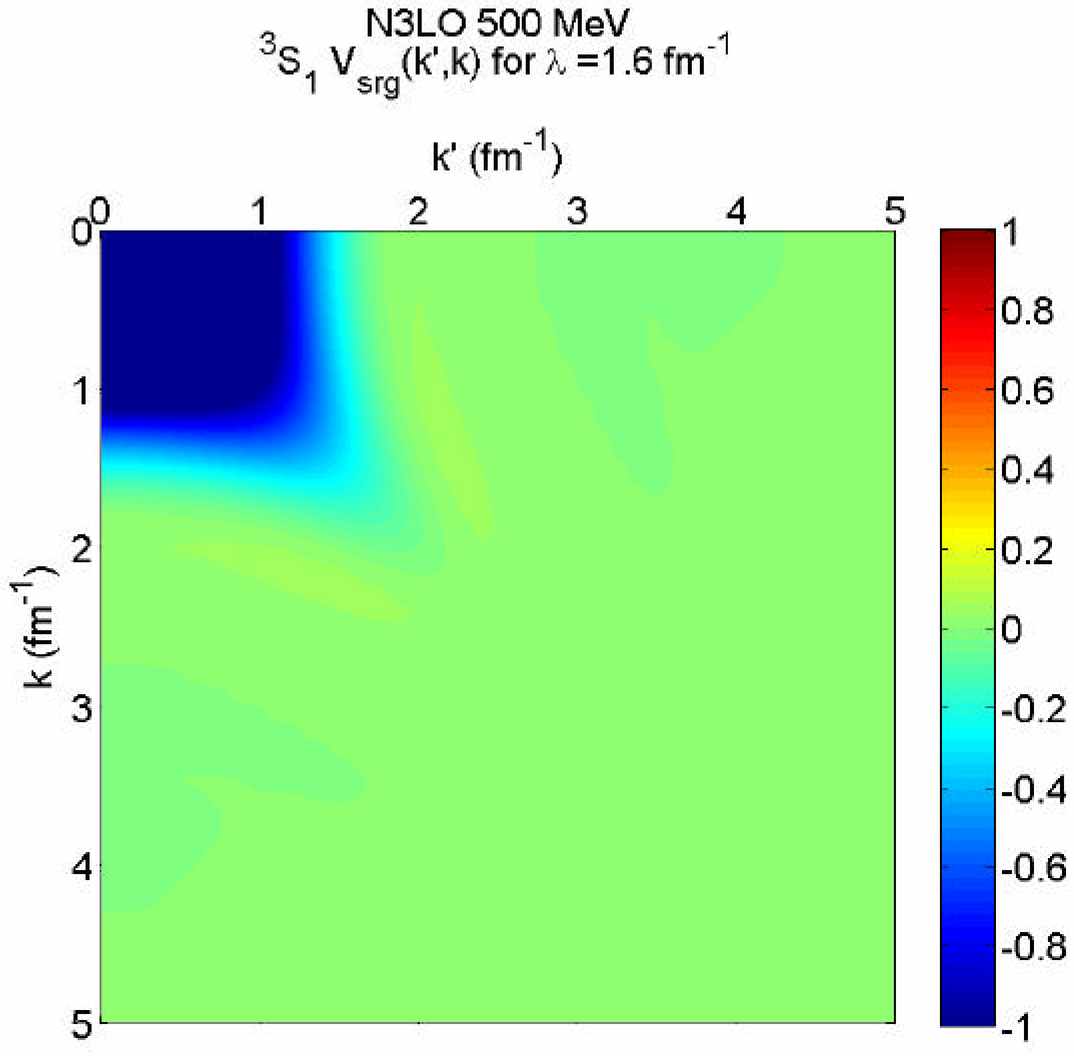
N3LO 500 MeV

$^3S_1$ $V_{srg}(k',k)$ for λ =2.2 fm^{-1}

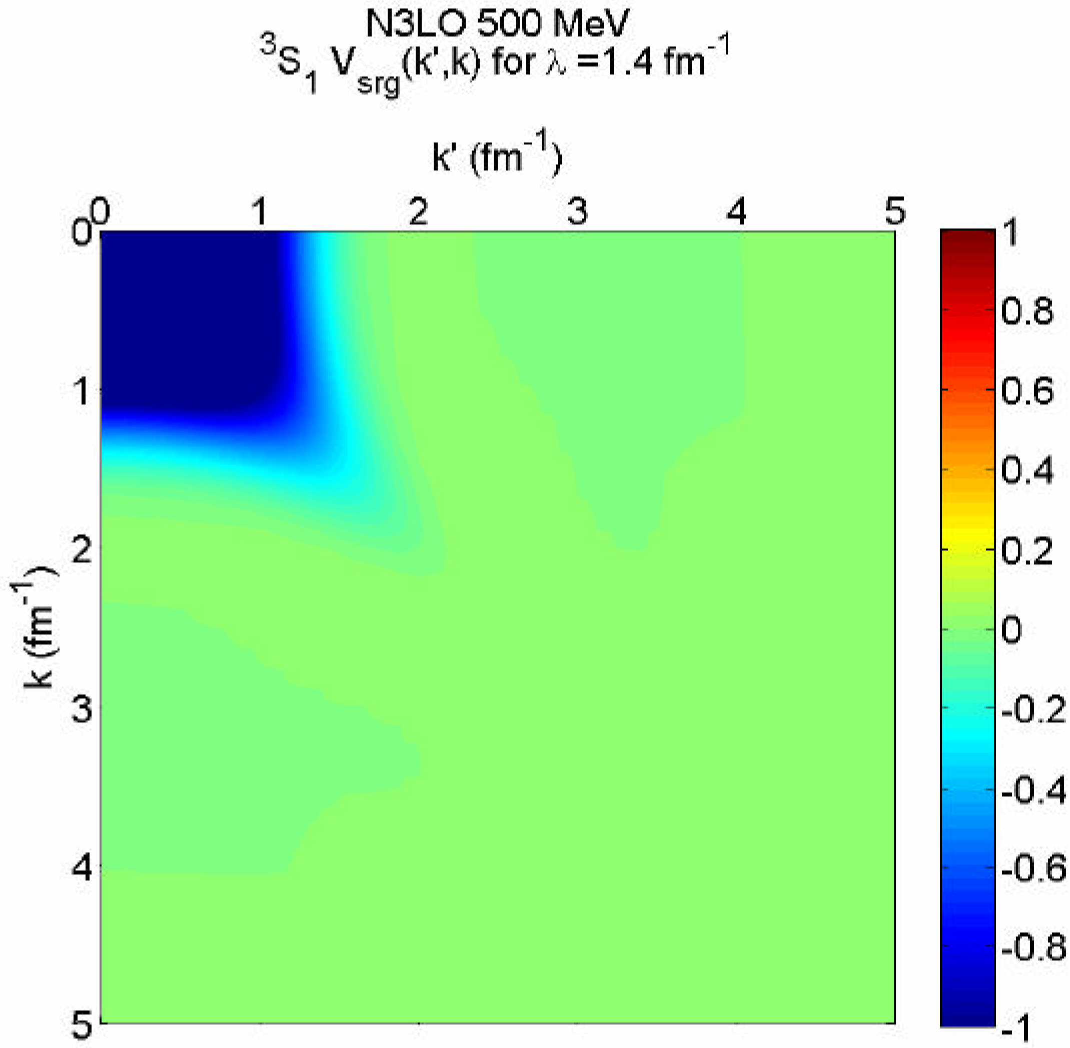


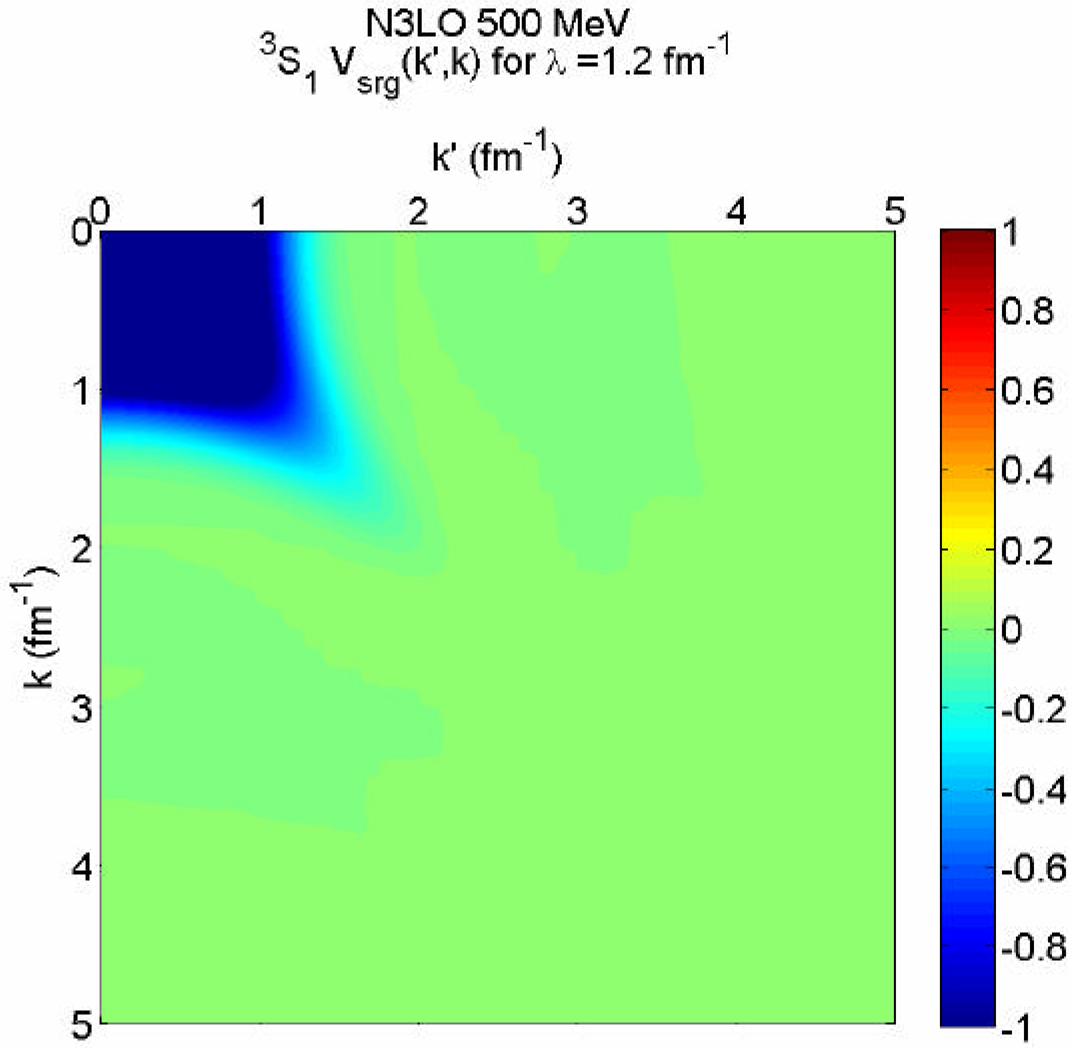

N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =2.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

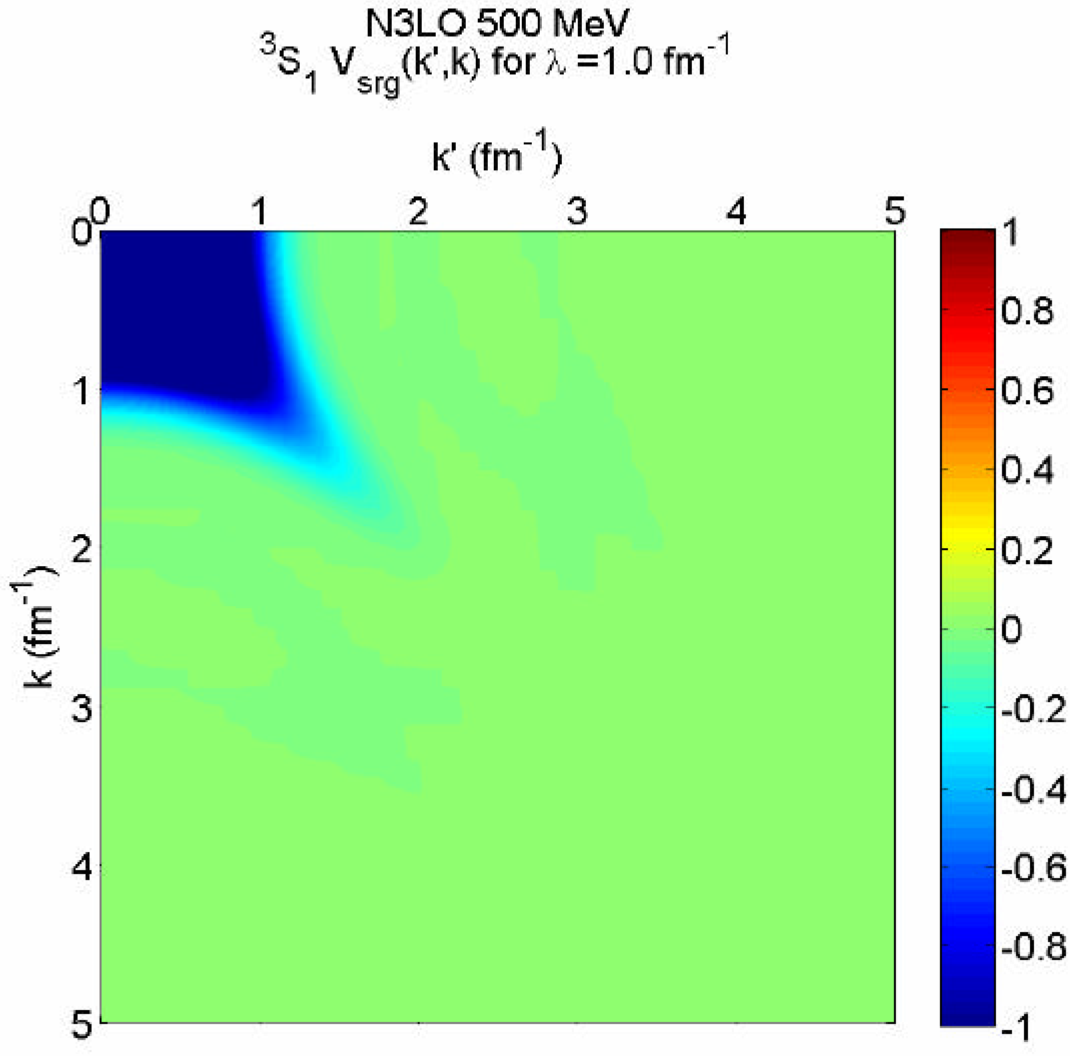

N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =1.8 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



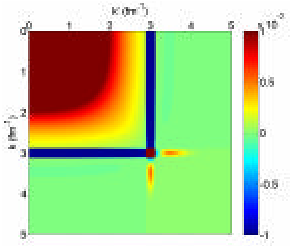
N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =1.6 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =1.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

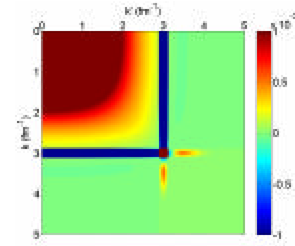

N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for $\lambda = 1.2$ fm^{-1}
k' (fm^{-1})
0 1 2 3 4 5
k (fm^{-1})
0 1 2 3 4 5
1 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6 -0.8 -1



N3LO 500 MeV
$^3S_1$ $V_{srg}(k',k)$ for λ =1.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

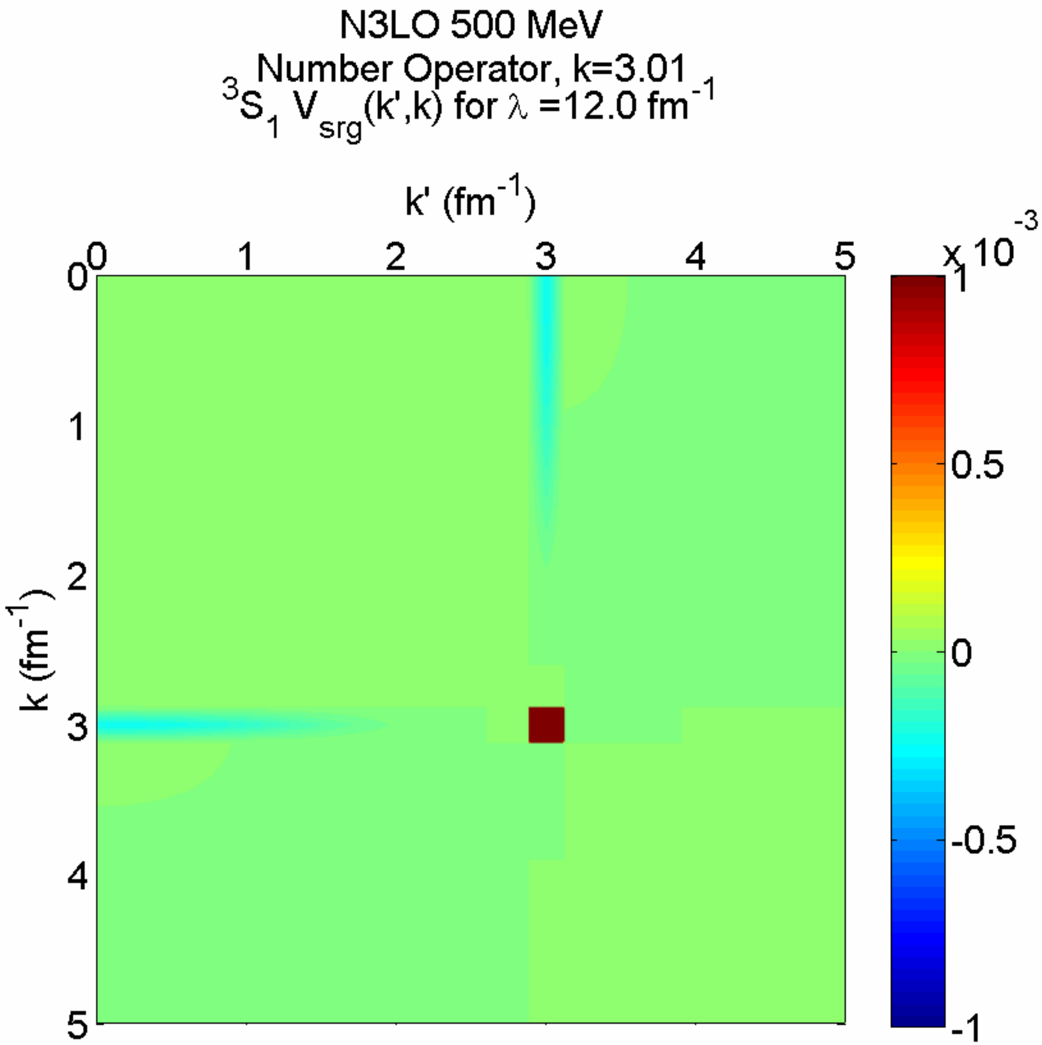


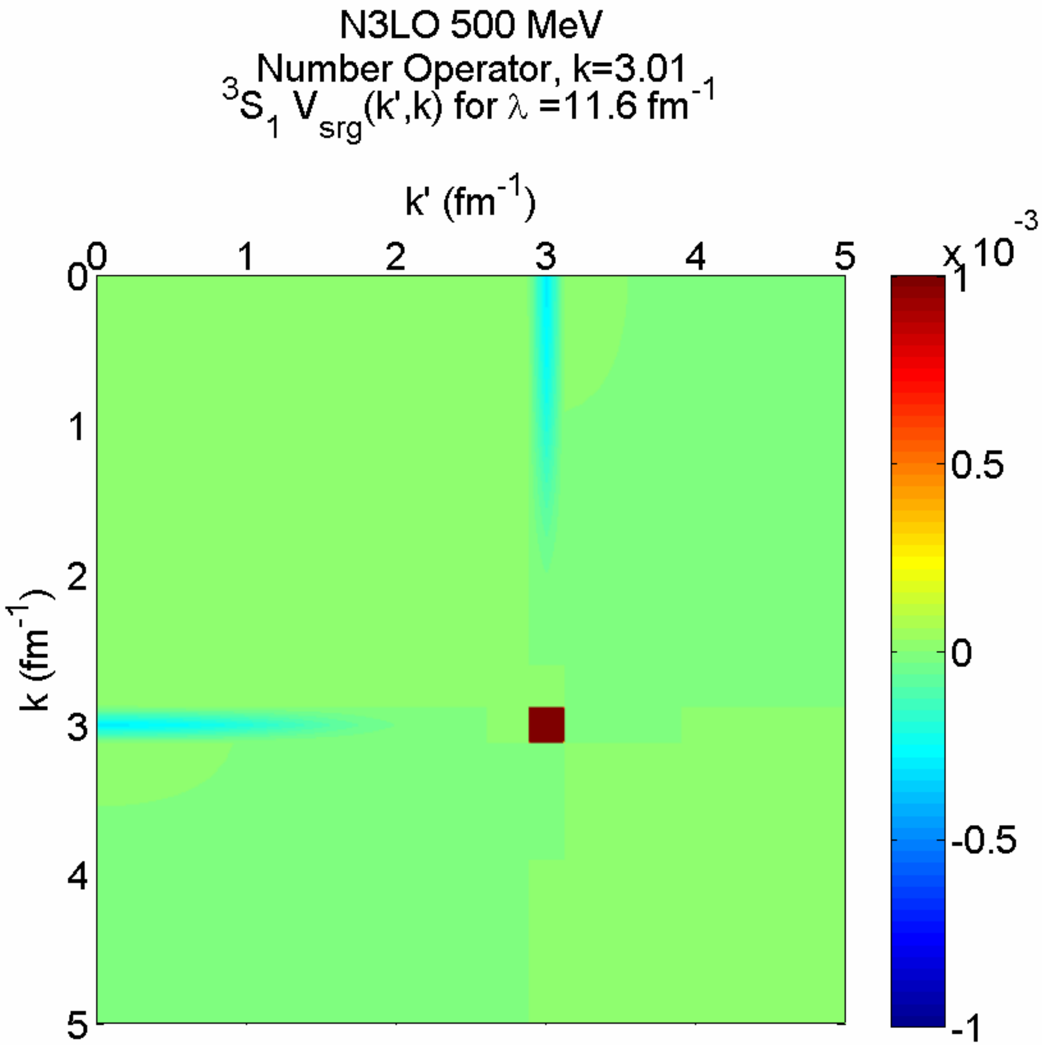
Operator Evolution

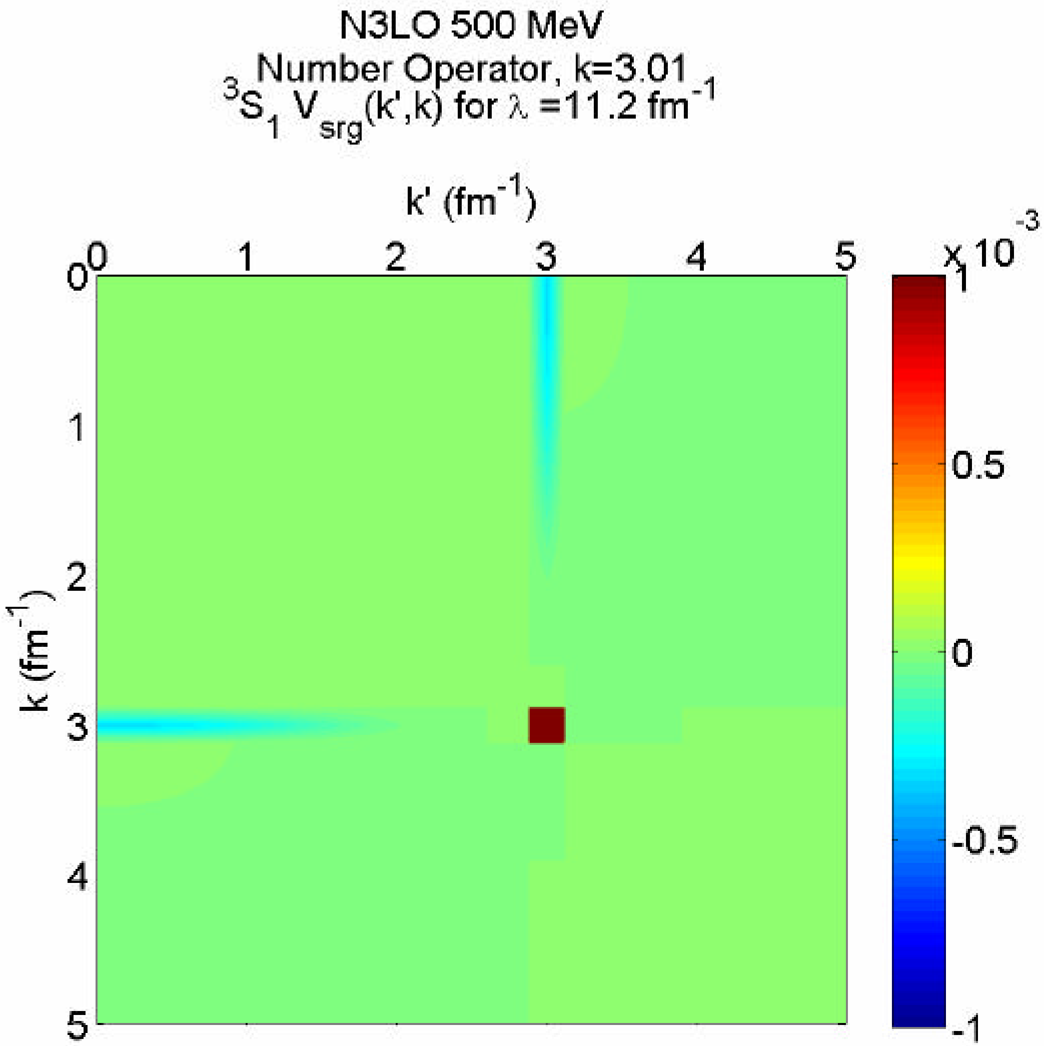


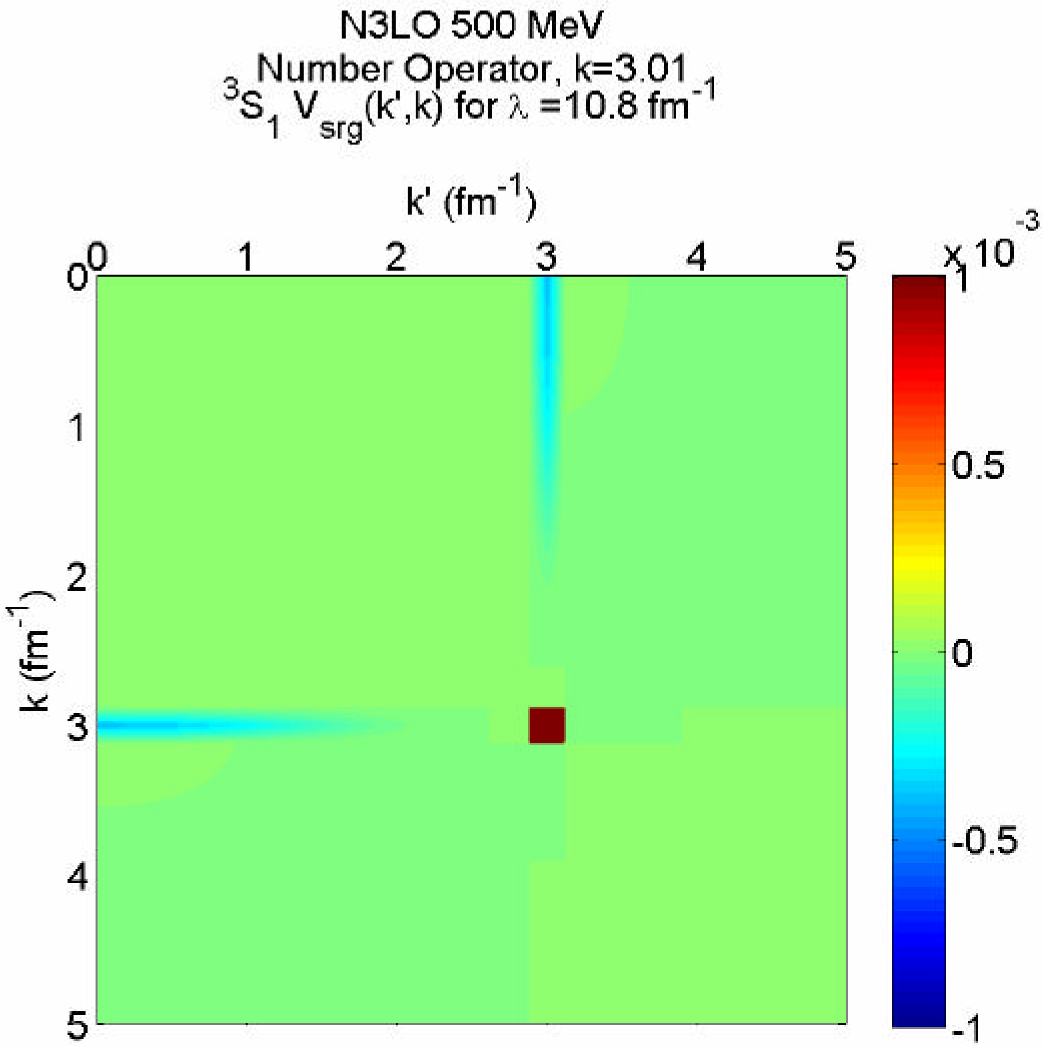
$$\frac{\mathrm{d}\mathrm{O}_s}{\mathrm{d}s} = \left[\eta(s), \mathrm{O}(s)\right] = \left[\left[T_{rel}, V_s\right], \mathrm{O}_s\right]$$

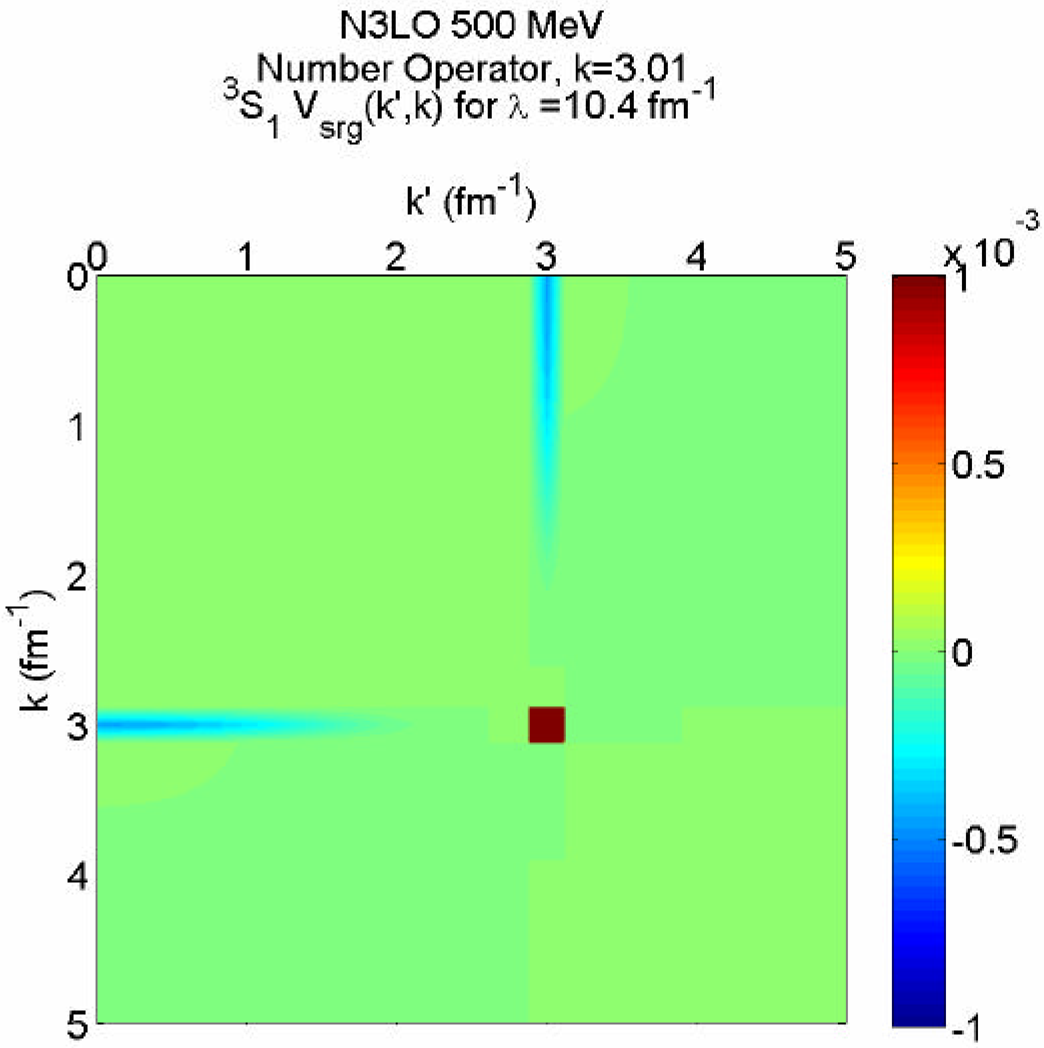
$$\frac{\mathrm{d}\mathrm{O}_s(k, k')}{\mathrm{d}s} = \frac{2}{\pi}\int_0^\infty q^2\,\mathrm{d}q\left[\left(k^2 - q^2\right)V_s(k, \mathrm{q})\mathrm{O}_s(q, k') + \left(k'^2 - q^2\right)\mathrm{O}_s(k, q)V_s(q, k')\right]$$

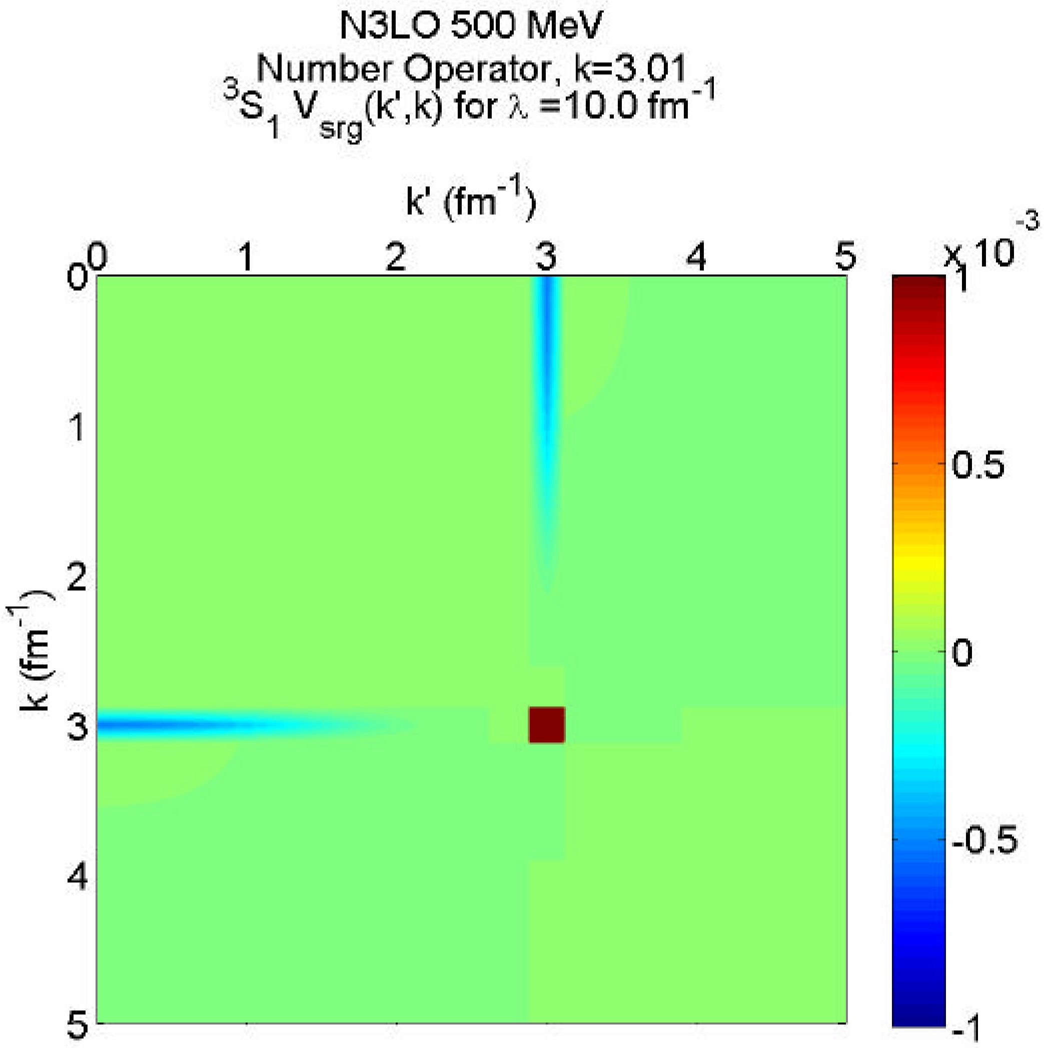

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =12.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =11.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
1
0.5
0
-0.5
-1
0
1
2
3
4
5

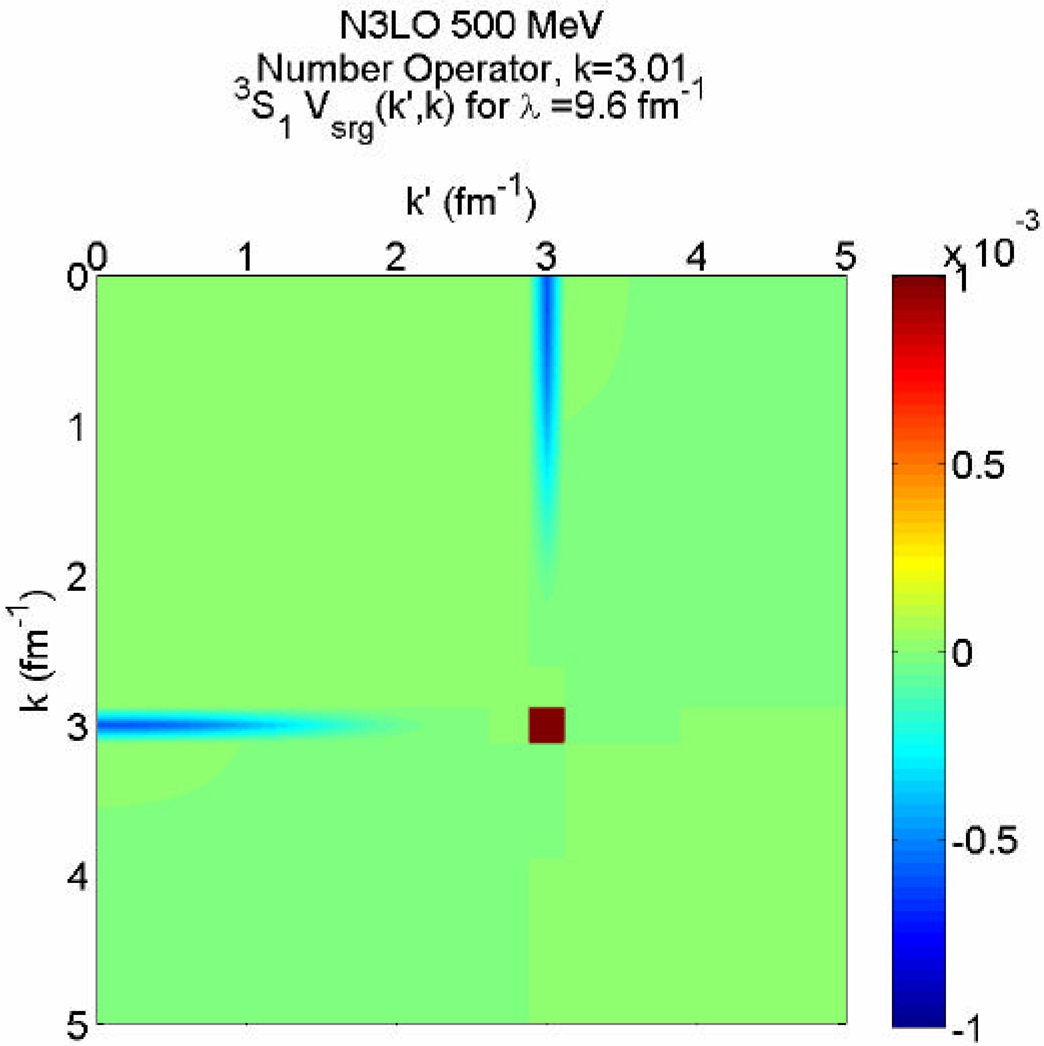

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =11.2 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =10.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

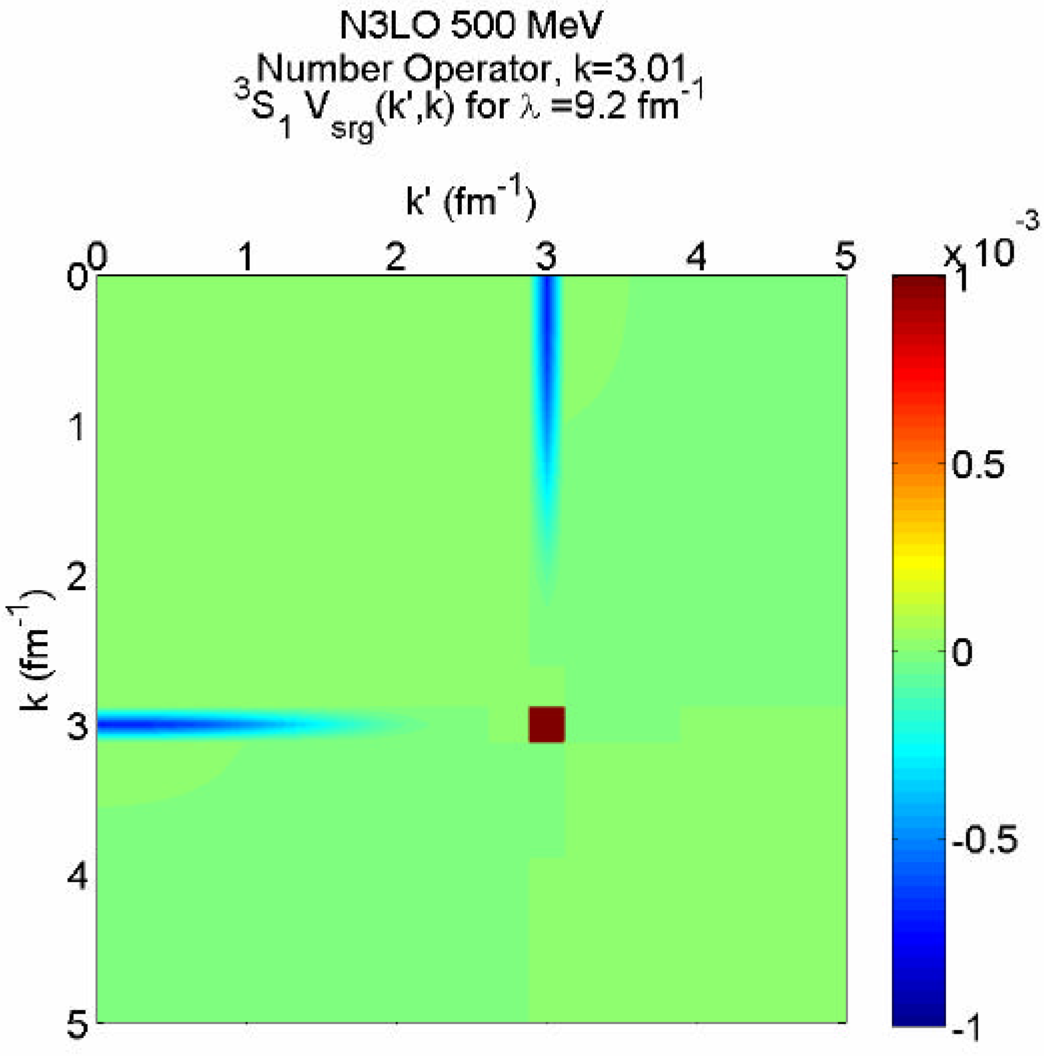

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =10.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



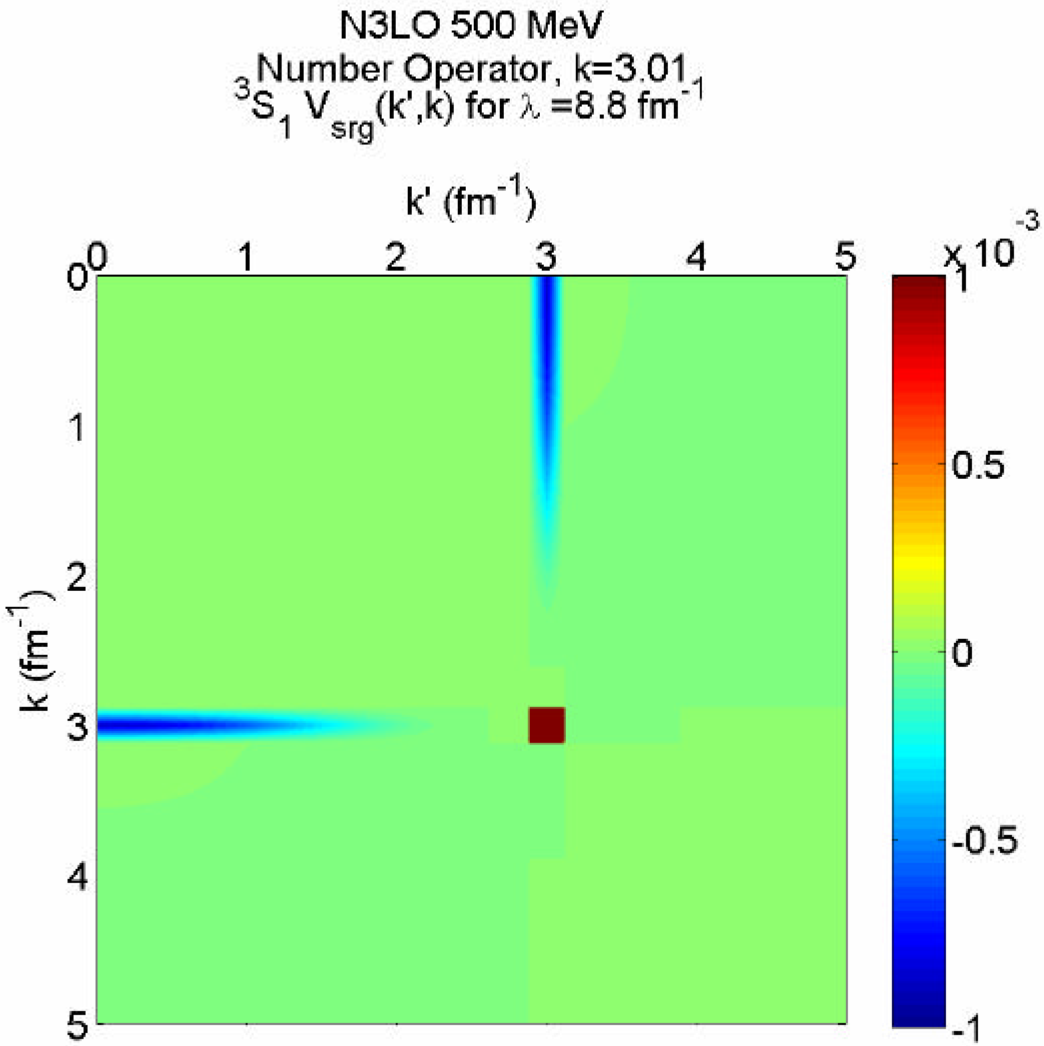
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =10.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =9.6 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



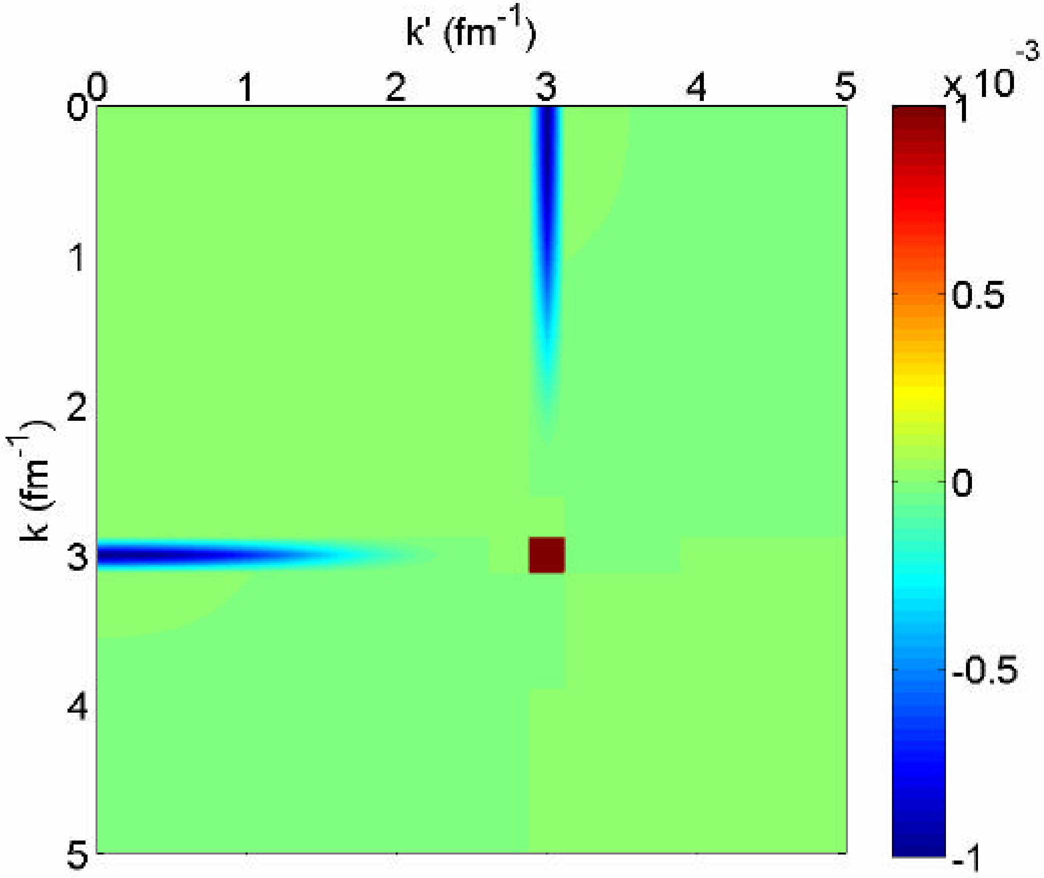
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =9.2 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
x 10^{-3}
1
0.5
0
-0.5
-1

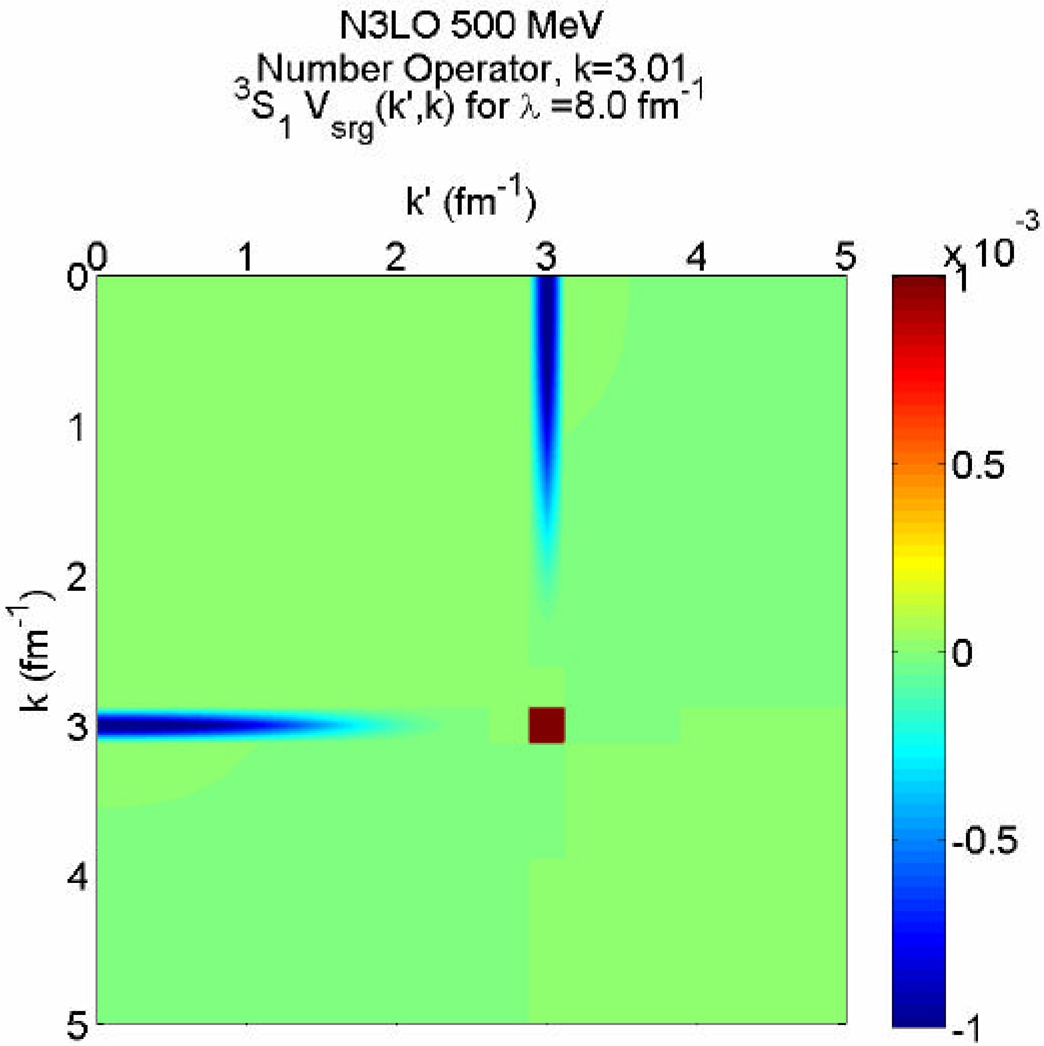

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =8.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
1
0.5
0
-0.5
-1

N3LO 500 MeV

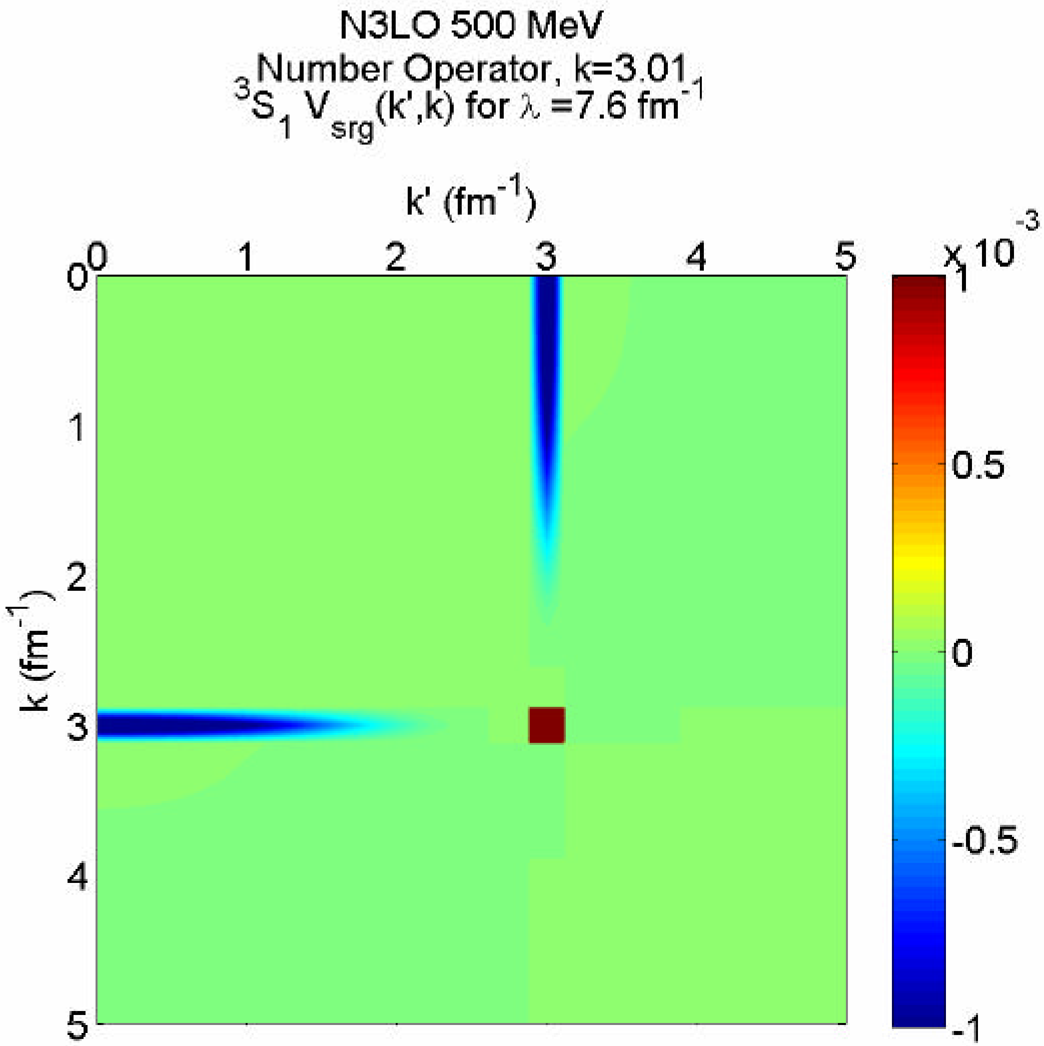
Number Operator, $k=3.01$

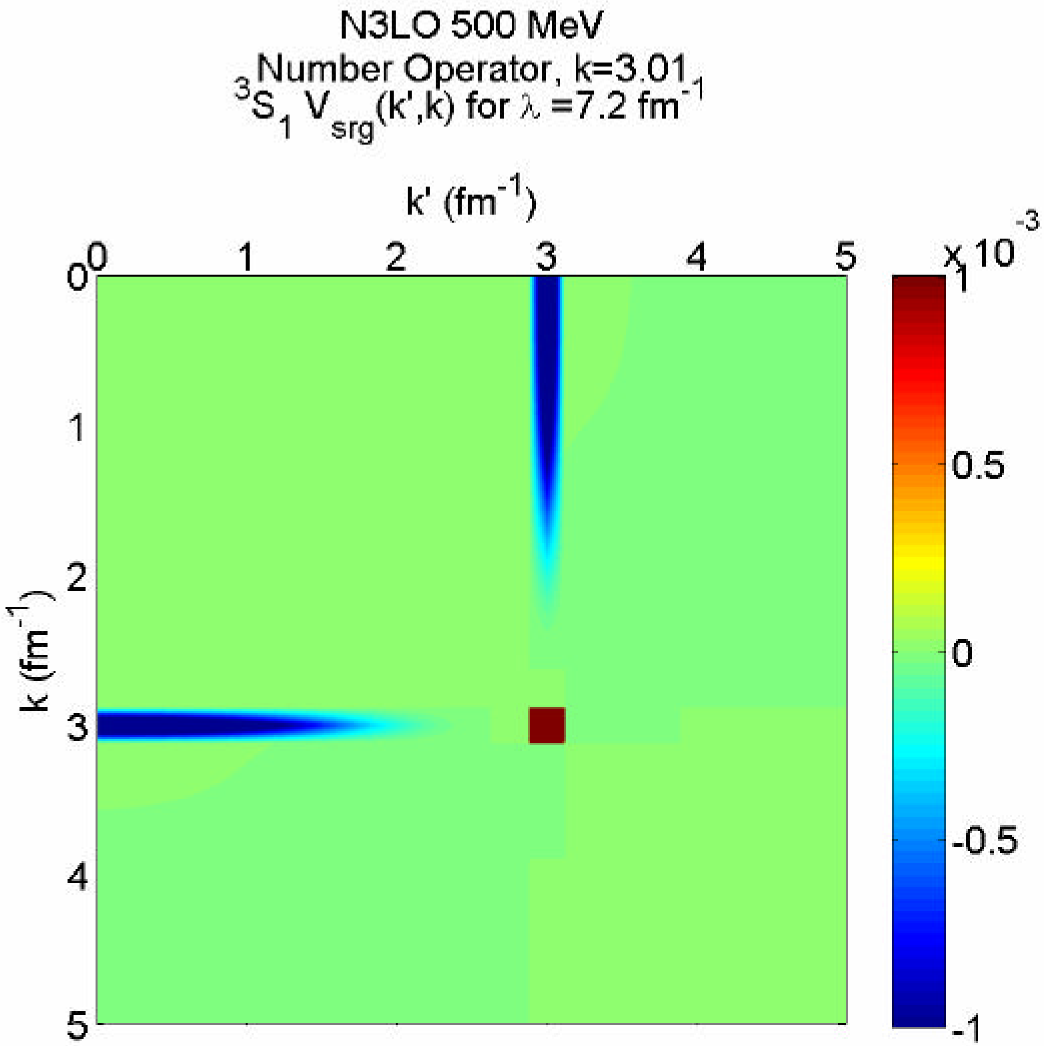
$^3S_1$ $V_{srg}(k',k)$ for $\lambda = 8.4$ fm^{-1}



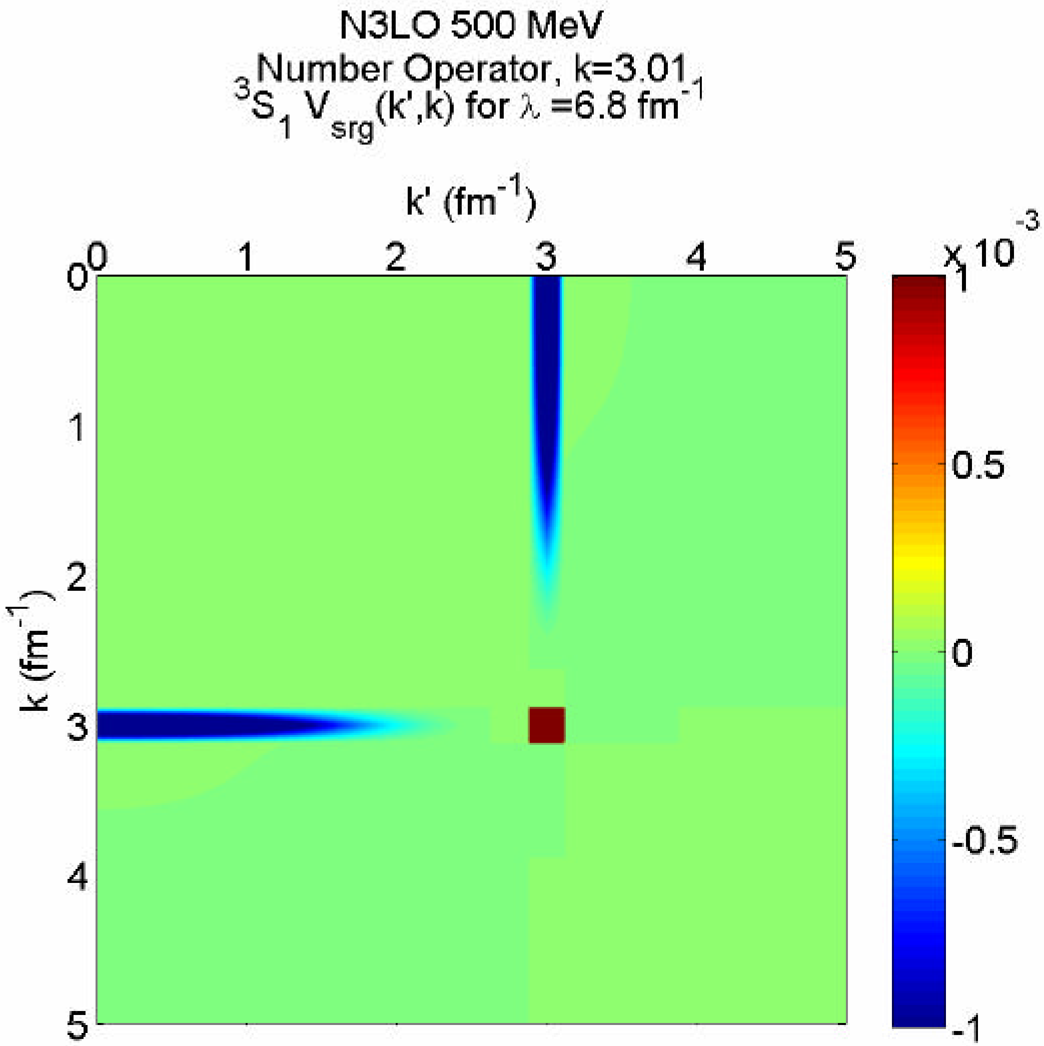


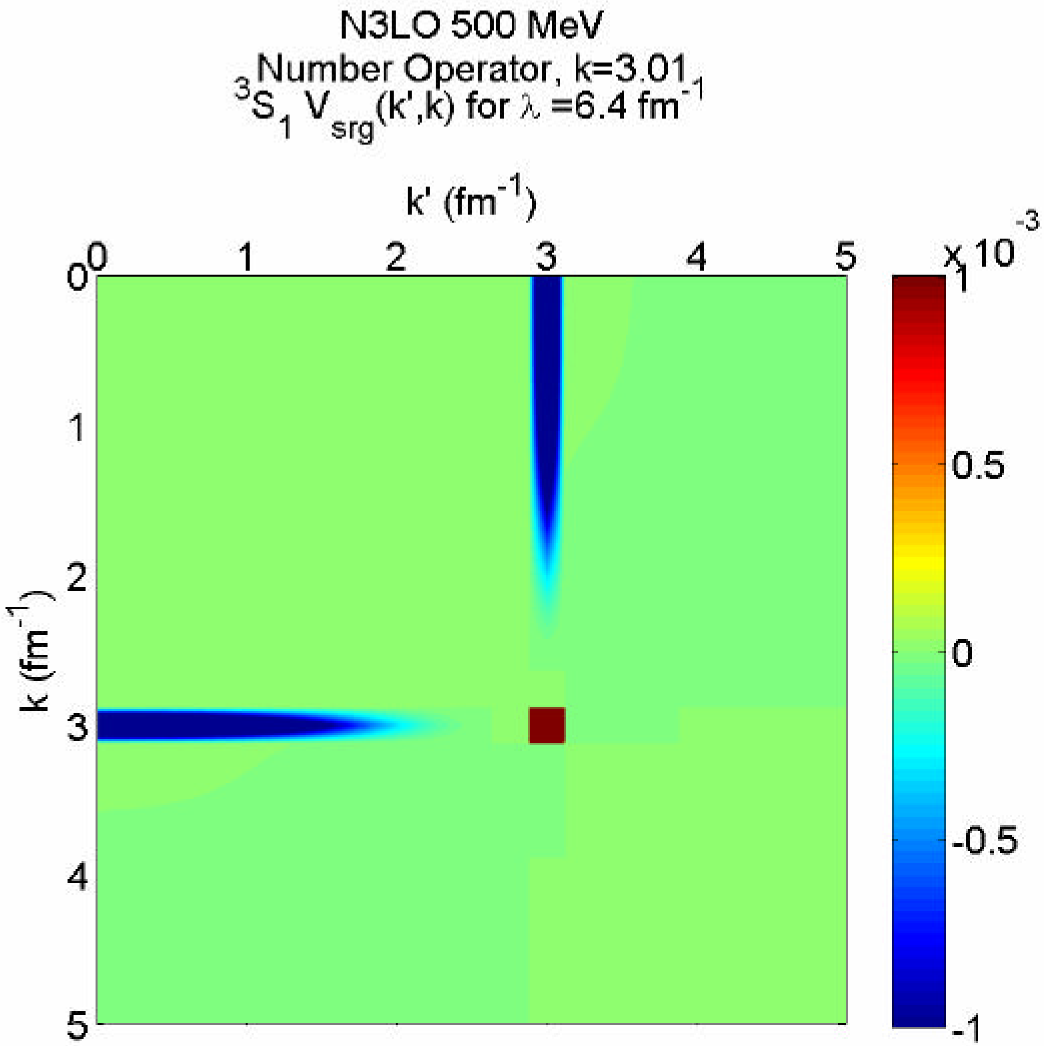
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =8.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =7.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

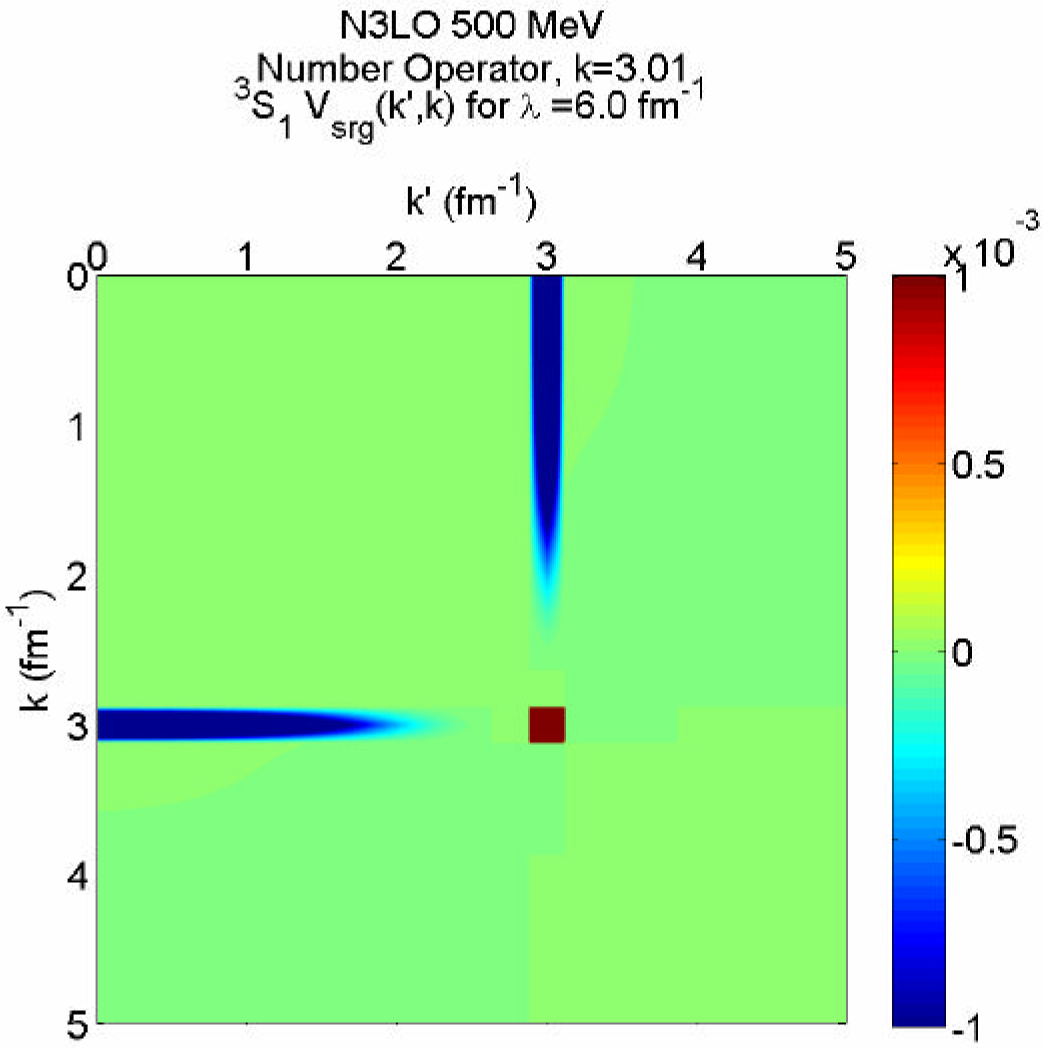


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =7.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



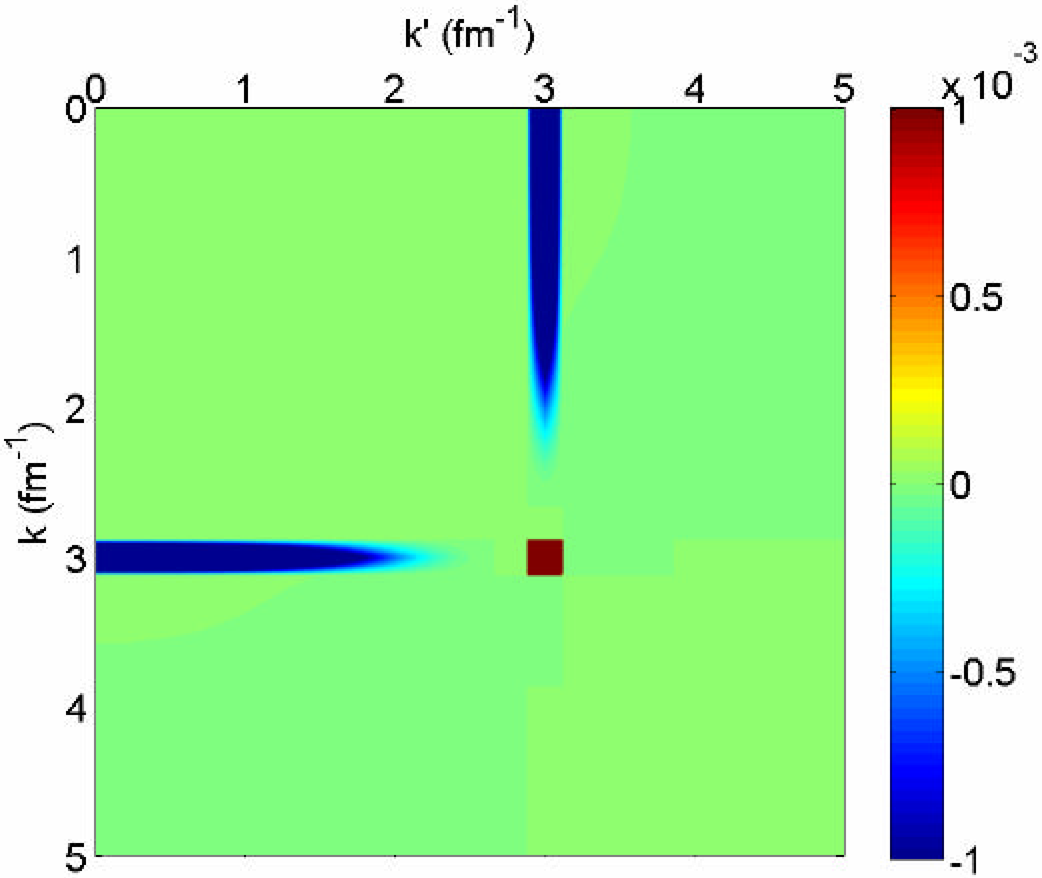
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.4 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

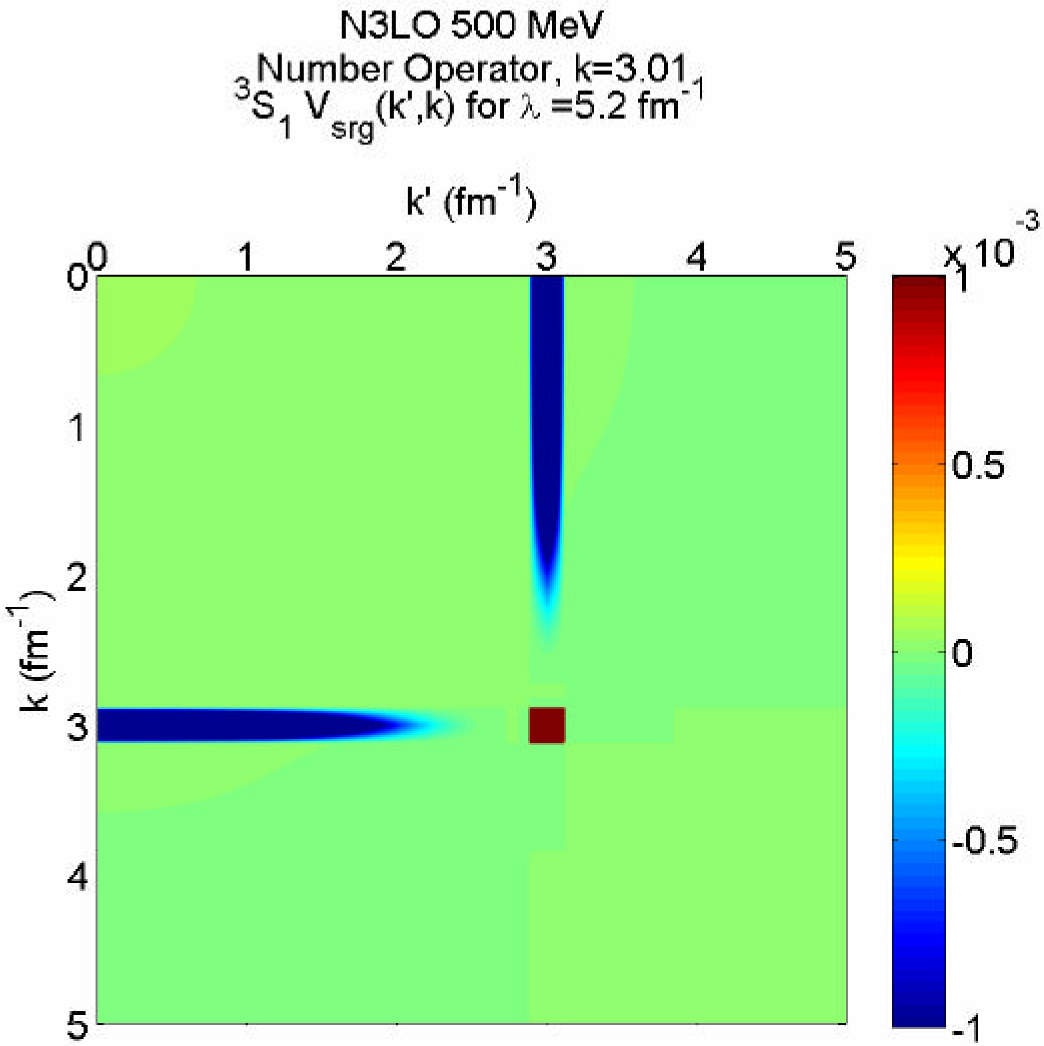

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

N3LO 500 MeV

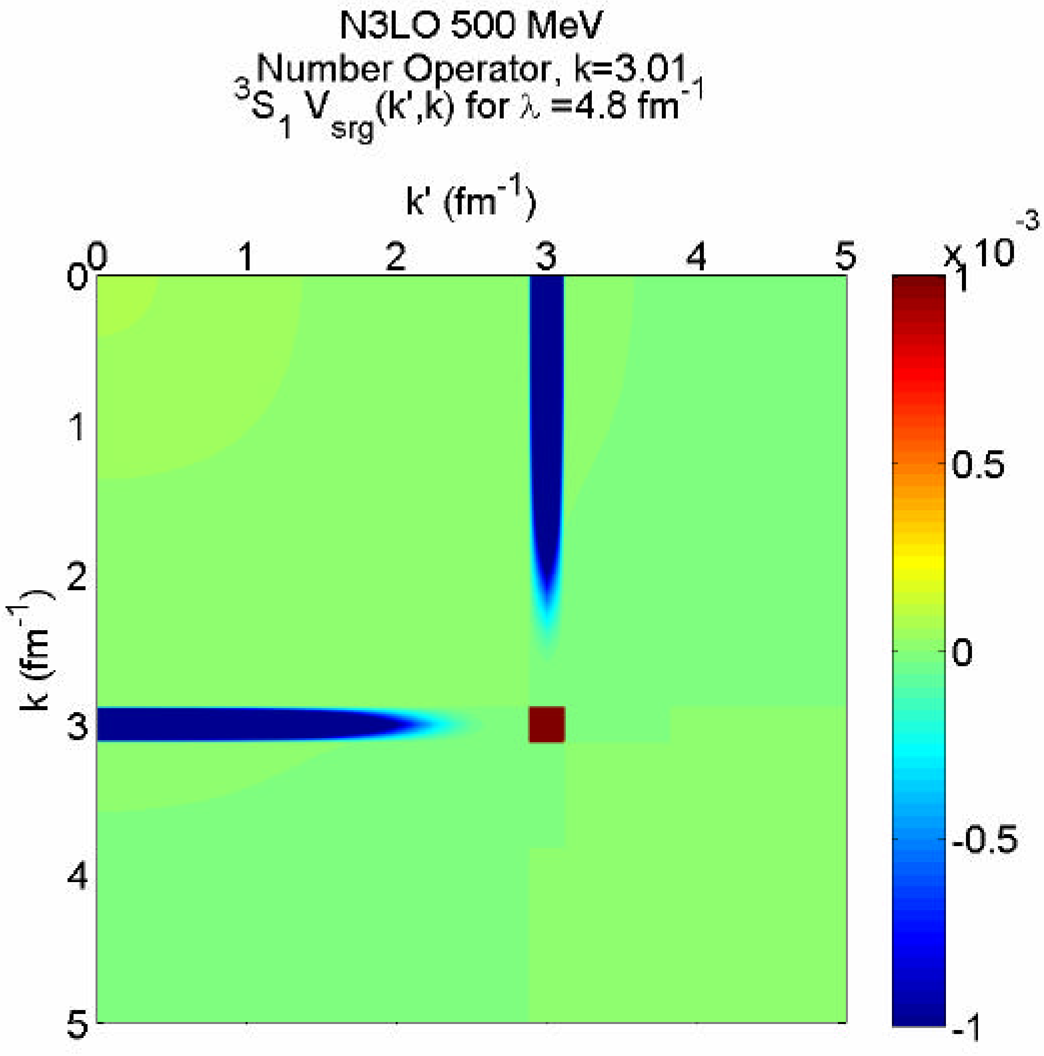
Number Operator, k=3.01

$^3S_1$ $V_{srg}(k',k)$ for λ =5.6 fm^{-1}

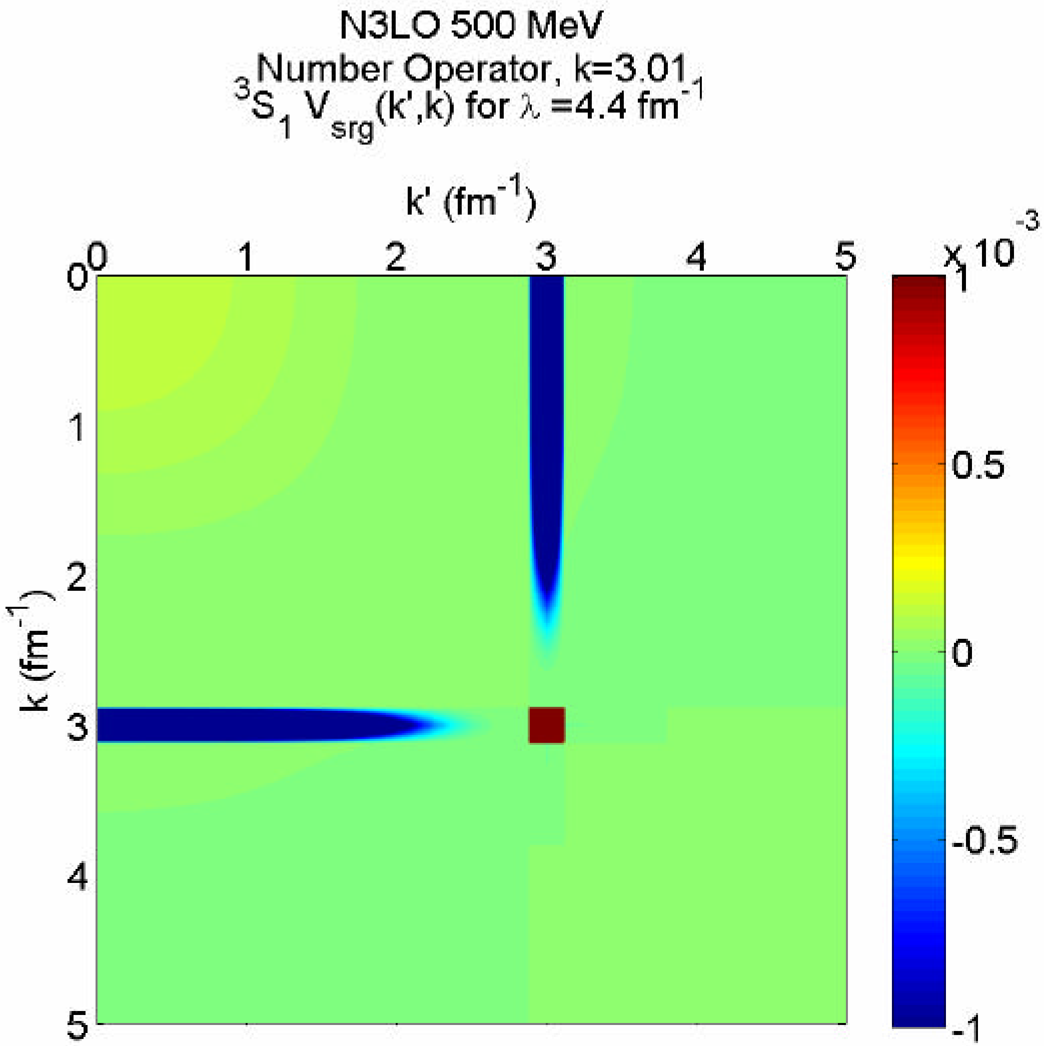




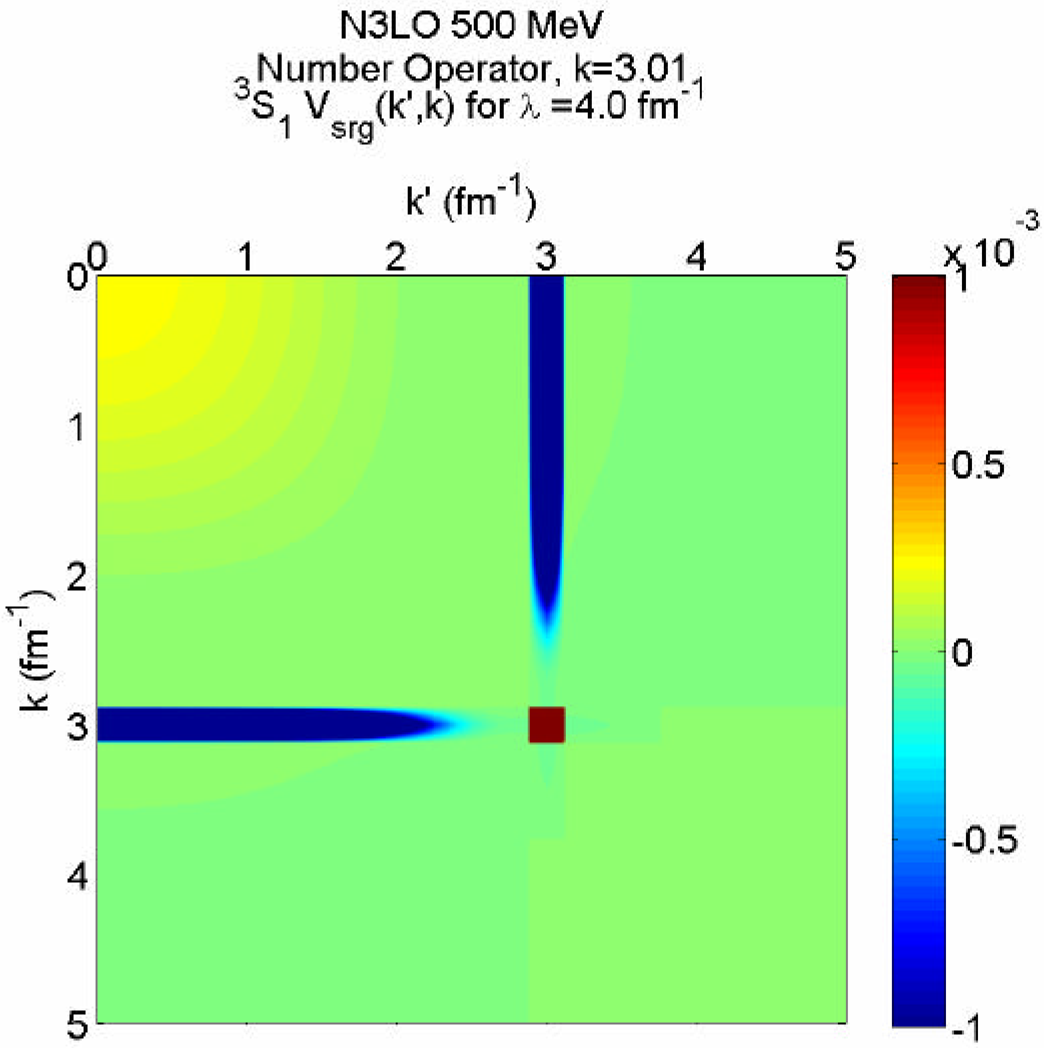
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =5.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



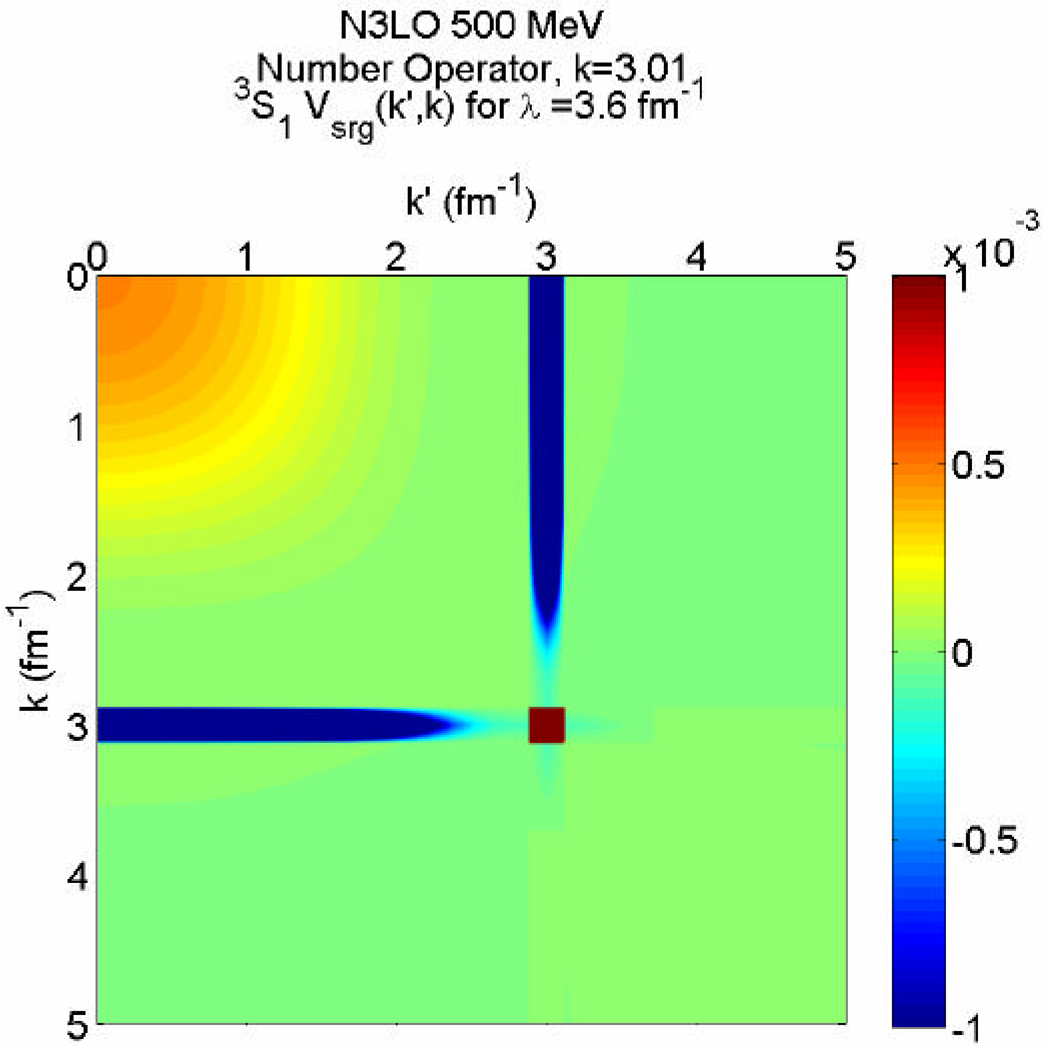
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

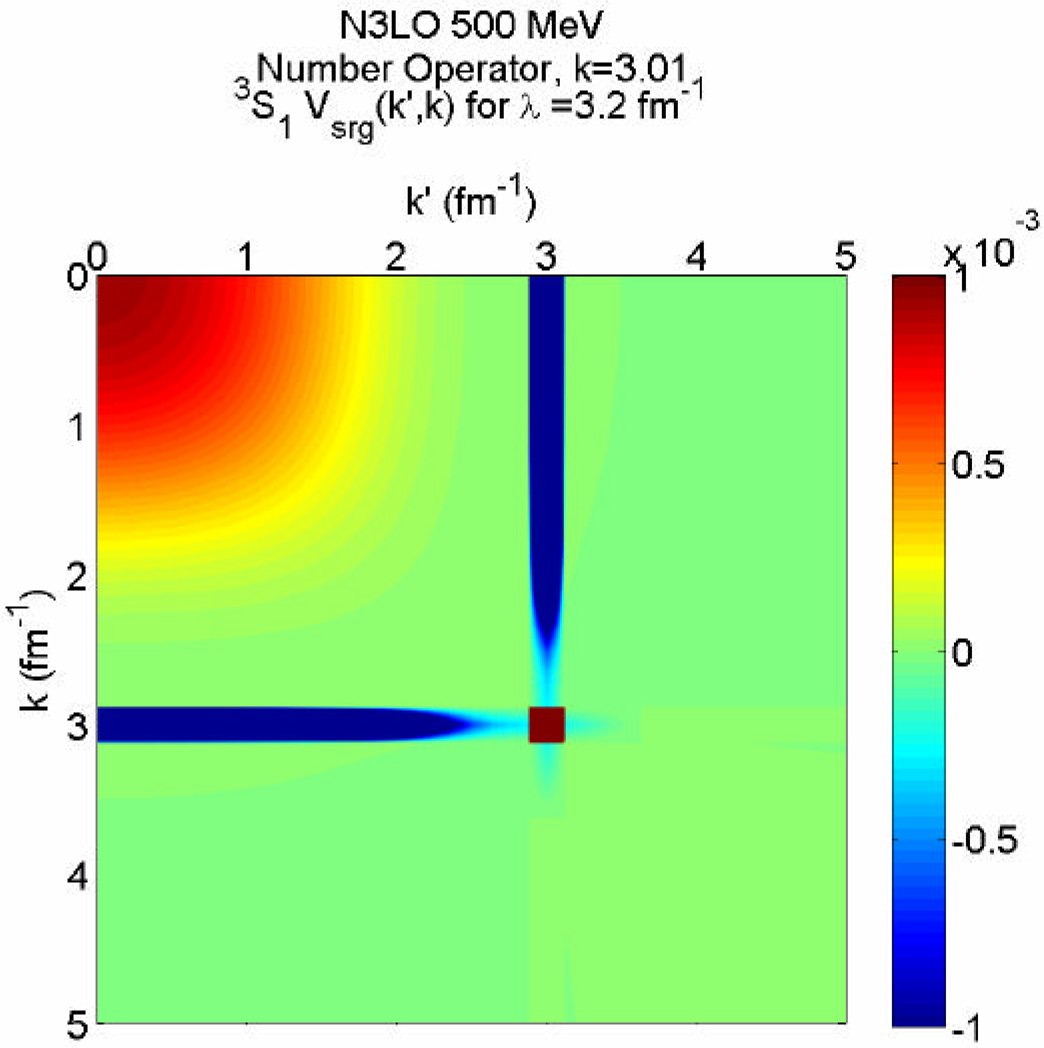


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

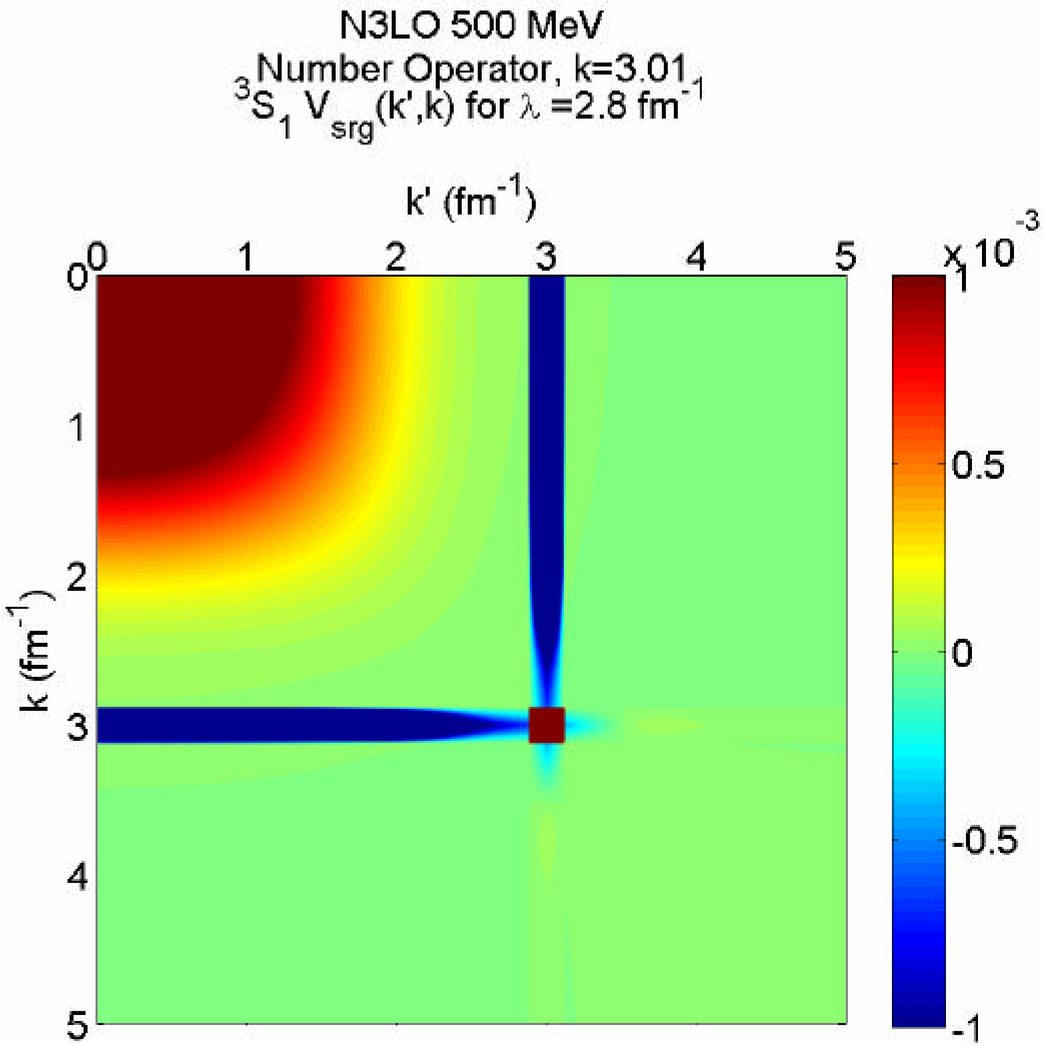


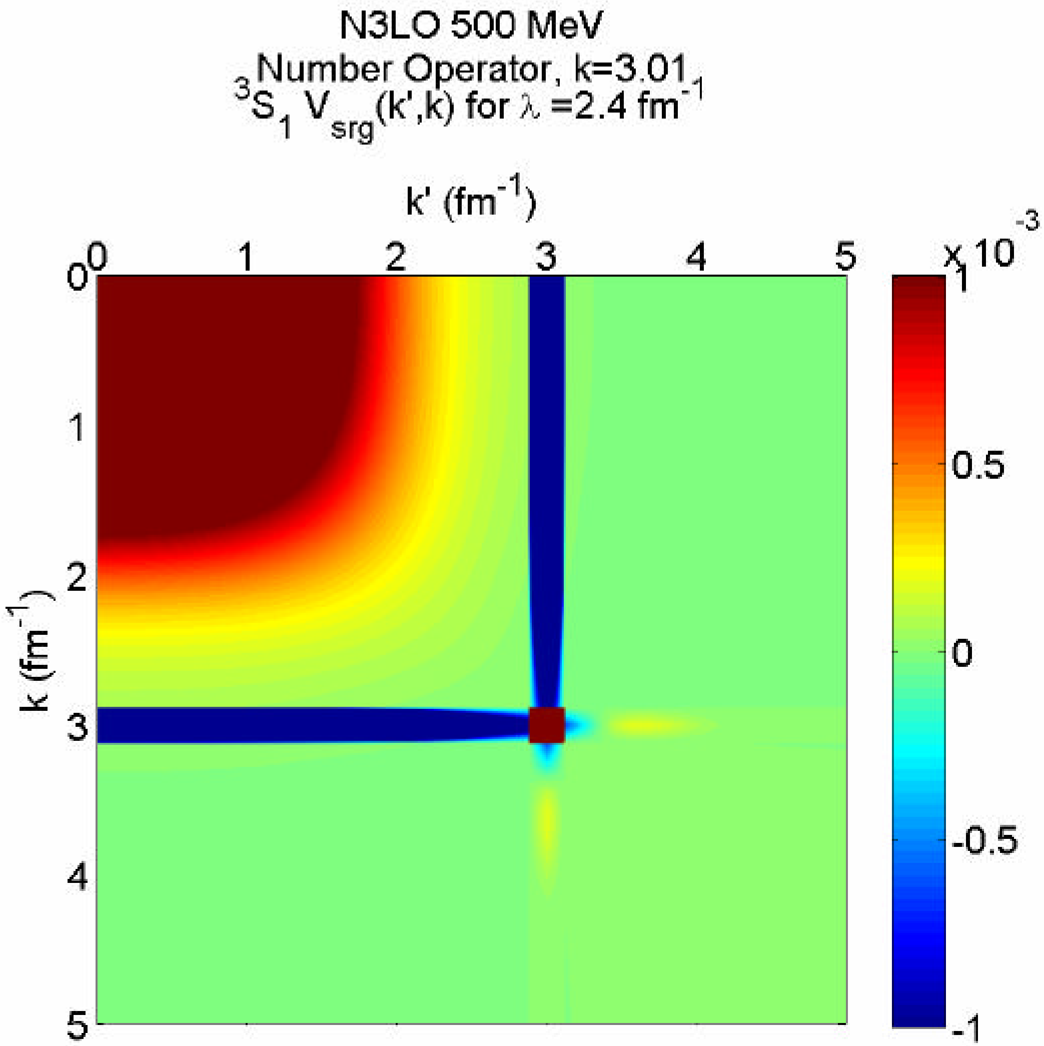
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

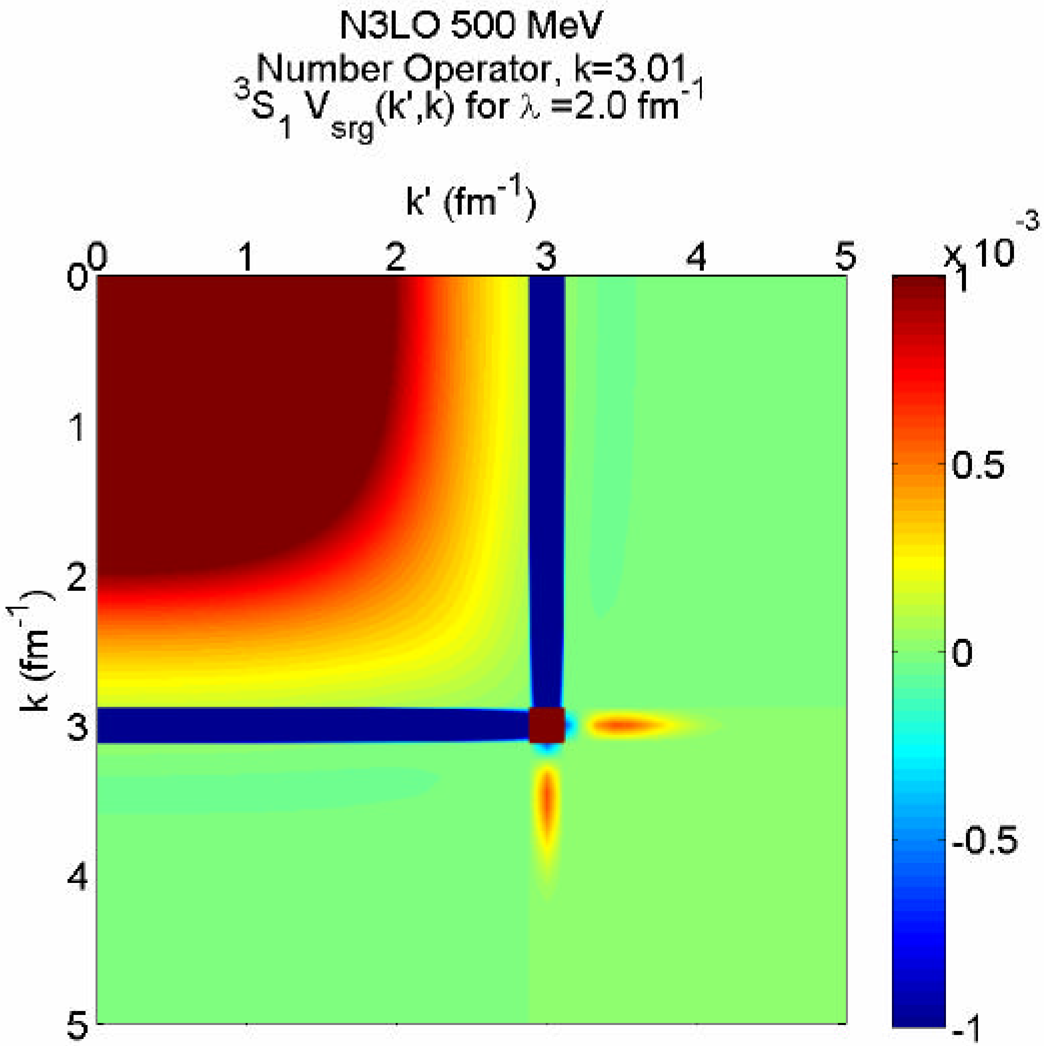

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



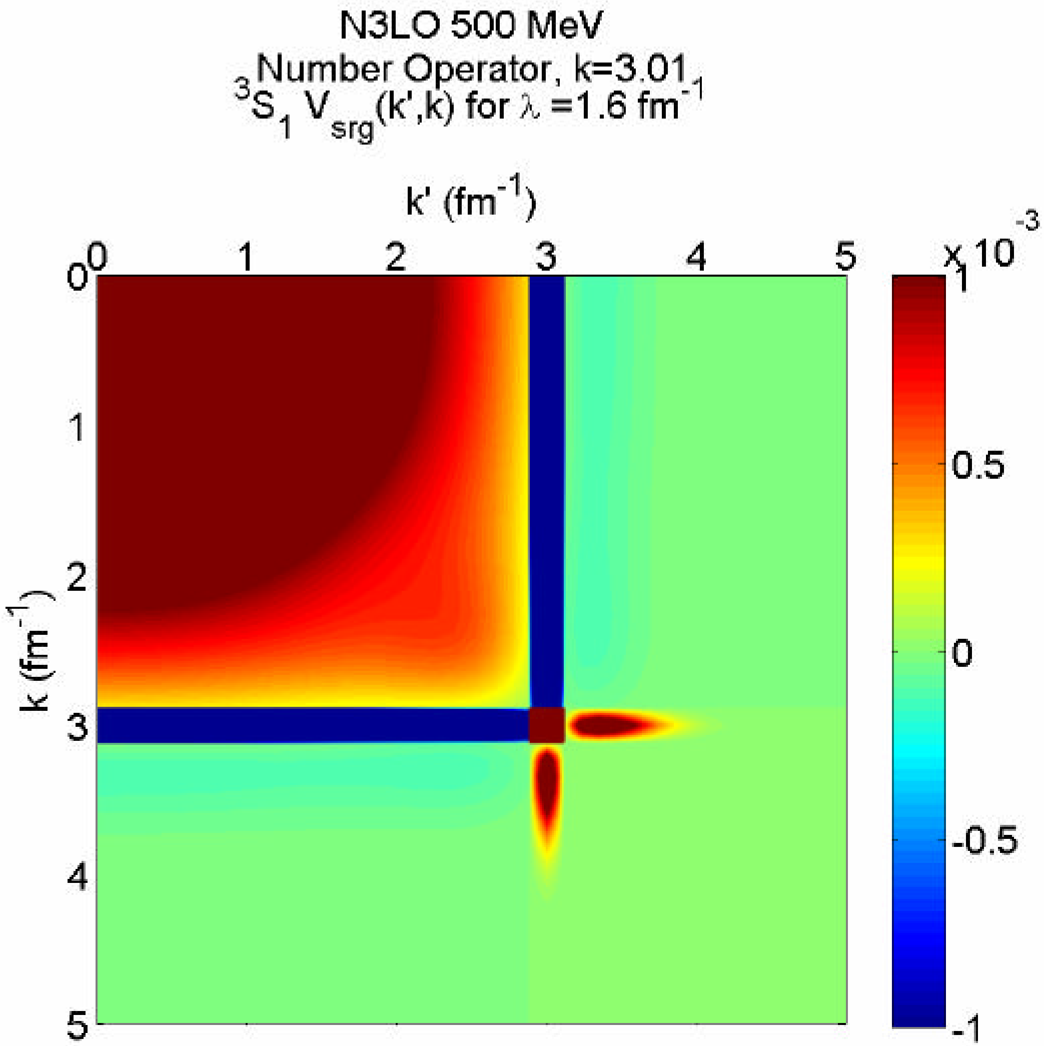
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}

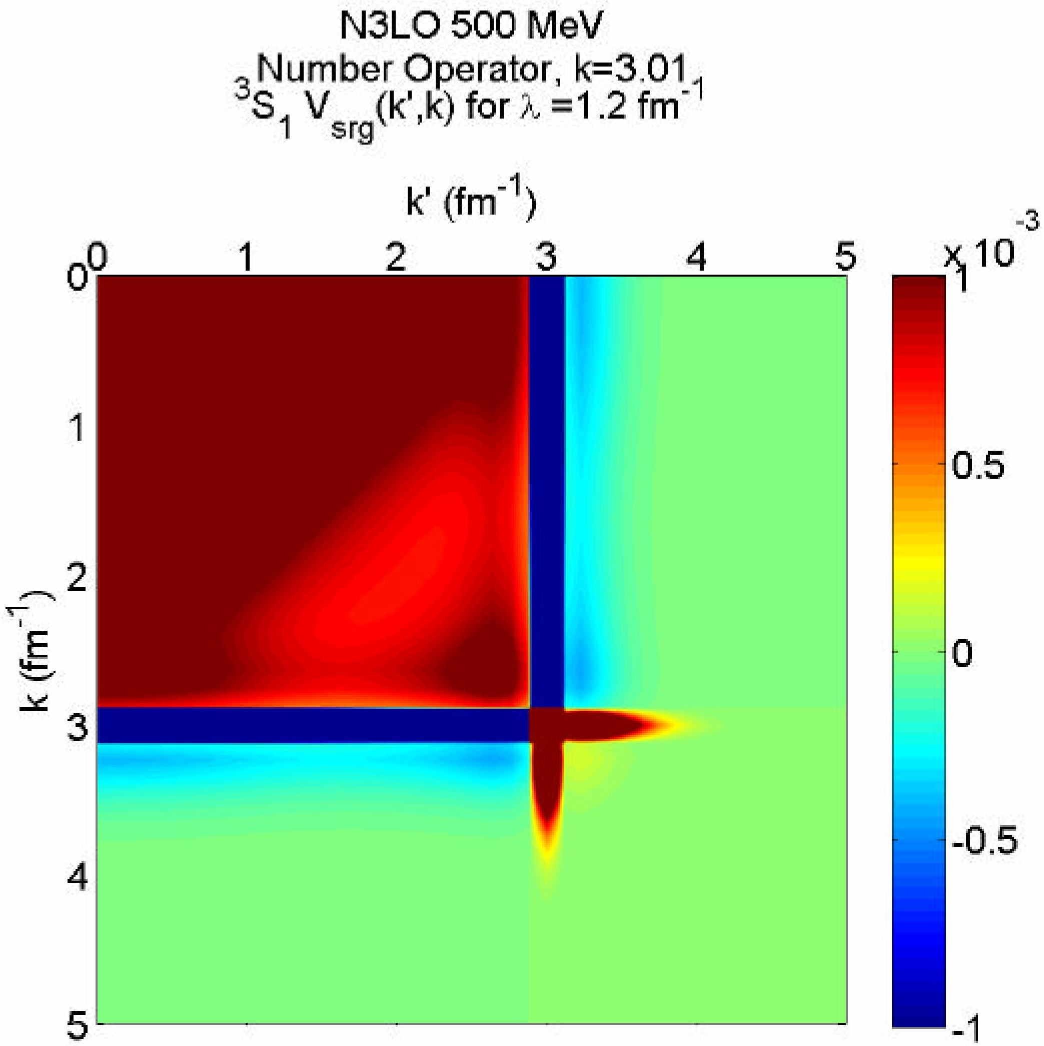

N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

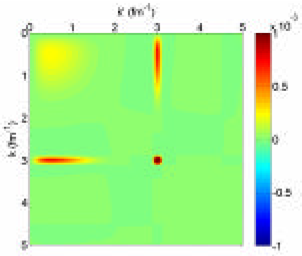


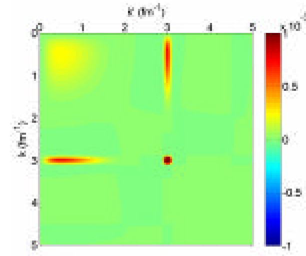
N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Number Operator, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =1.6 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



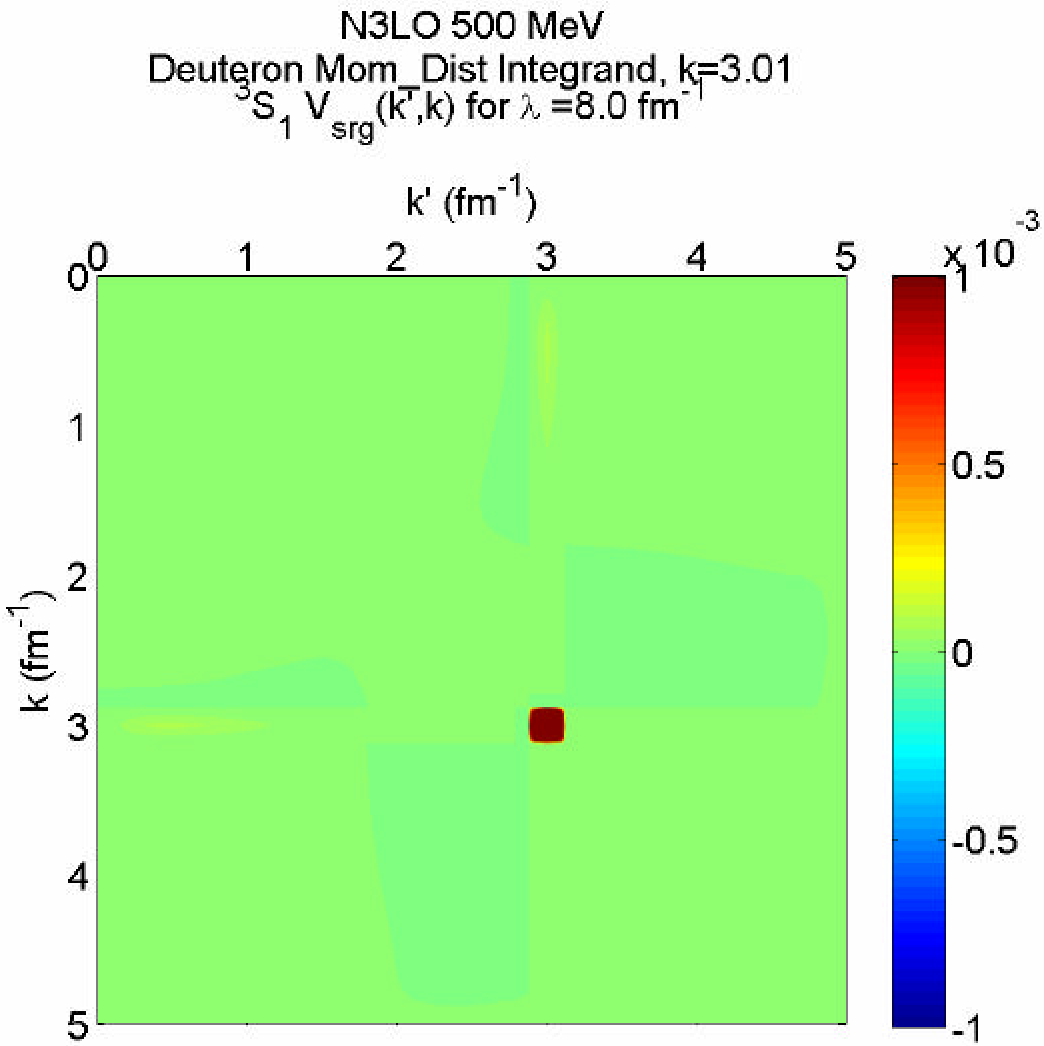
N3LO 500 MeV
Number Operator, k=3.01
3S1 Vsrg(k',k) for λ =1.2 fm-1
k' (fm-1)
k (fm-1)
0
1
2
3
4
5
x 10-3
1
0.5
0
-0.5
-1



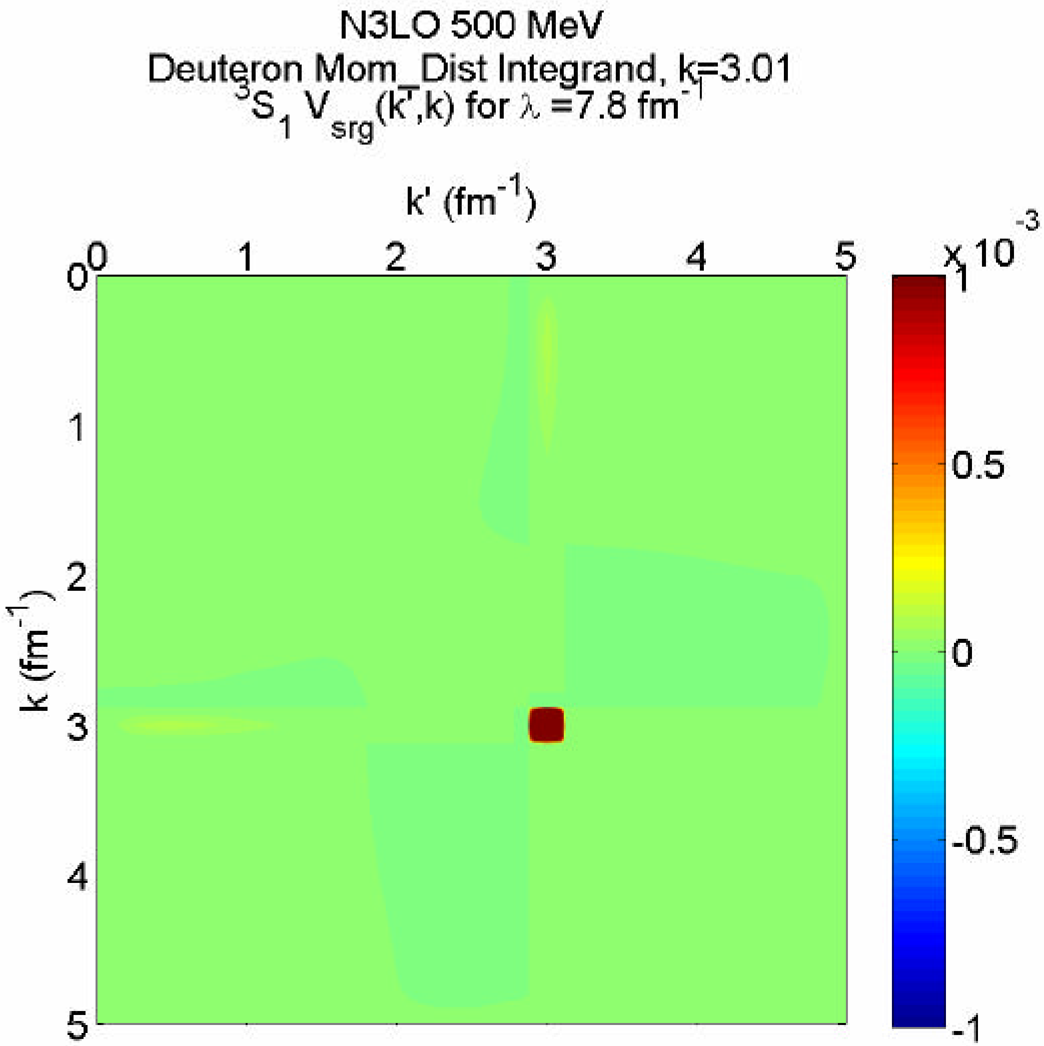


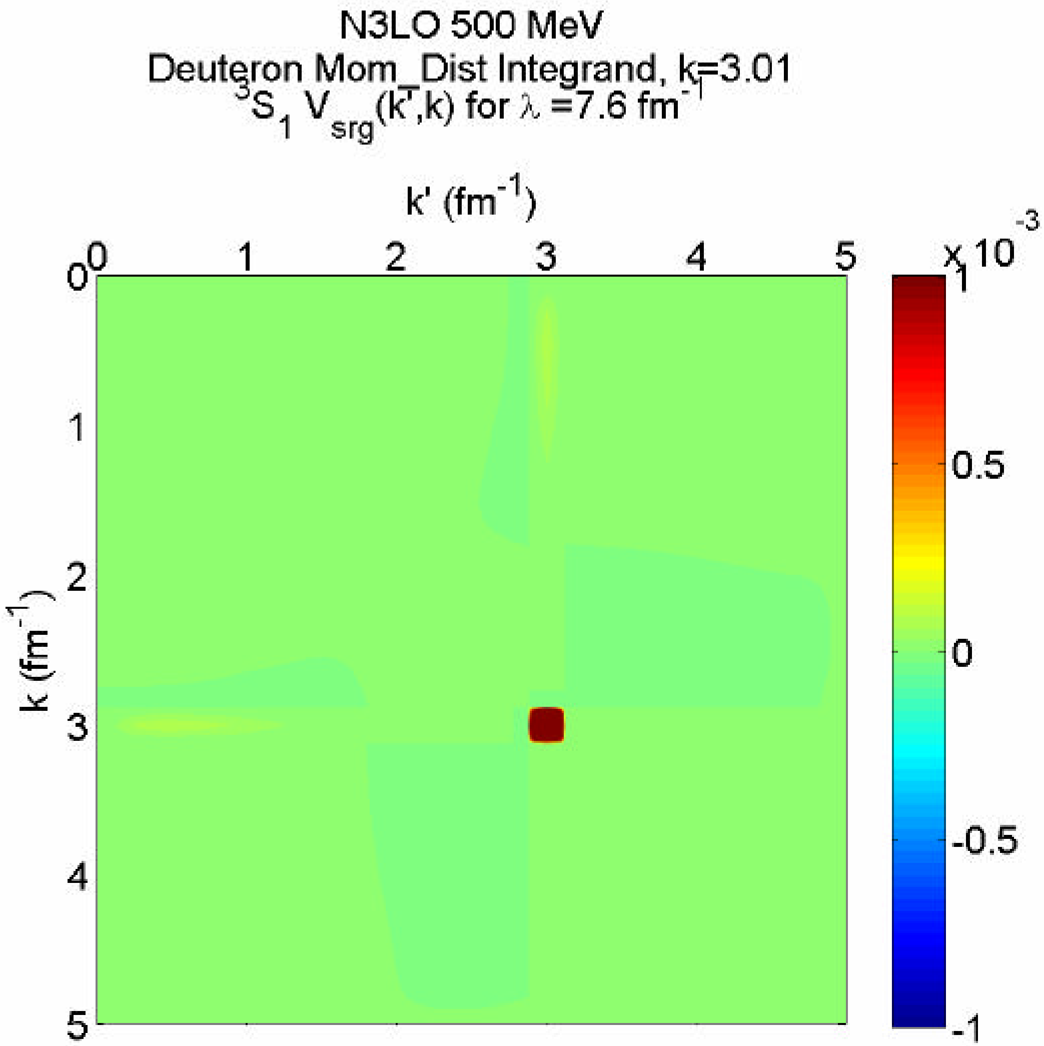
The Deuteron Integrand

$$\left\langle \psi_d \middle| U a_q^\dagger a_q U^\dagger \middle| \psi_d \right\rangle$$

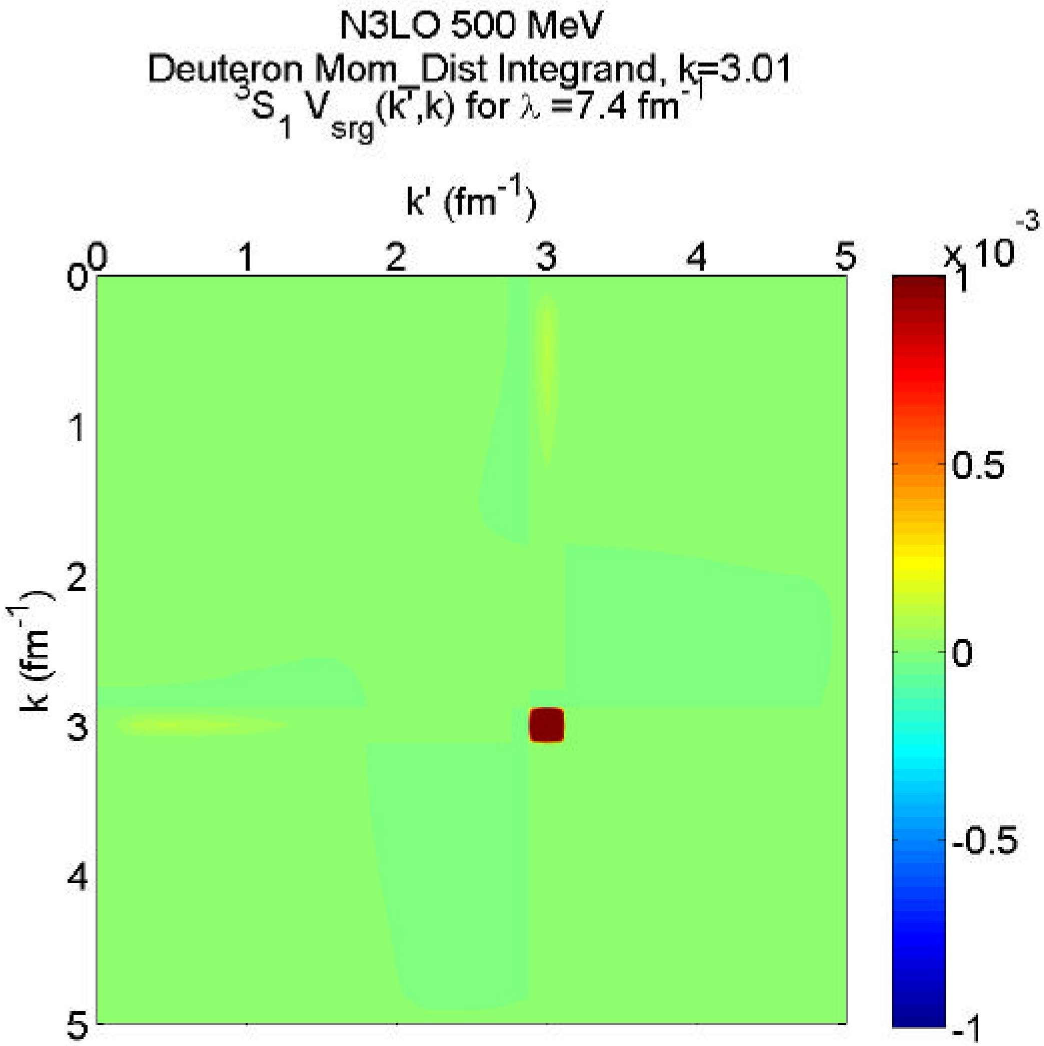


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =8.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
$x\,10^{-3}$
1
0.5
0
-0.5
-1


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =7.8 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =7.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

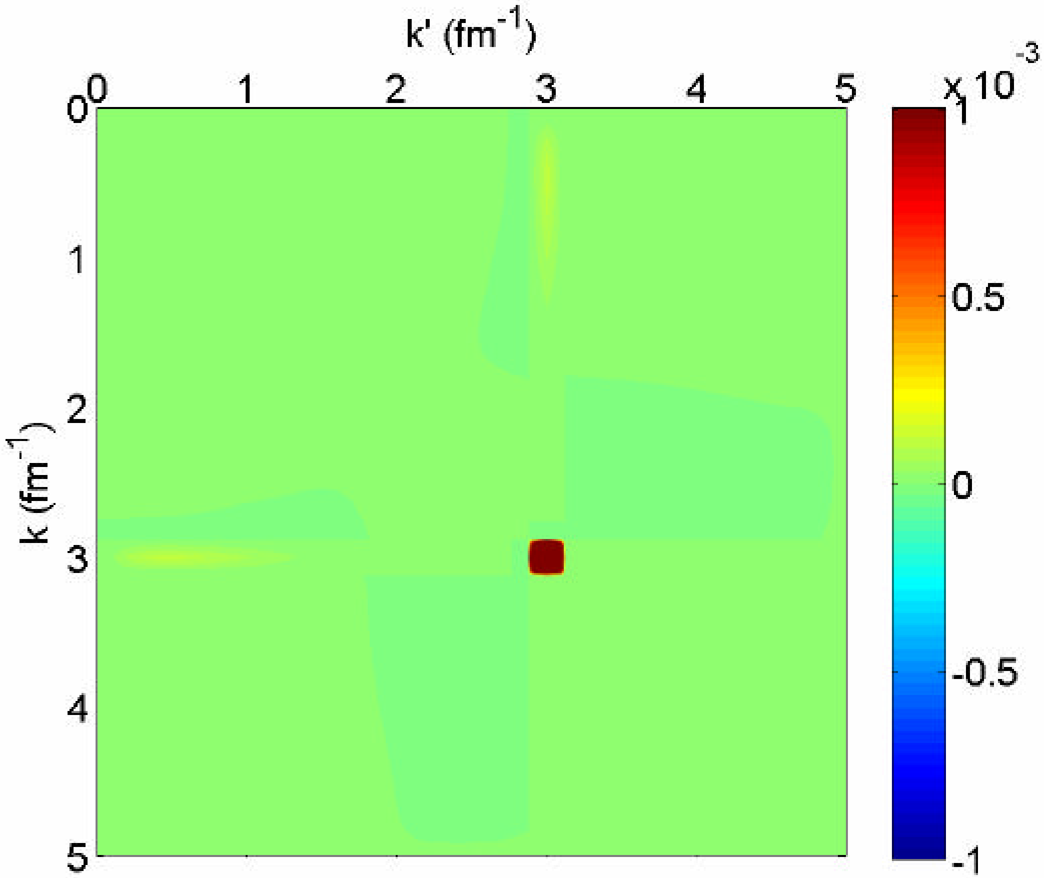


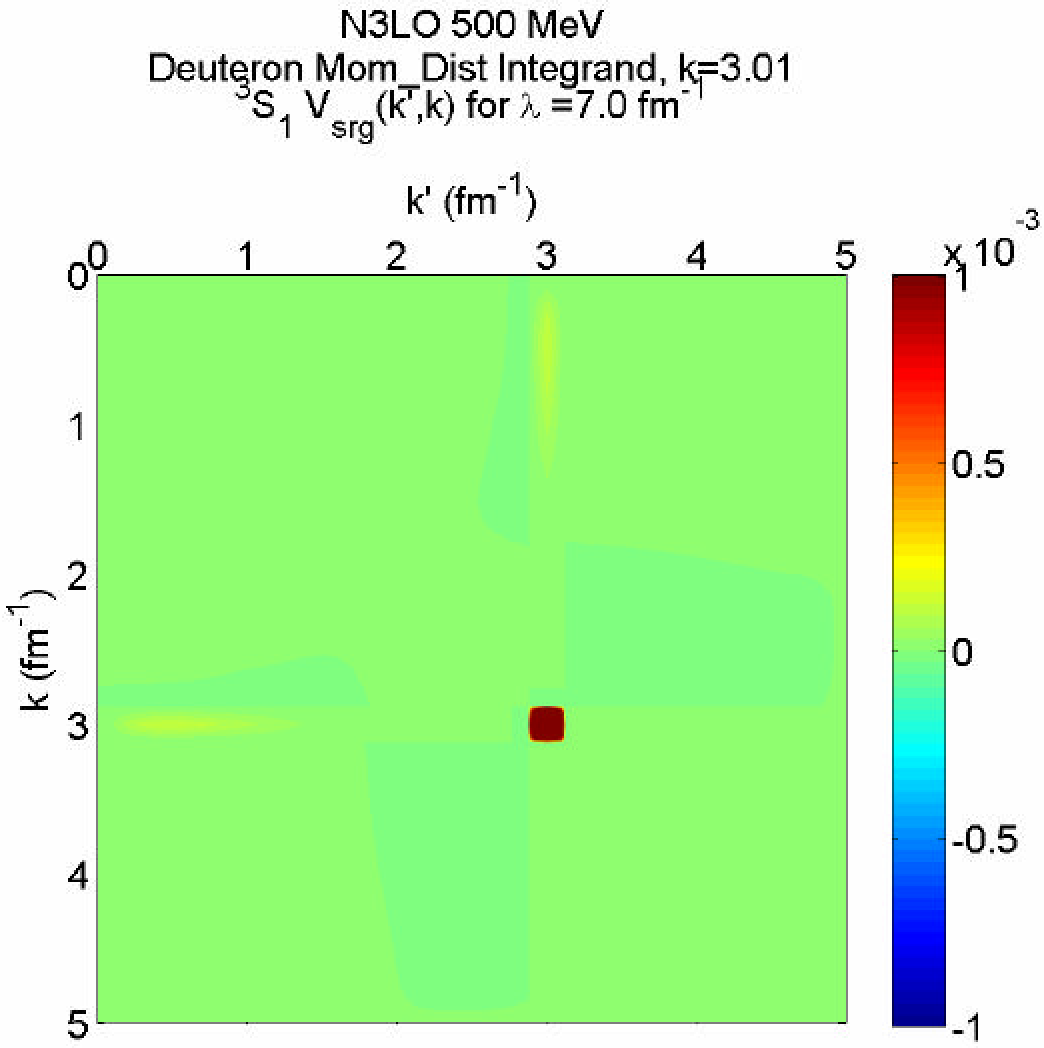
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ V_{srg}(k',k) for λ =7.4 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

N3LO 500 MeV

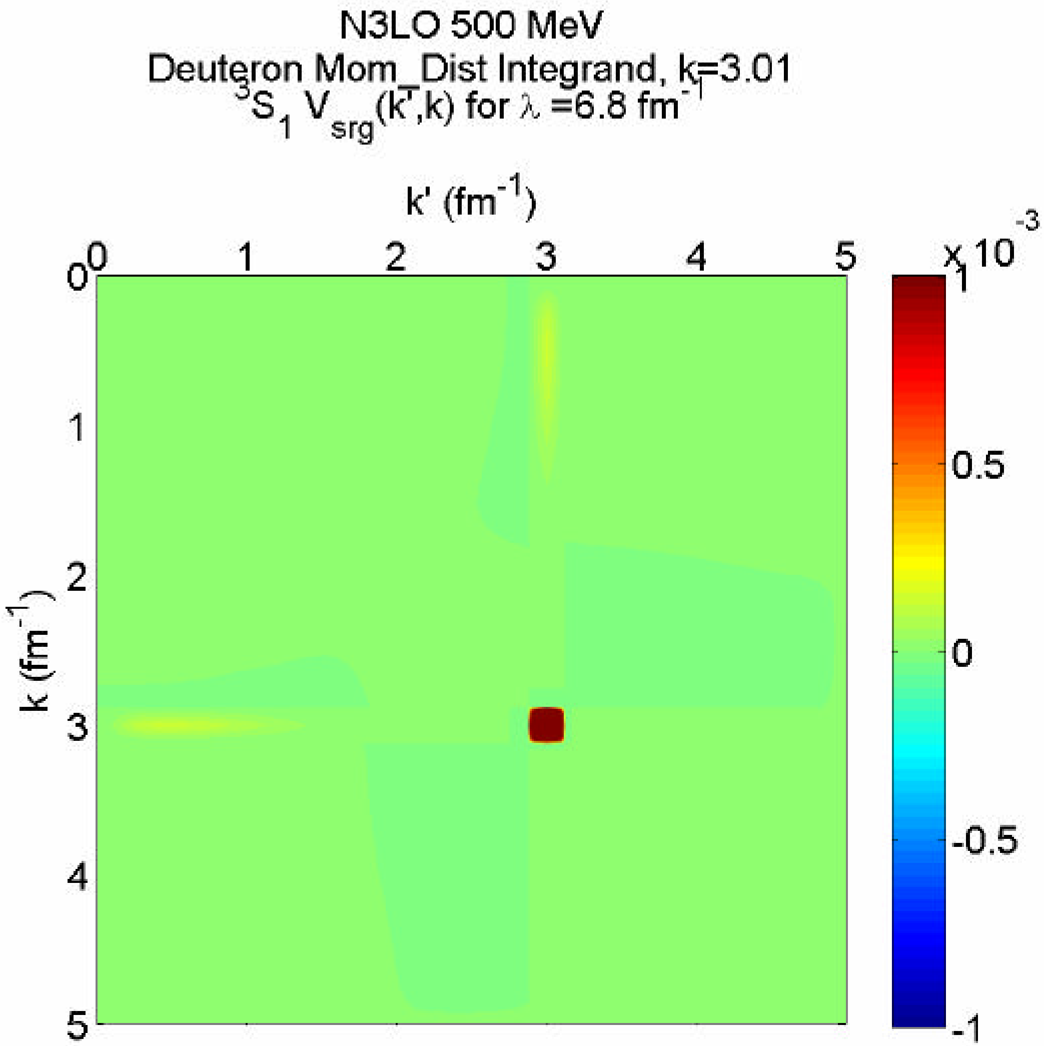
Deuteron Mom_Dist Integrand, k=3.01

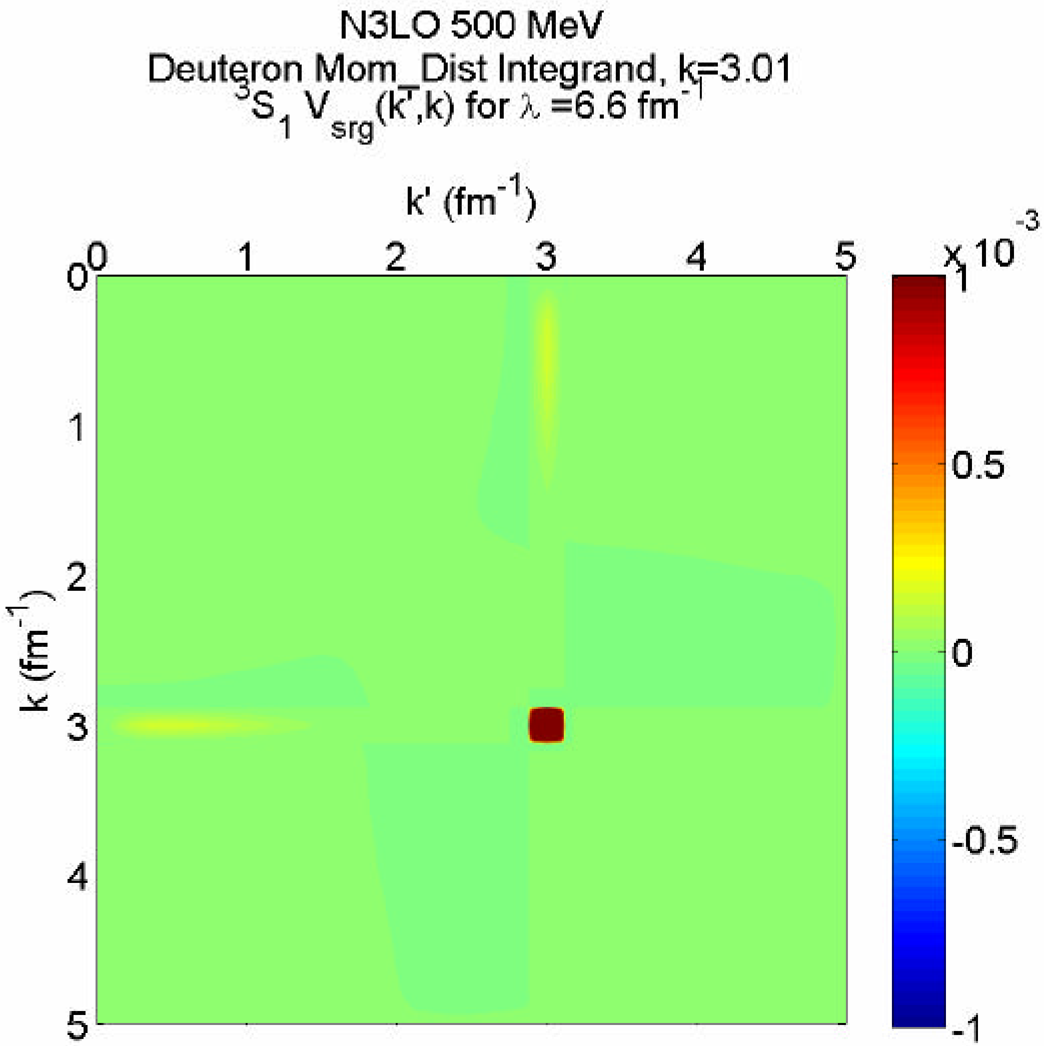
$^{3}S_{1}$ $V_{srg}(k',k)$ for λ =7.2 fm^{-1}

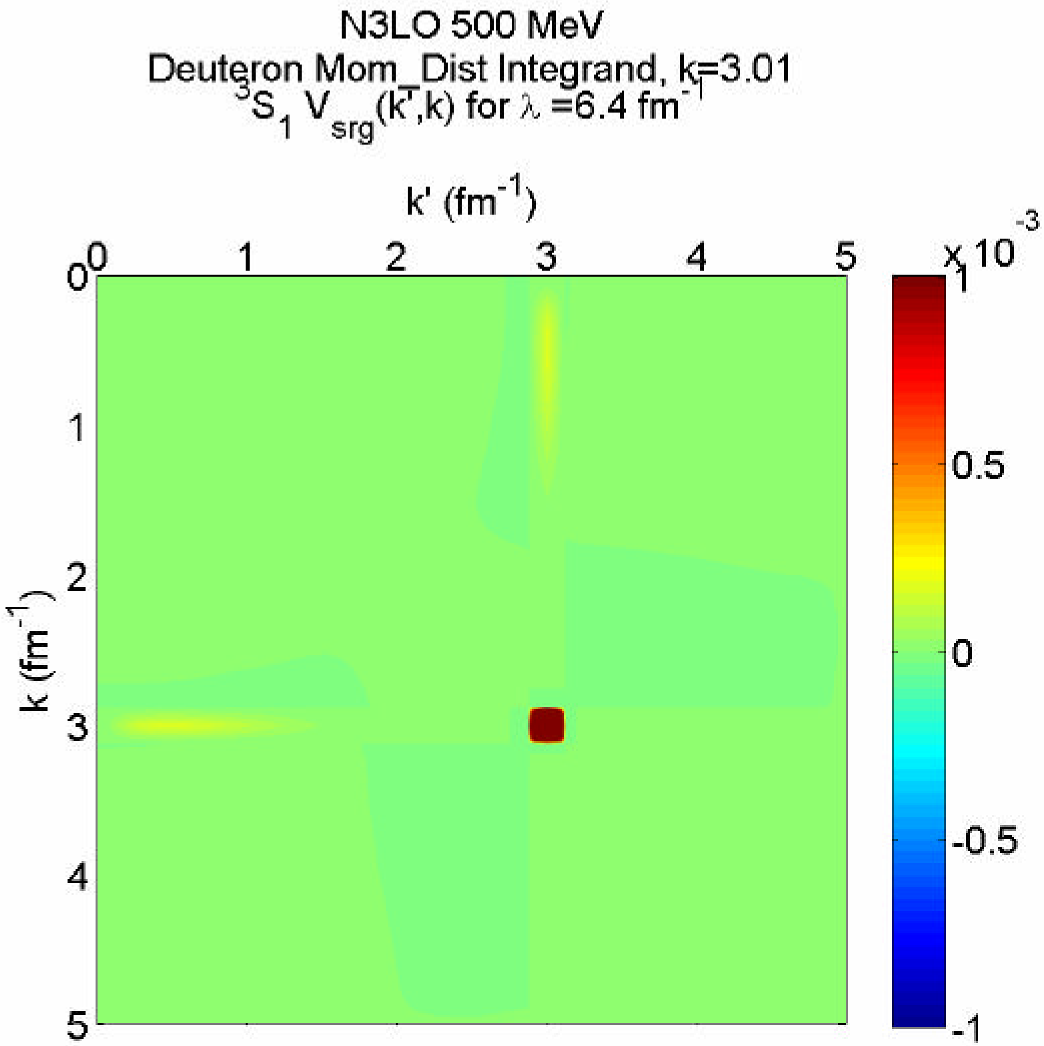




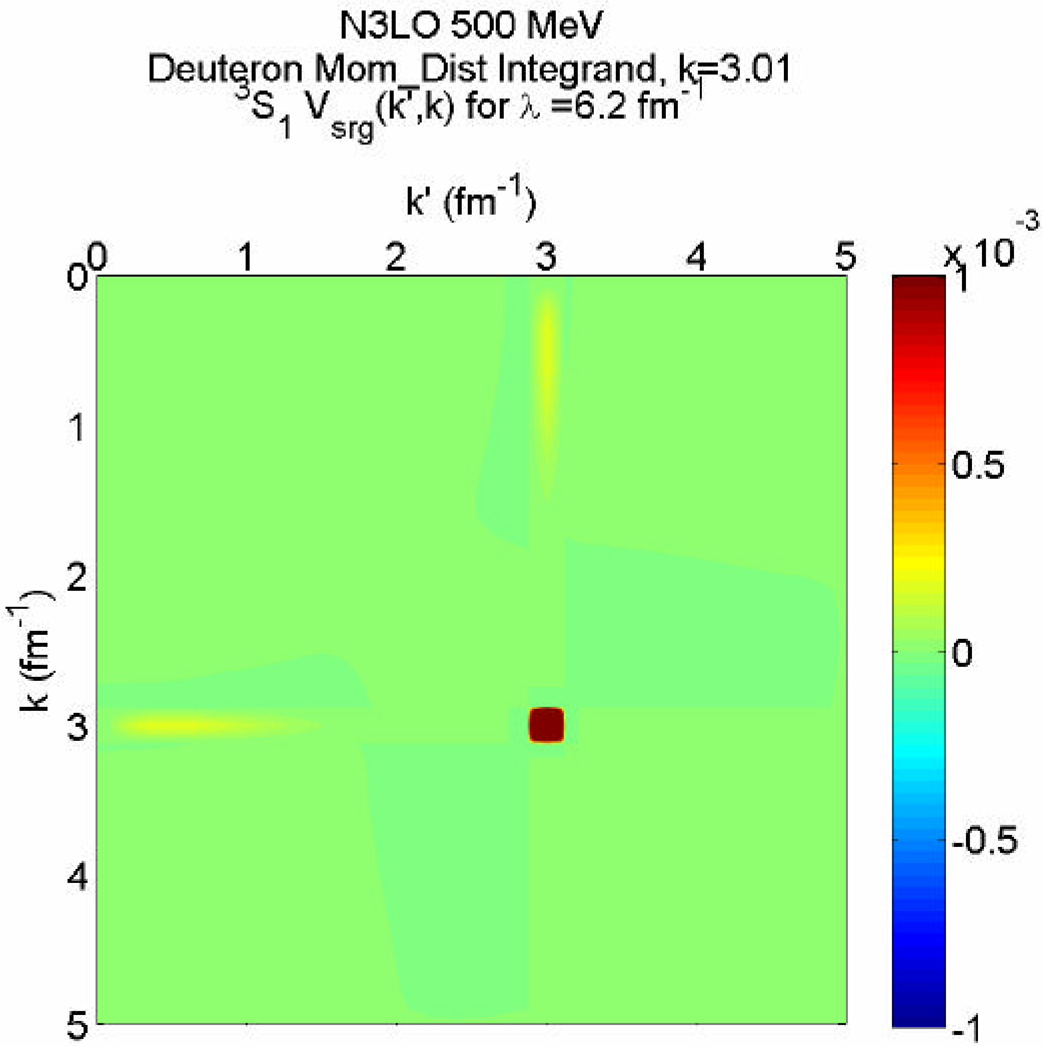
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =7.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

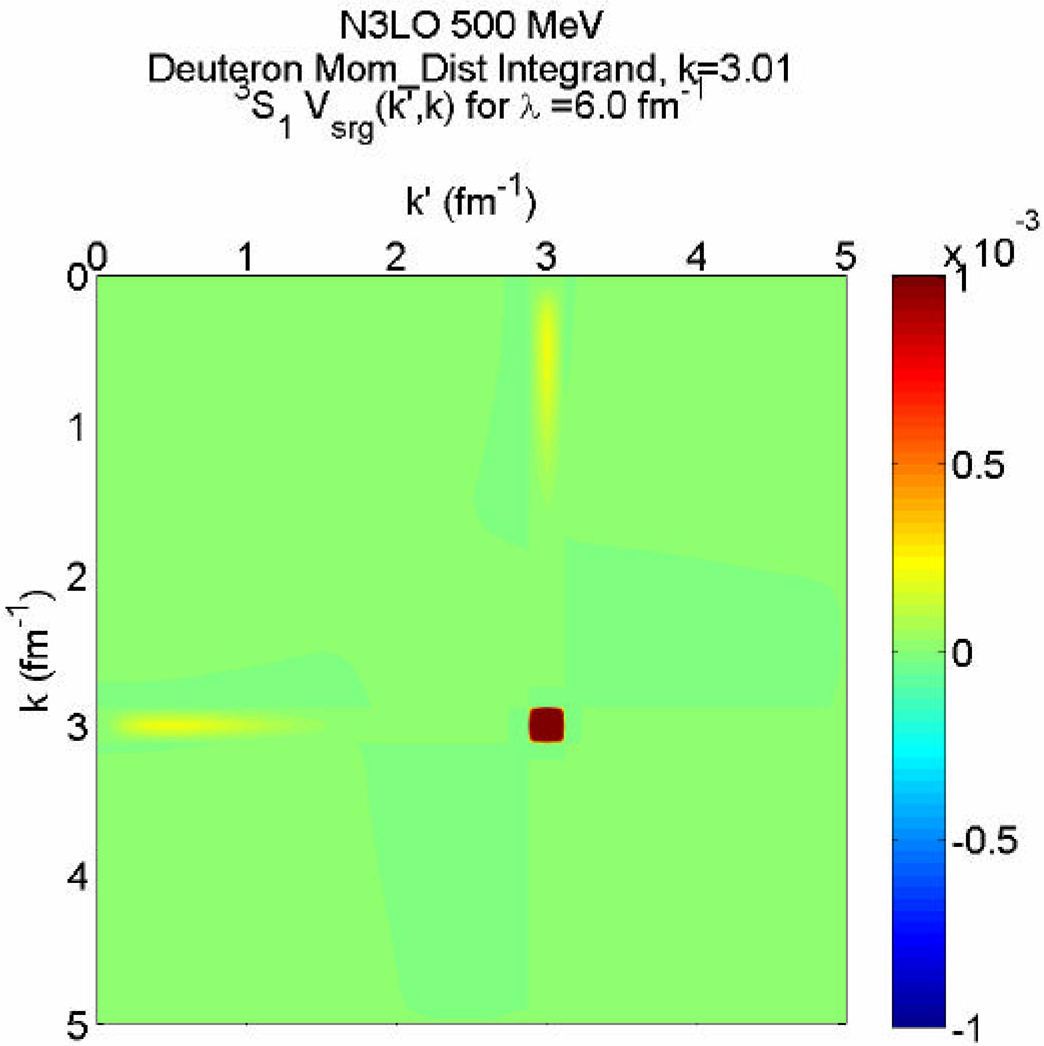

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
3S1 Vsrg(k',k) for λ =6.6 fm-1
k' (fm-1)
k (fm-1)
0
1
2
3
4
5
x 10-3
1
0.5
0
-0.5
-1



N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
1
0.5
0
-0.5
-1

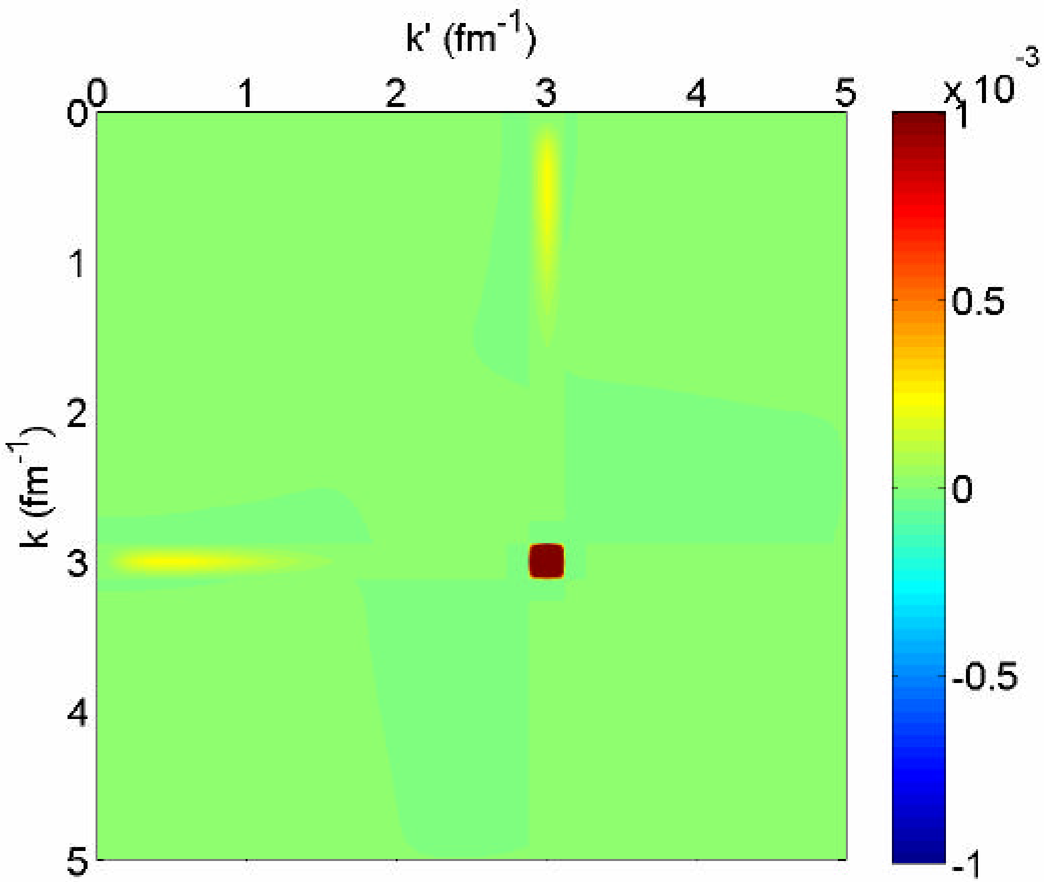


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
0.5
-0.5
-1



N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =6.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

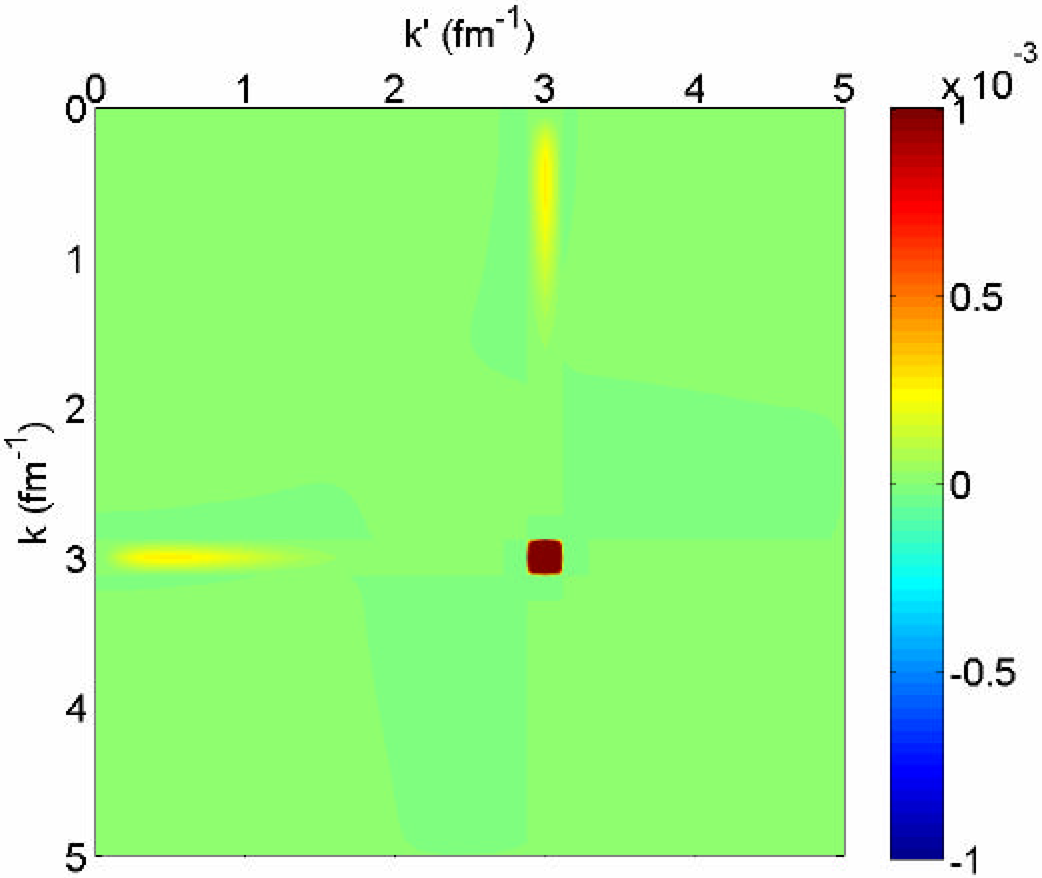
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =5.8 fm^{-1}

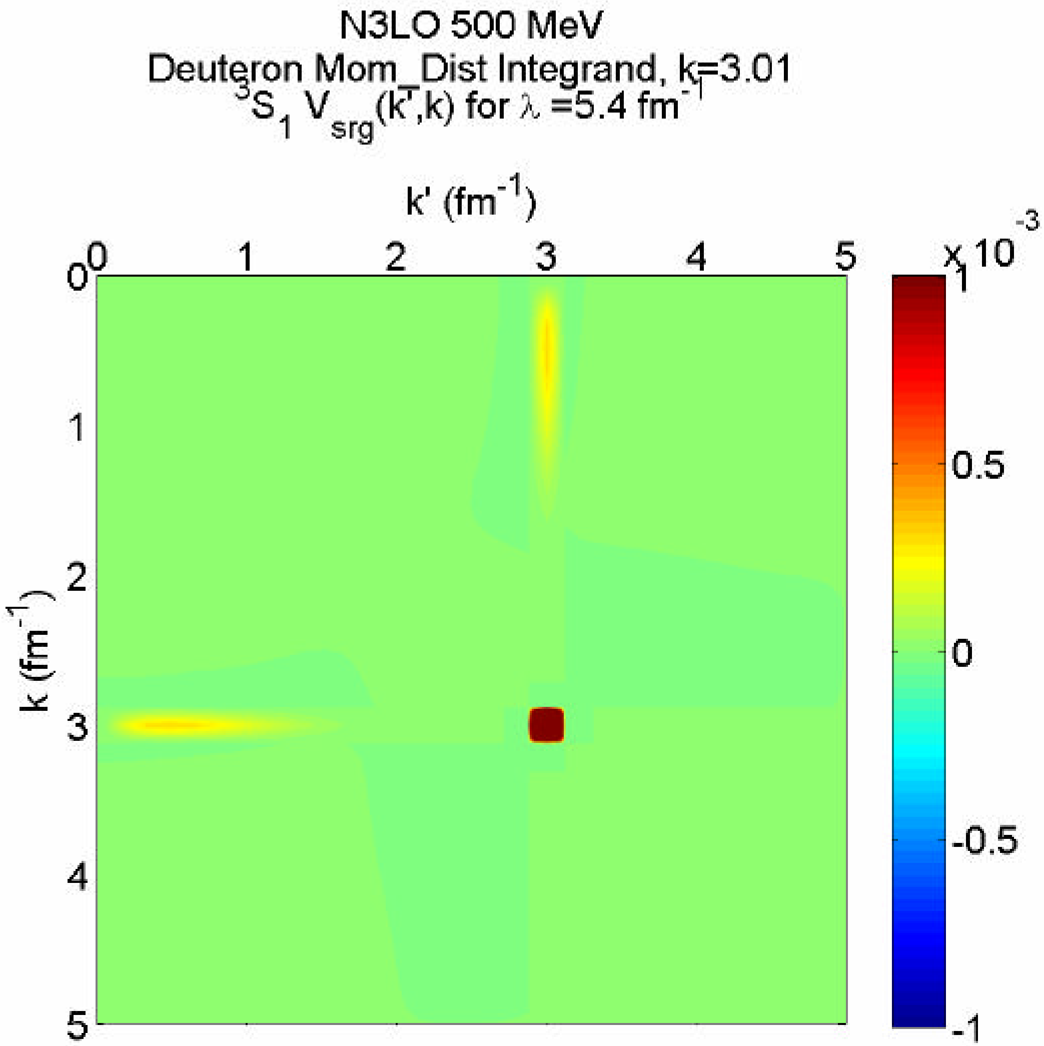


N3LO 500 MeV

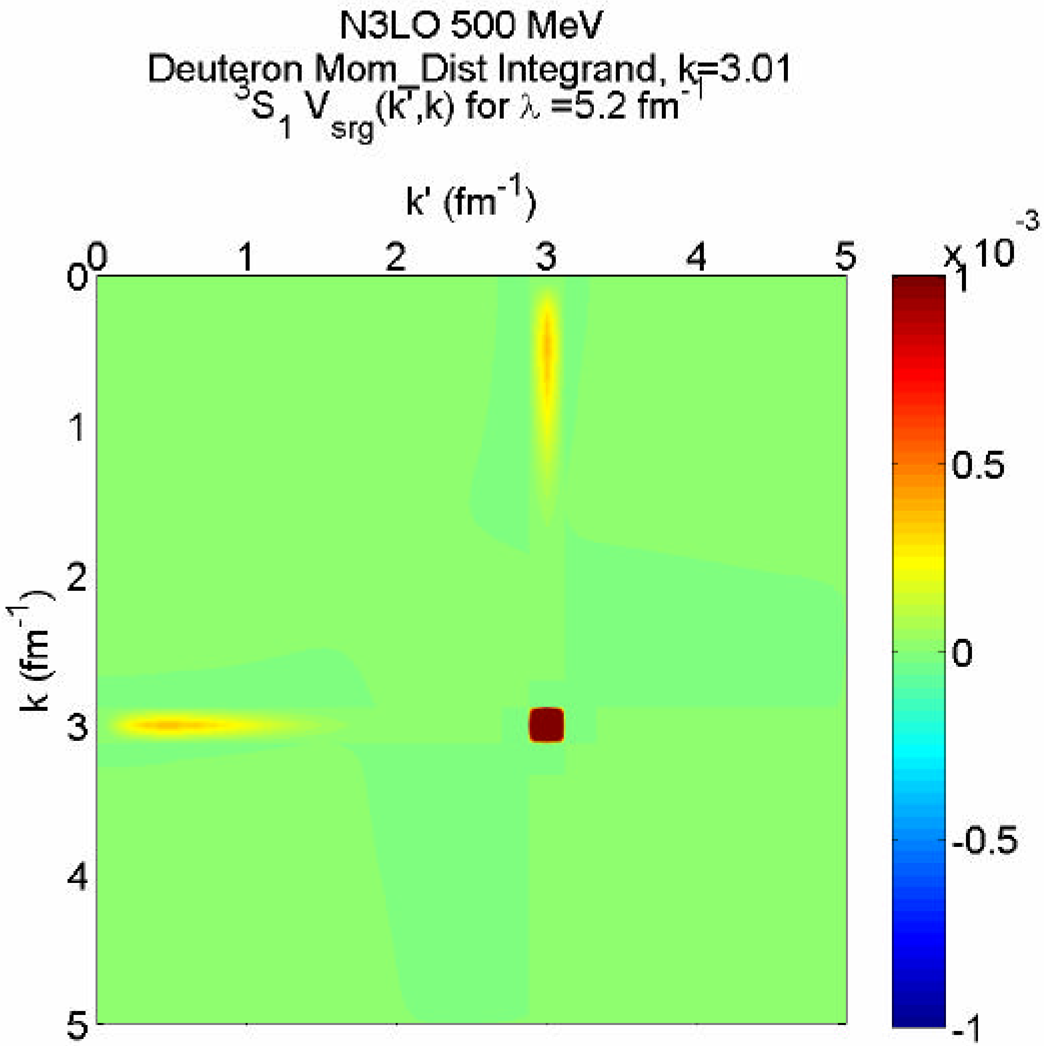
Deuteron Mom_Dist Integrand, k=3.01

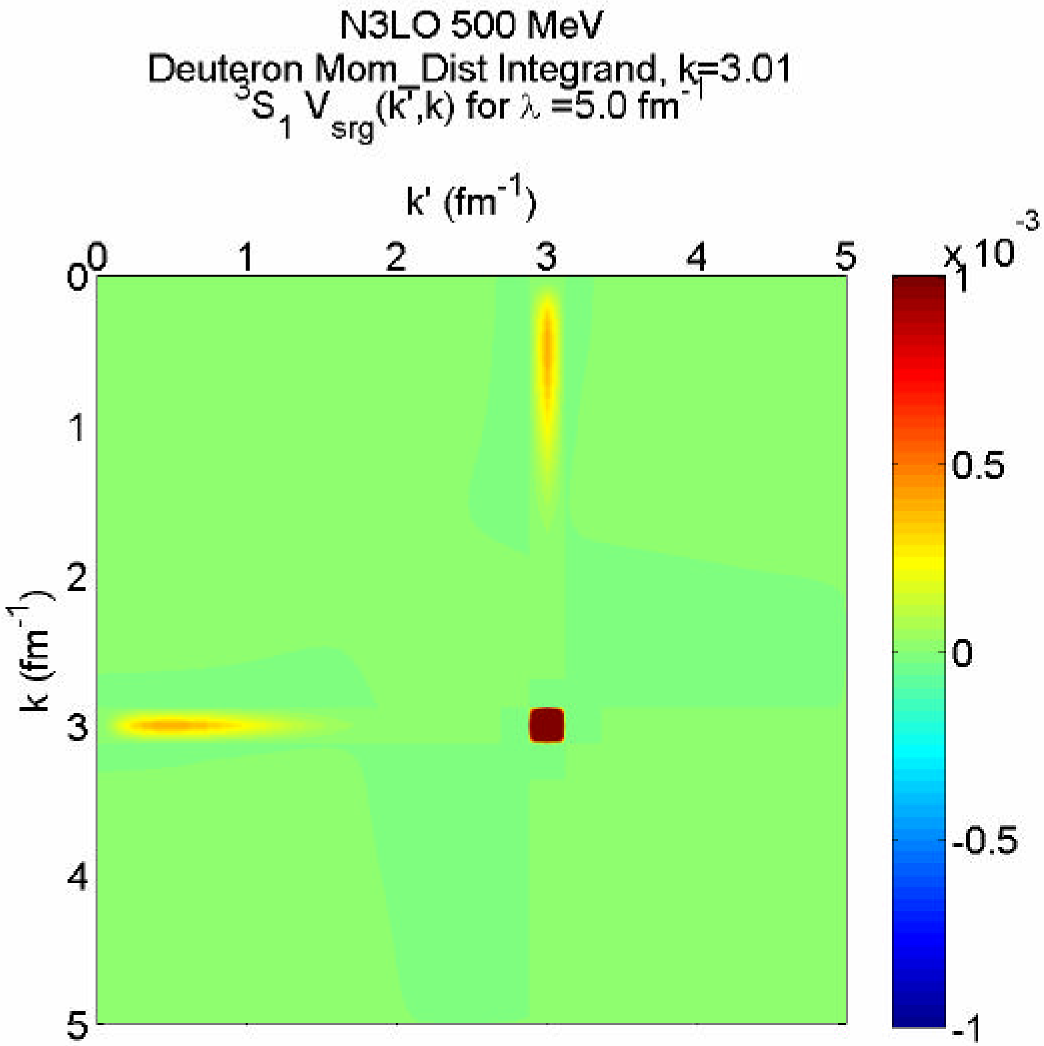
$^{3}S_{1}$ $V_{srg}(k',k)$ for λ =5.6 fm^{-1}



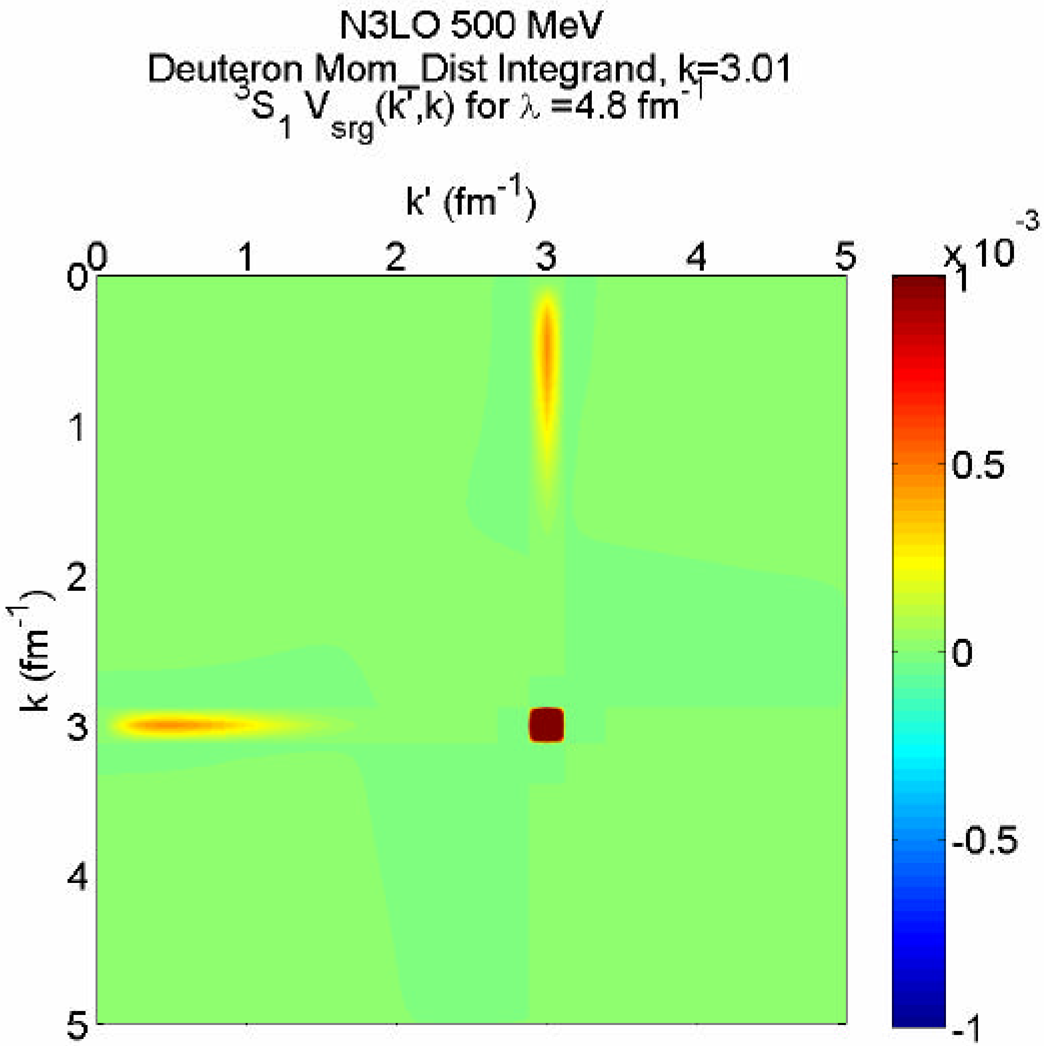


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =5.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

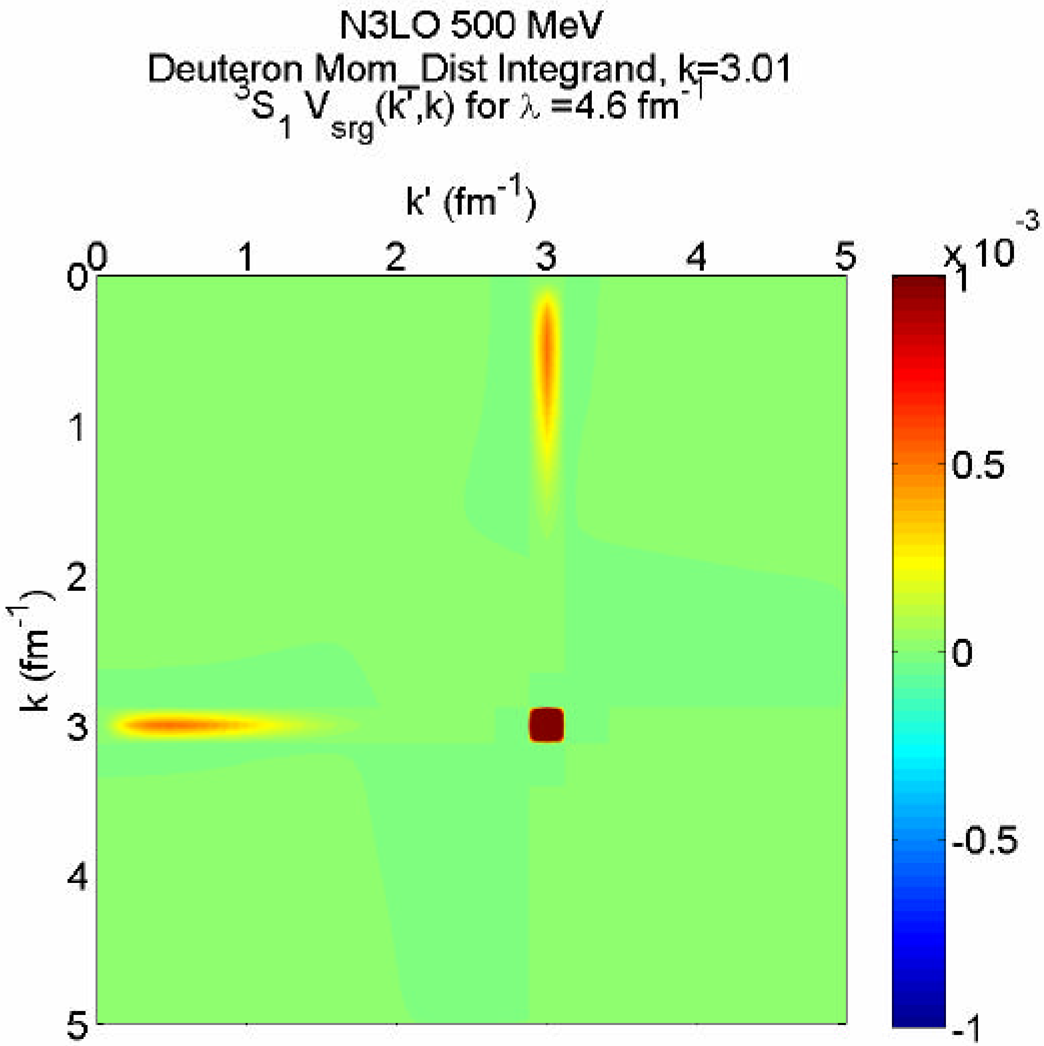

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =5.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

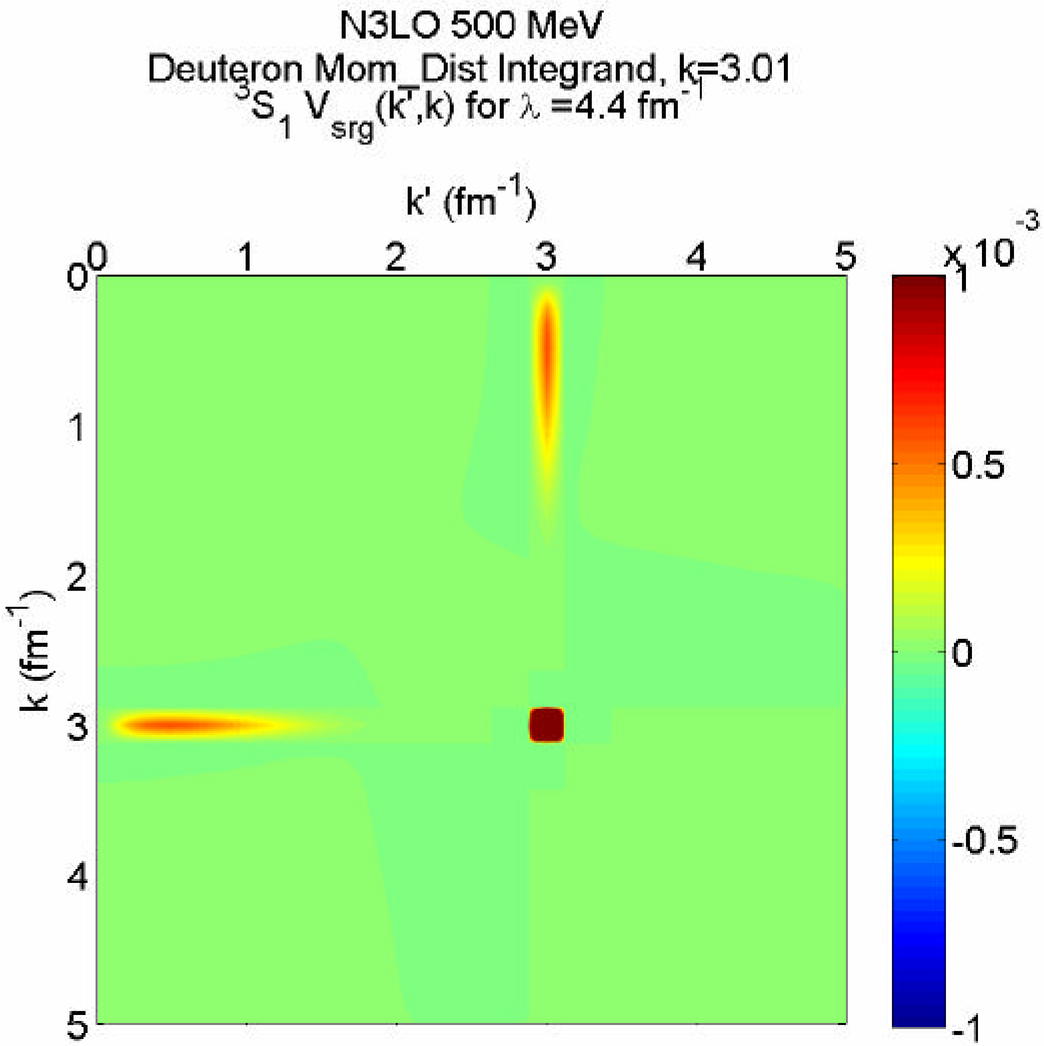


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =5.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

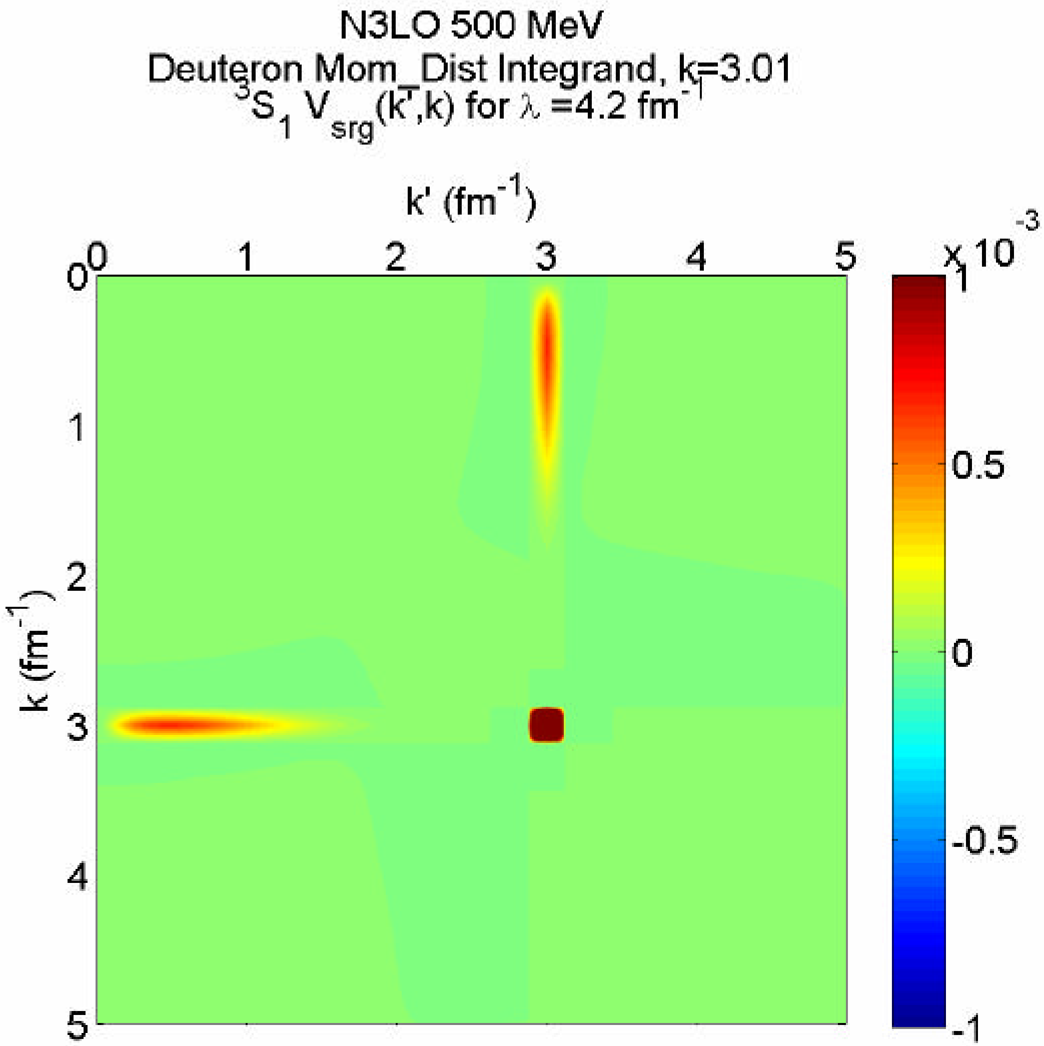


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ V_{srg}(k',k) for λ =4.8 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
x 10^{-3}
1
0.5
0
-0.5
-1

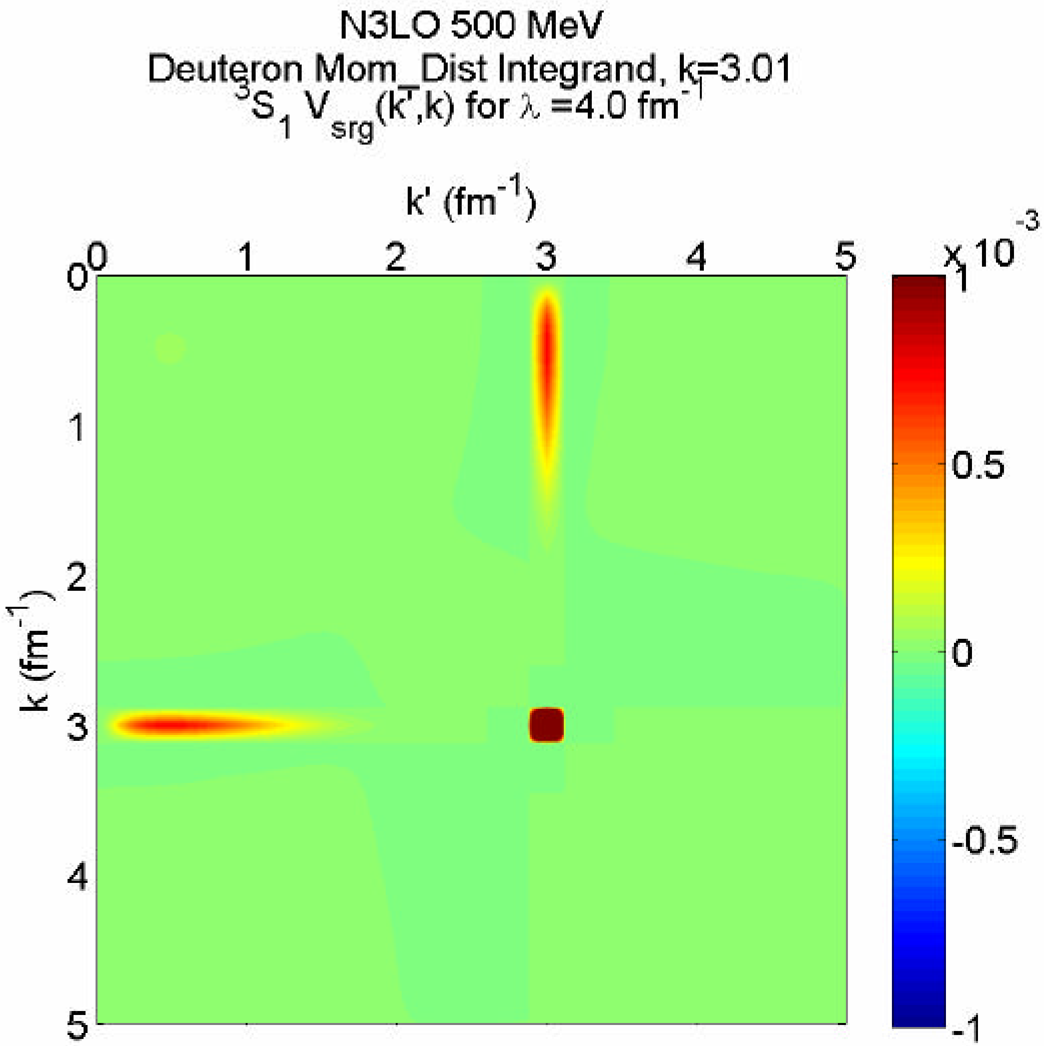

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
3S1 Vsrg(k',k) for λ =4.6 fm-1
k' (fm-1)
0
1
2
3
4
5
k (fm-1)
0
1
2
3
4
5
x 10-3
1
0.5
0
-0.5
-1

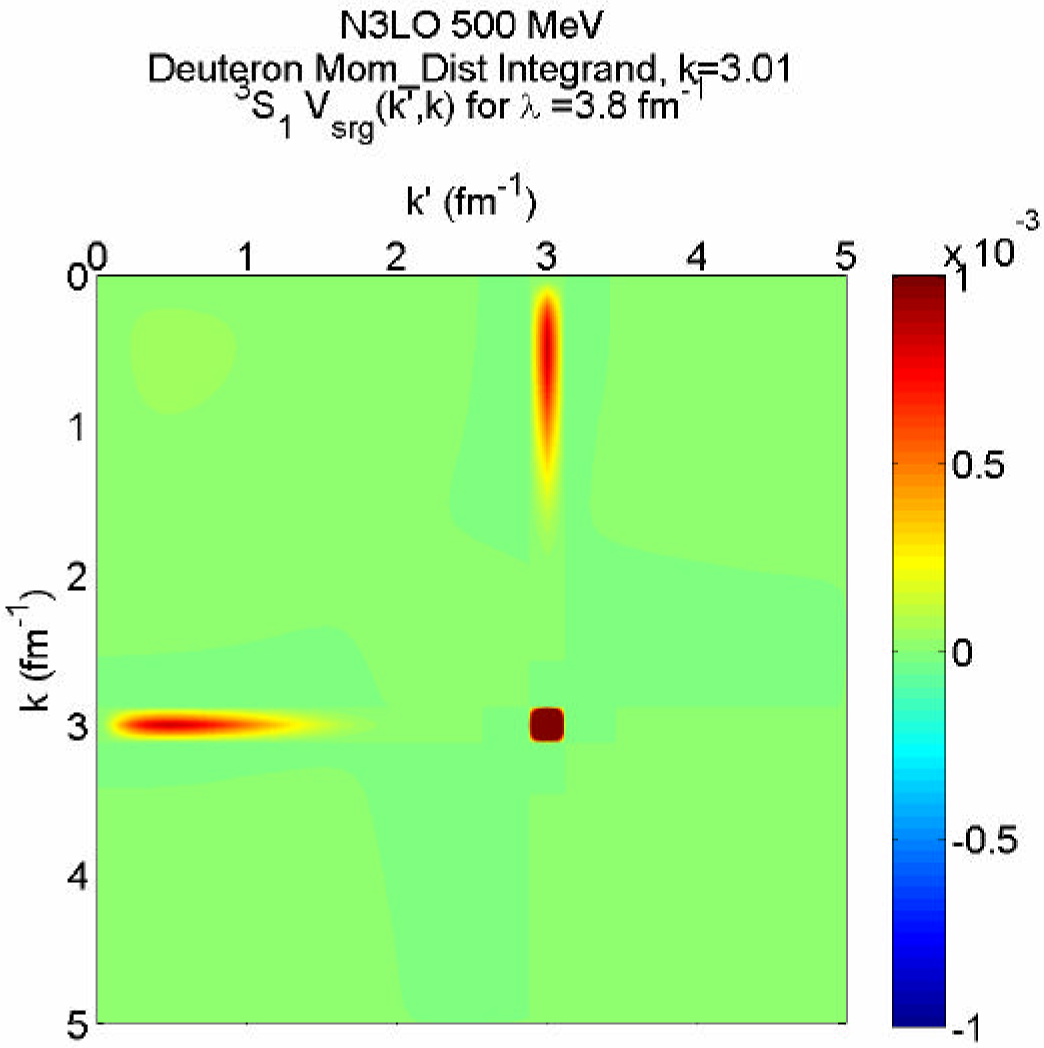


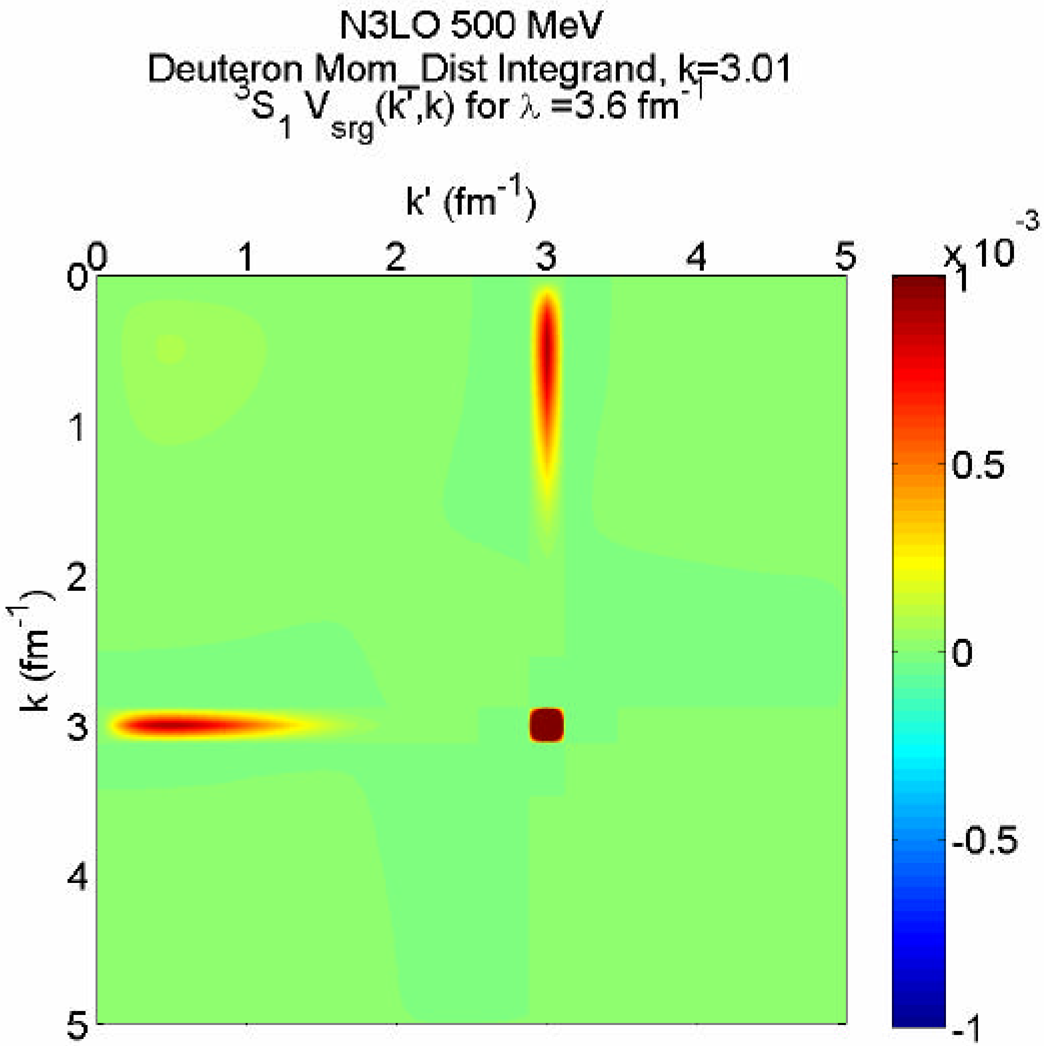
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.4 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



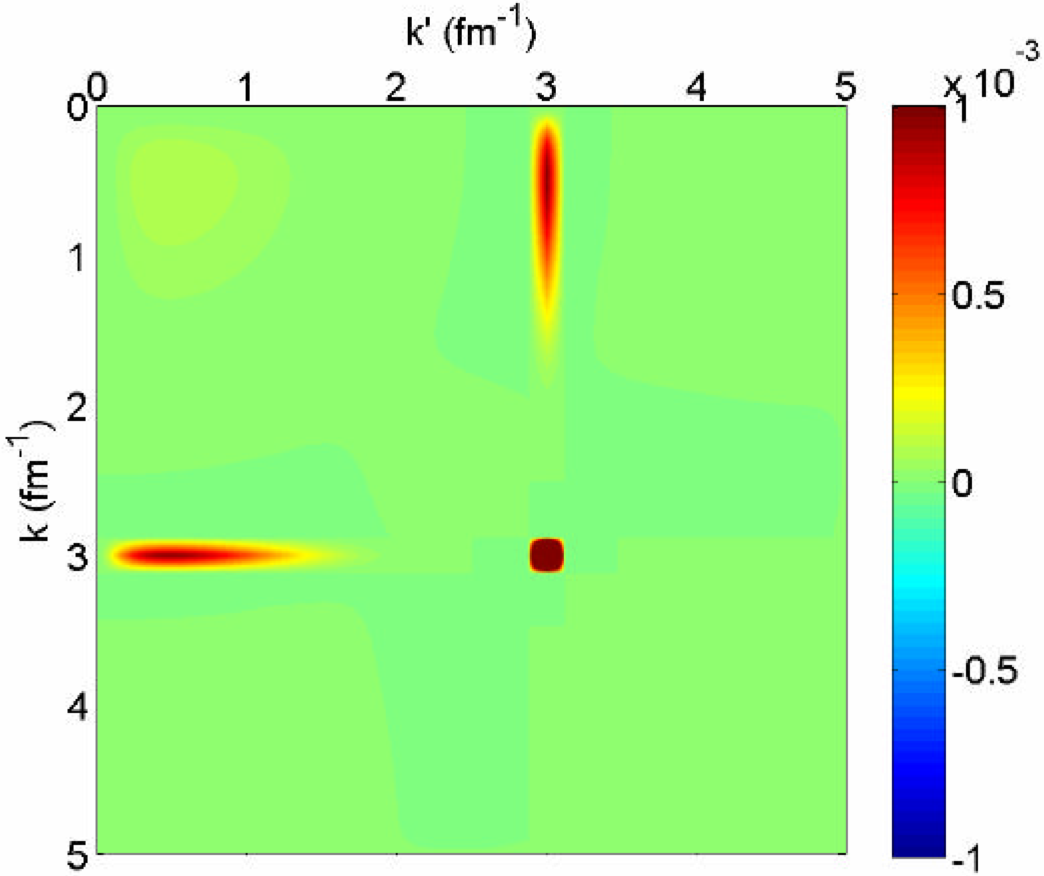
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}

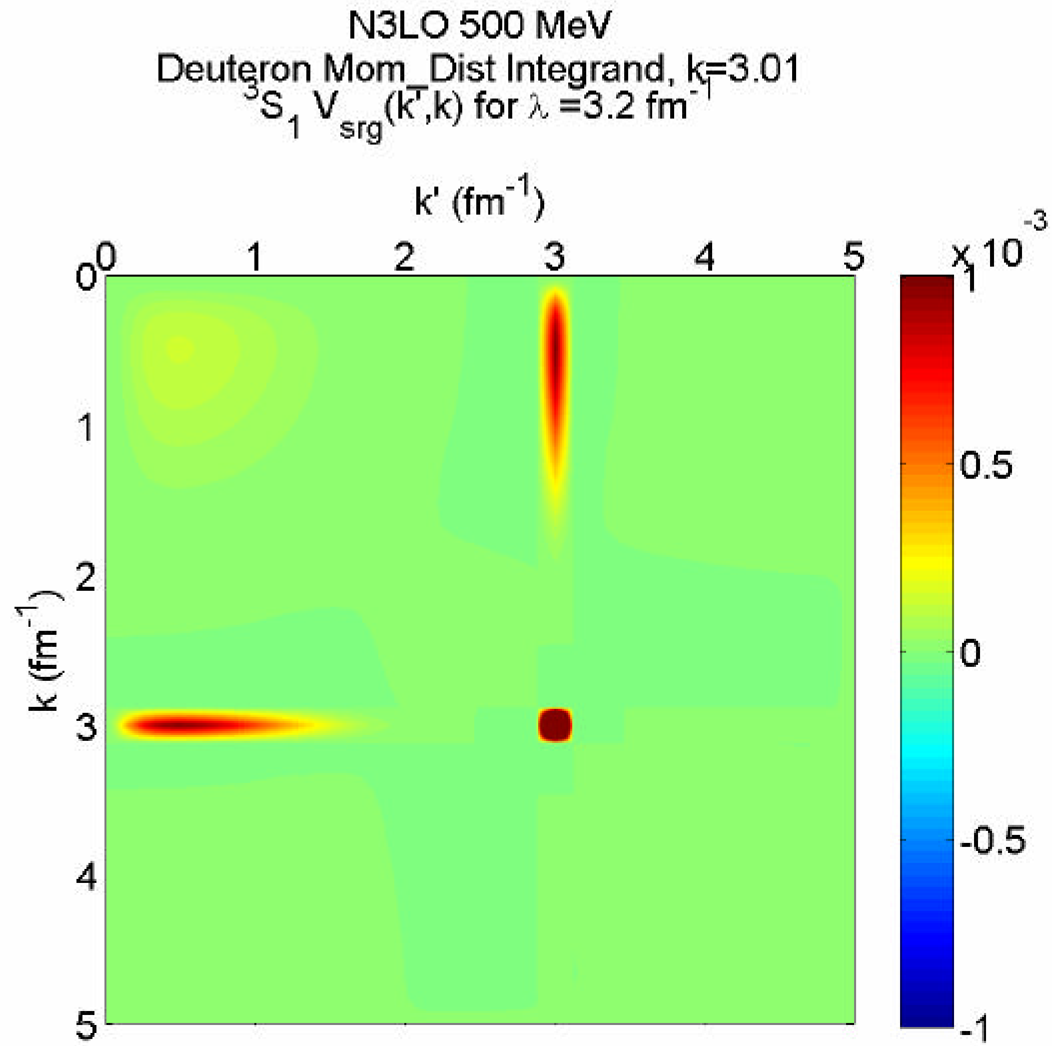

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =4.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
0
1
2
3
4
5
1
0.5
0
-0.5
-1

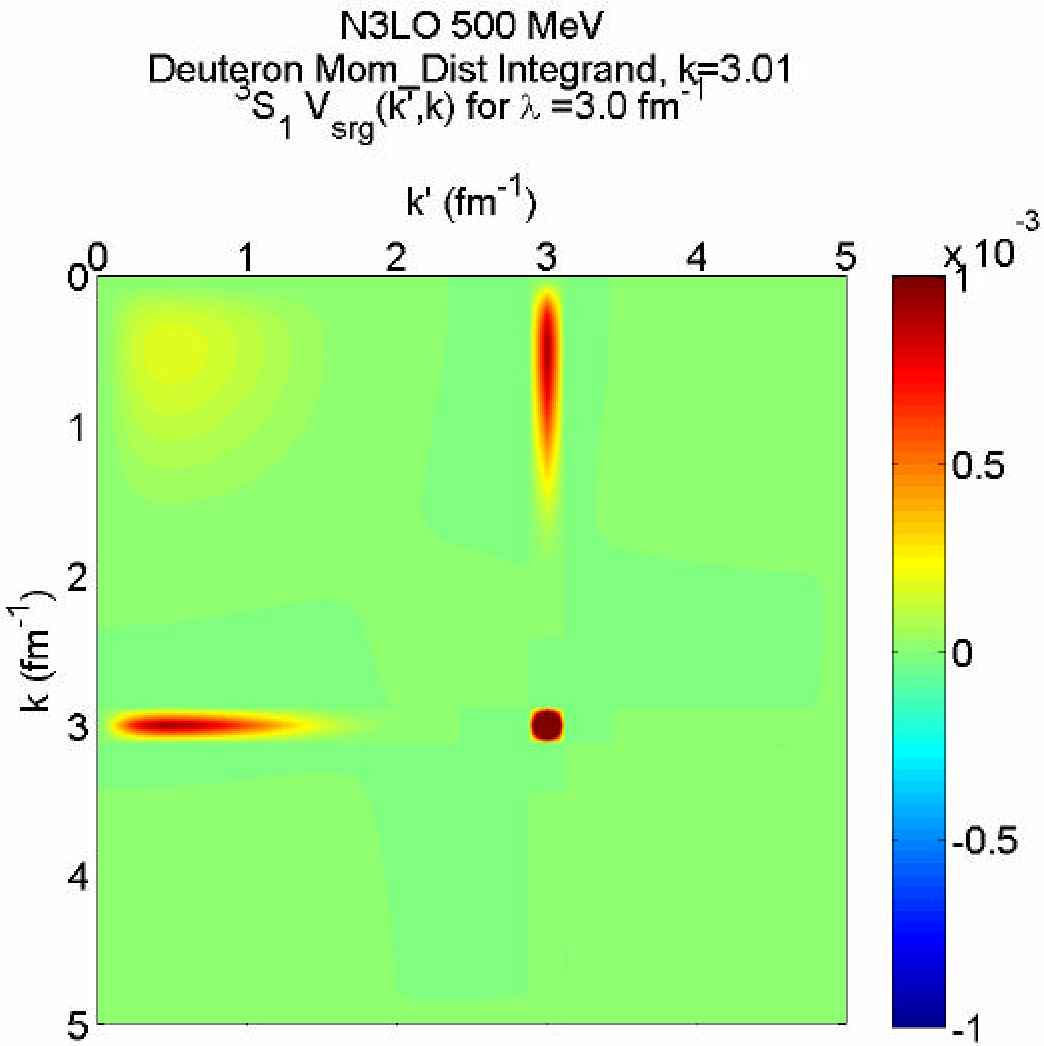

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

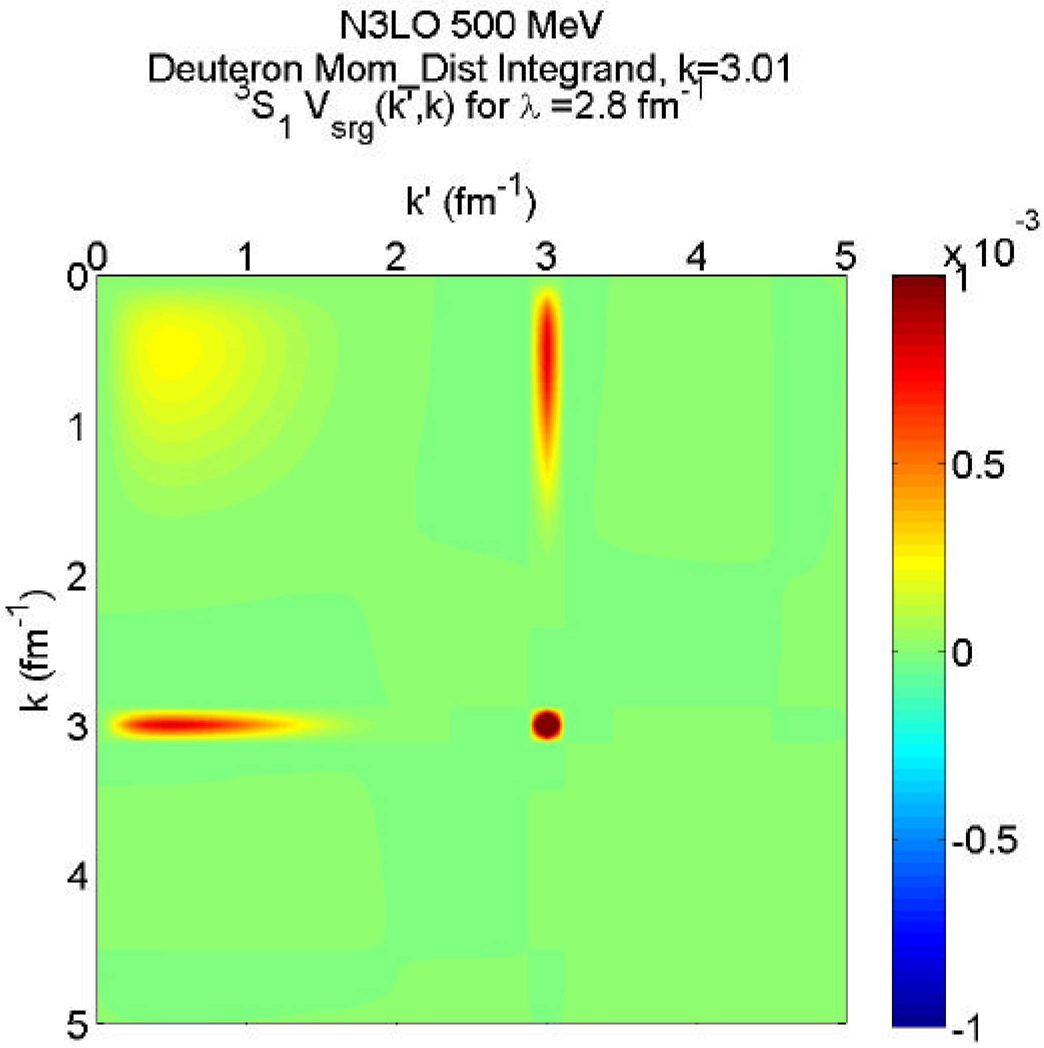
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.4 fm^{-1}




N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.2 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
0
1
2
3
4
5
$\times 10^{-3}$
1
0.5
0
-0.5
-1



N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =3.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}

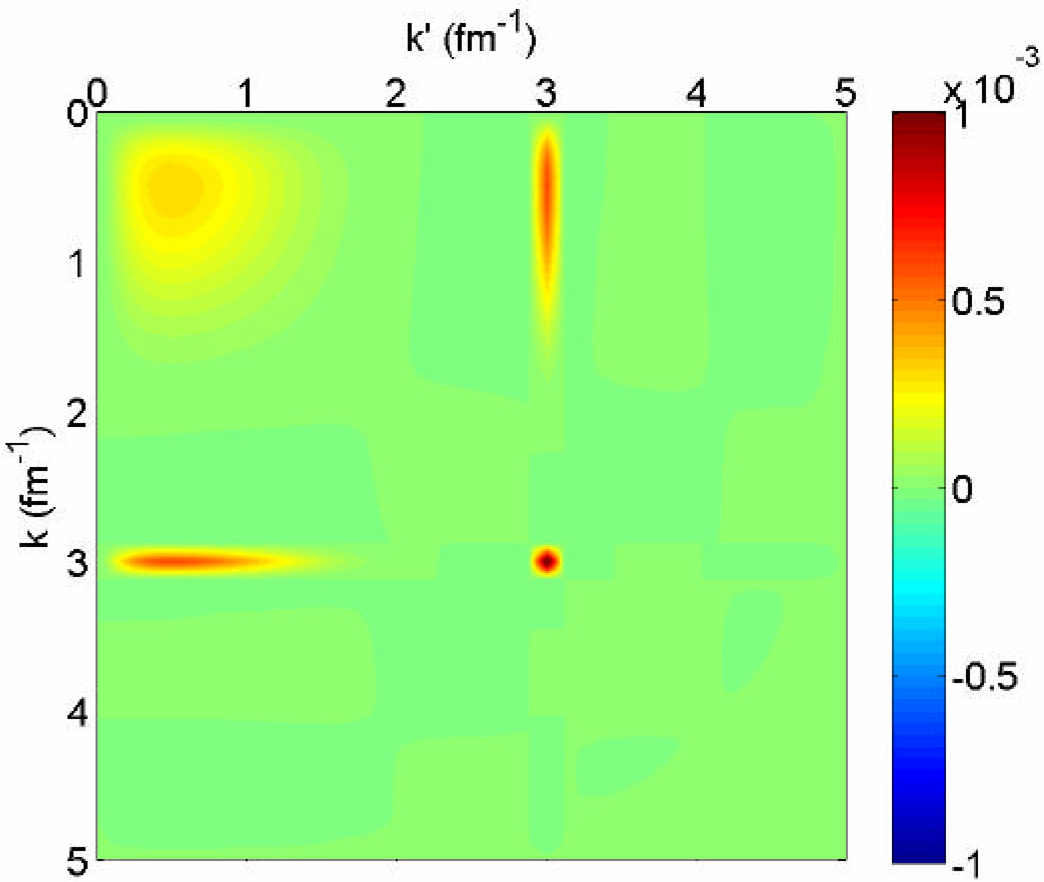


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.8 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

N3LO 500 MeV

Deuteron Mom_Dist Integrand, k=3.01

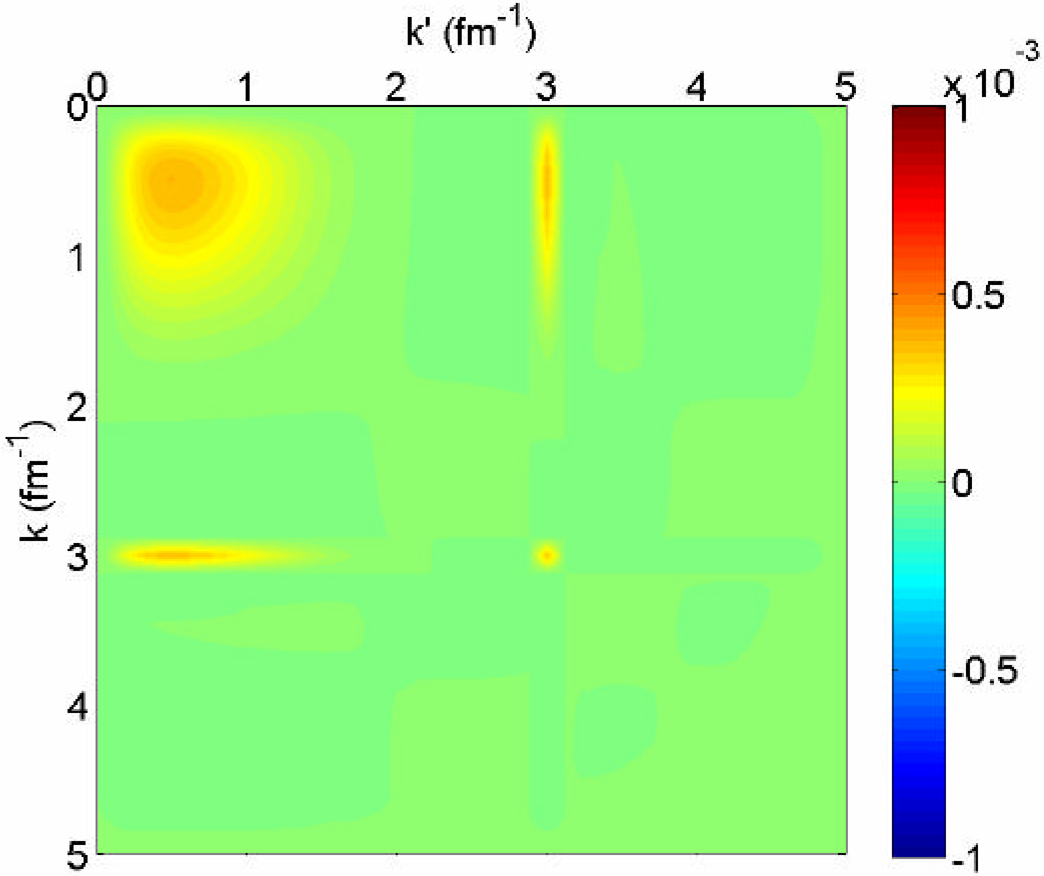
$^3S_1$ $V_{srg}(k',k)$ for λ =2.6 fm^{-1}

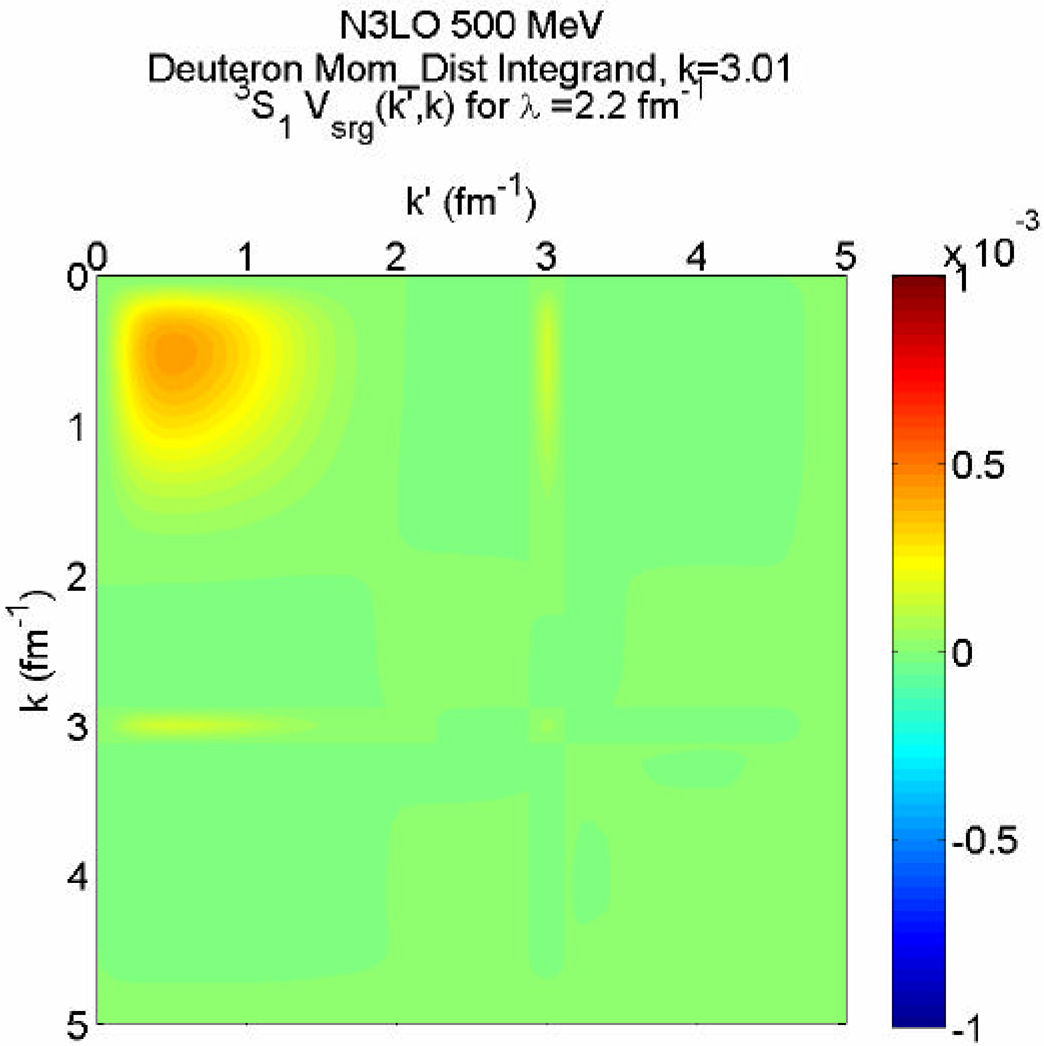


N3LO 500 MeV

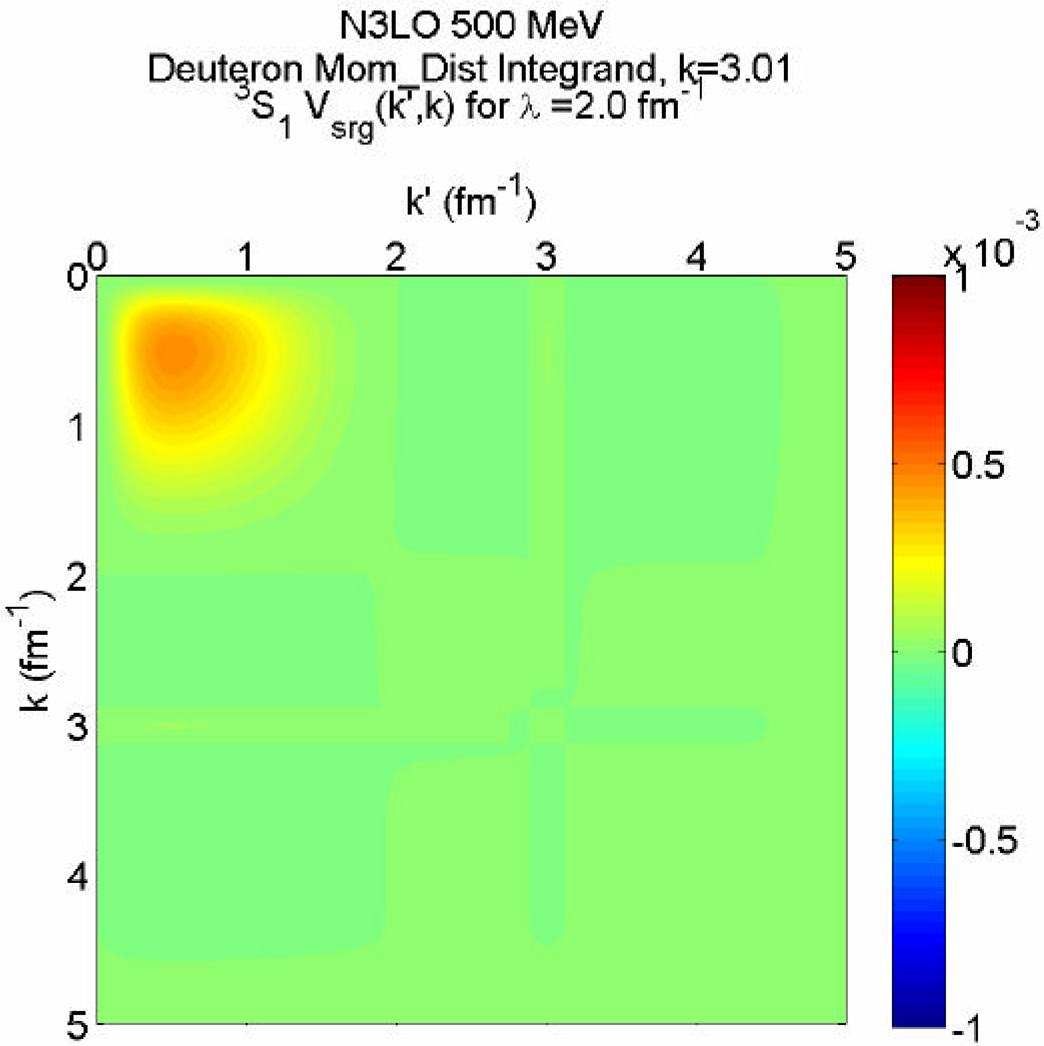
Deuteron Mom_Dist Integrand, k=3.01

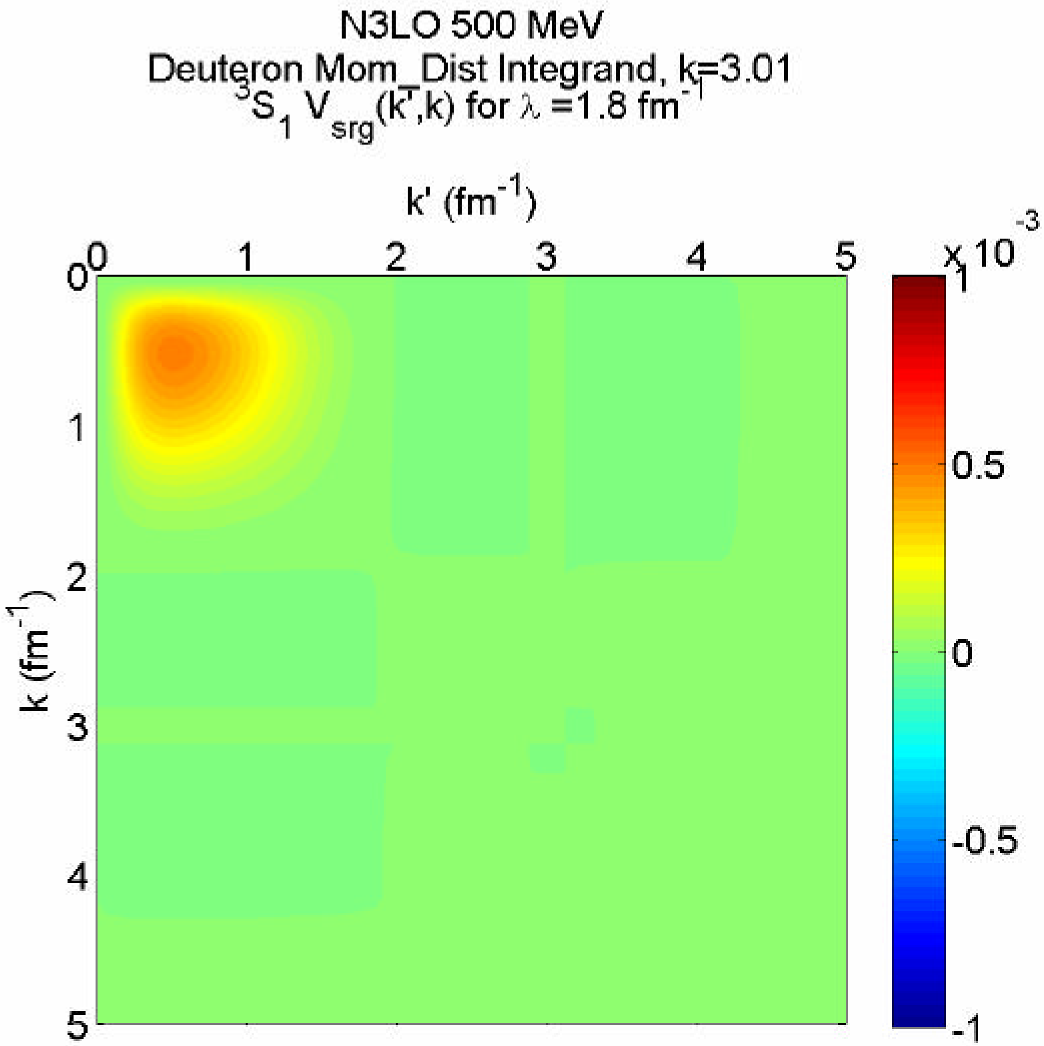
$^3S_1$ $V_{srg}(k',k)$ for λ =2.4 fm^{-1}



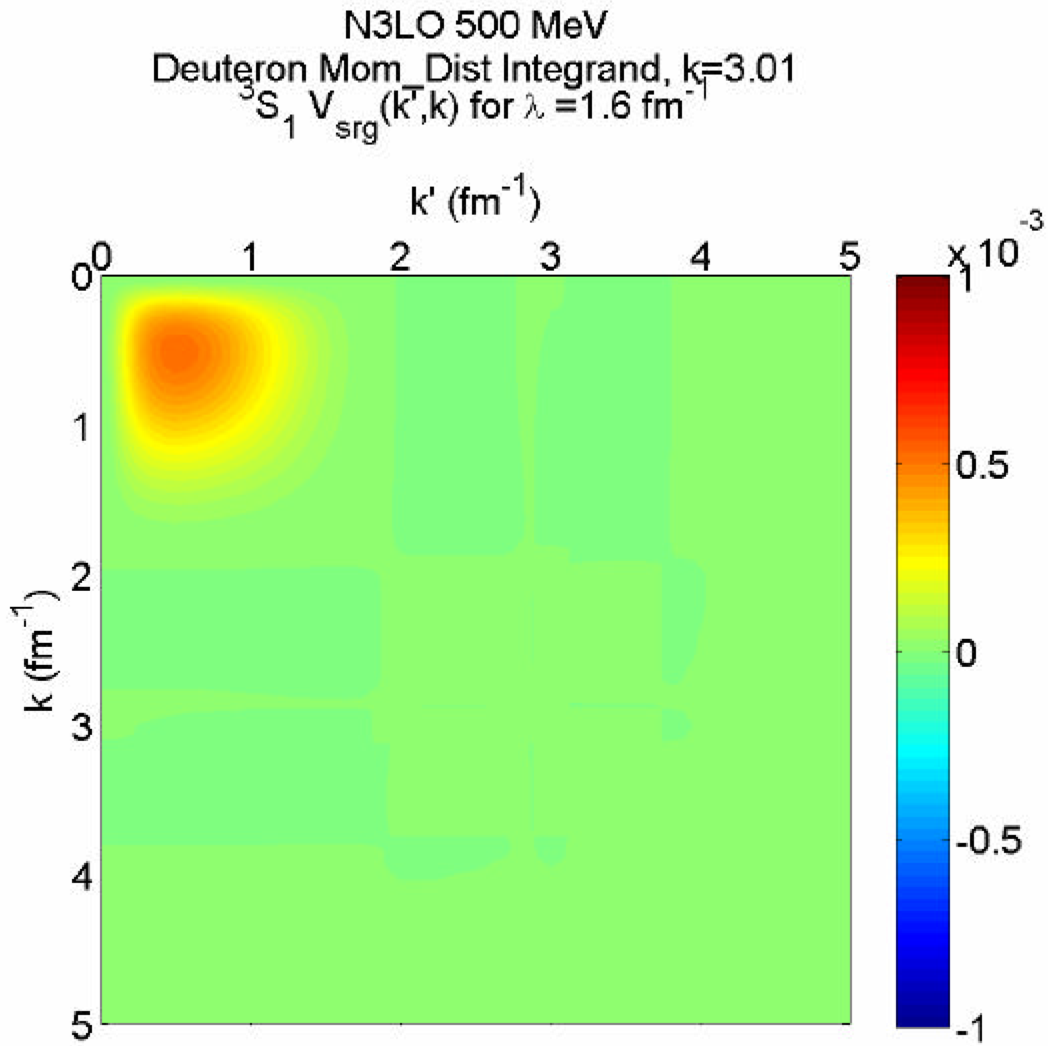


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.2 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

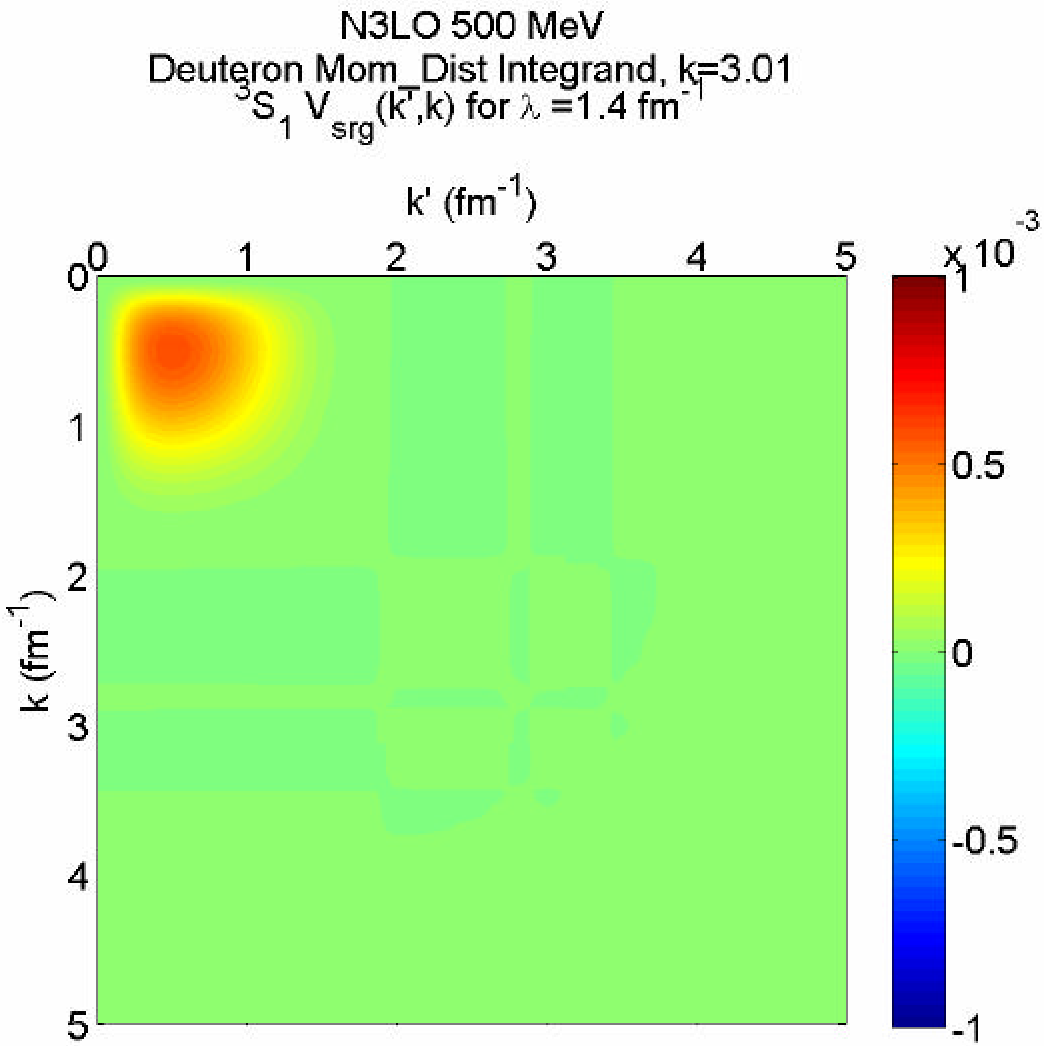

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =2.0 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1



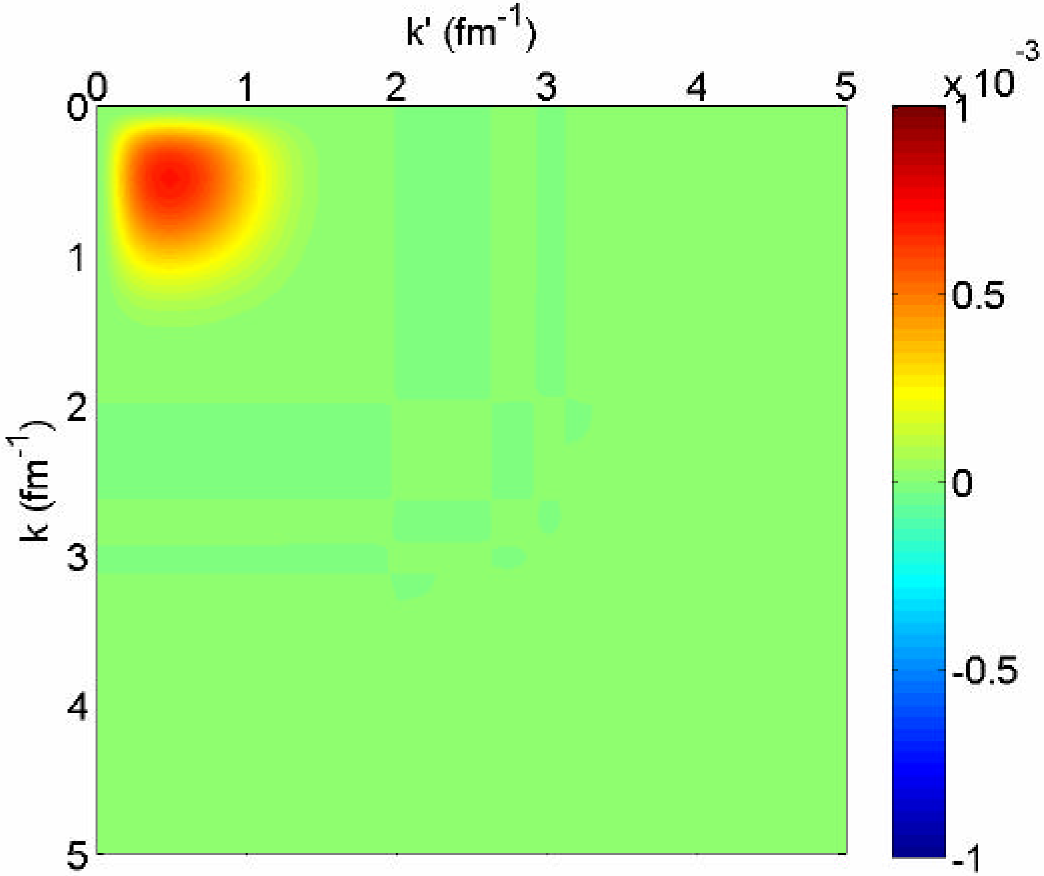
N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
³S₁ V_srg(k',k) for λ =1.8 fm⁻¹
k' (fm⁻¹)
k (fm⁻¹)
0
1
2
3
4
5
x 10⁻³
1
0.5
0
-0.5
-1

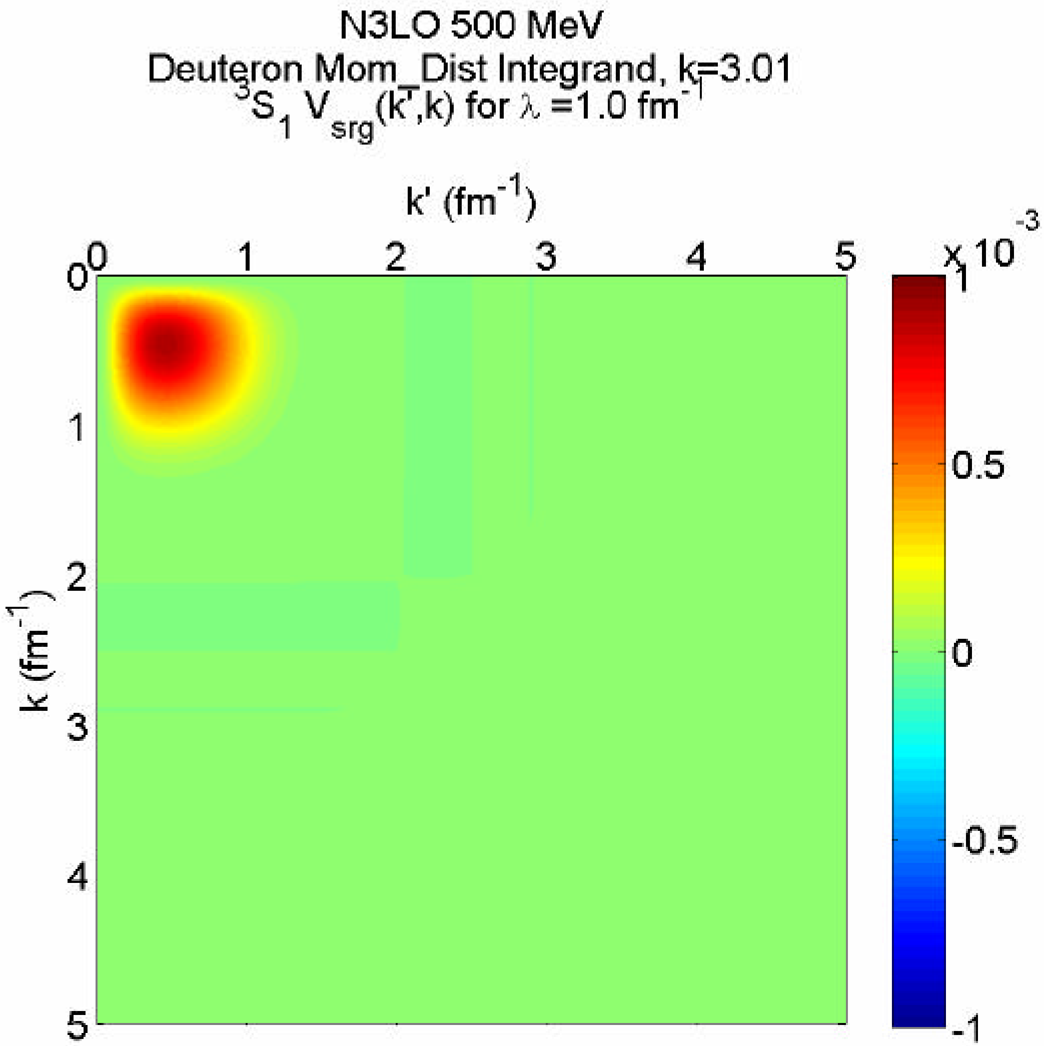


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =1.6 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
x 10^{-3}
1
0.5
0
-0.5
-1
0
1
2
3
4
5


N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =1.4 fm^{-1}
k' (fm^{-1})
k (fm^{-1})
0
1
2
3
4
5
x 10^{-3}
1
0.5
0
-0.5
-1

N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =1.2 fm^{-1}





N3LO 500 MeV
Deuteron Mom_Dist Integrand, k=3.01
$^3S_1$ $V_{srg}(k',k)$ for λ =1.0 fm^{-1}
k' (fm^{-1})
0
1
2
3
4
5
k (fm^{-1})
x 10^{-3}
1
0.5
0
-0.5
-1

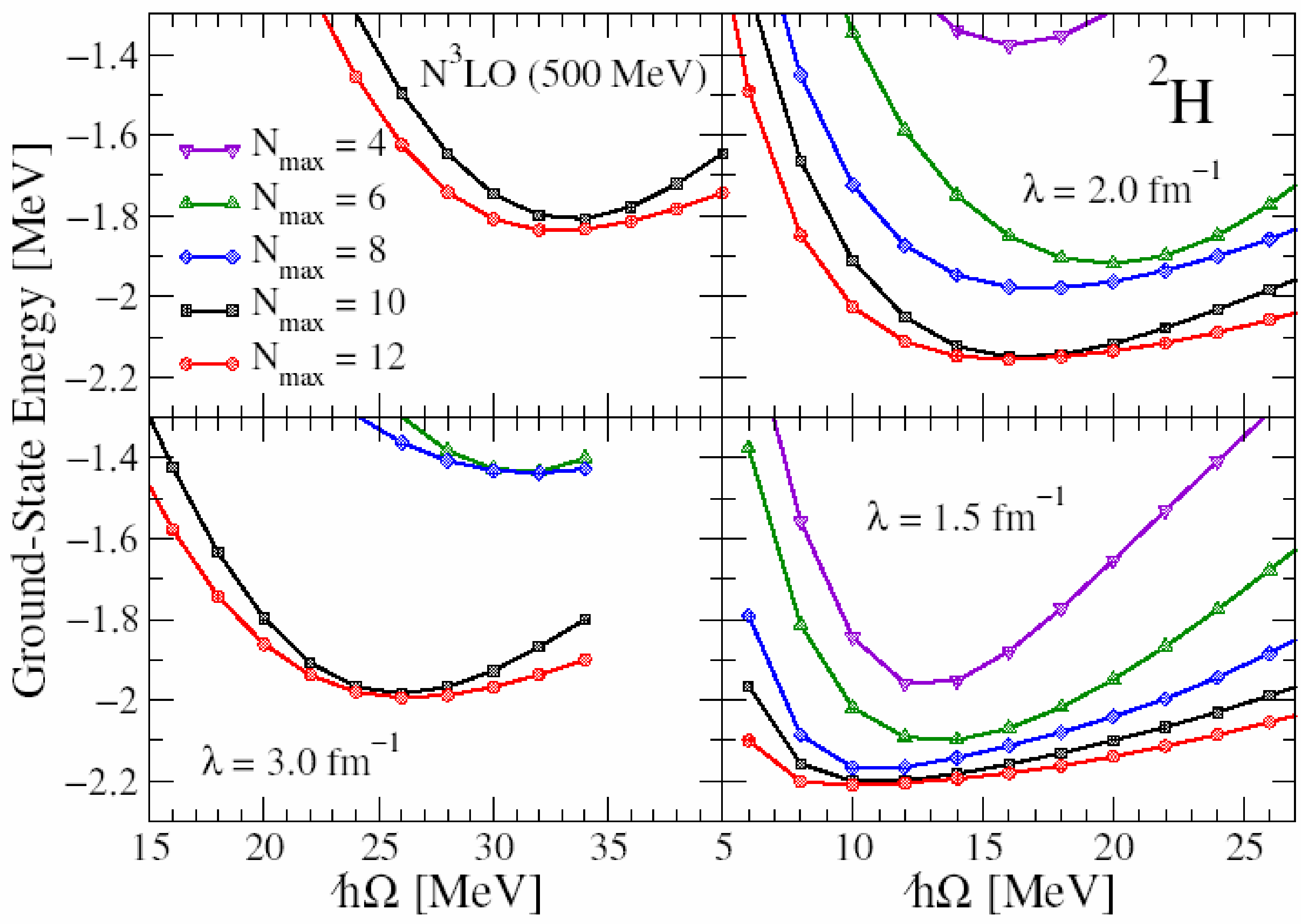
Observations - Recap

- Potentials are indeed evolving towards diagonalized form
  - --> The coupling of high and low momentum matrix elements is being eliminated
- Operator flow remains smooth as they are evolved
- Deuteron integrand flow of probability is toward low momentum in new space

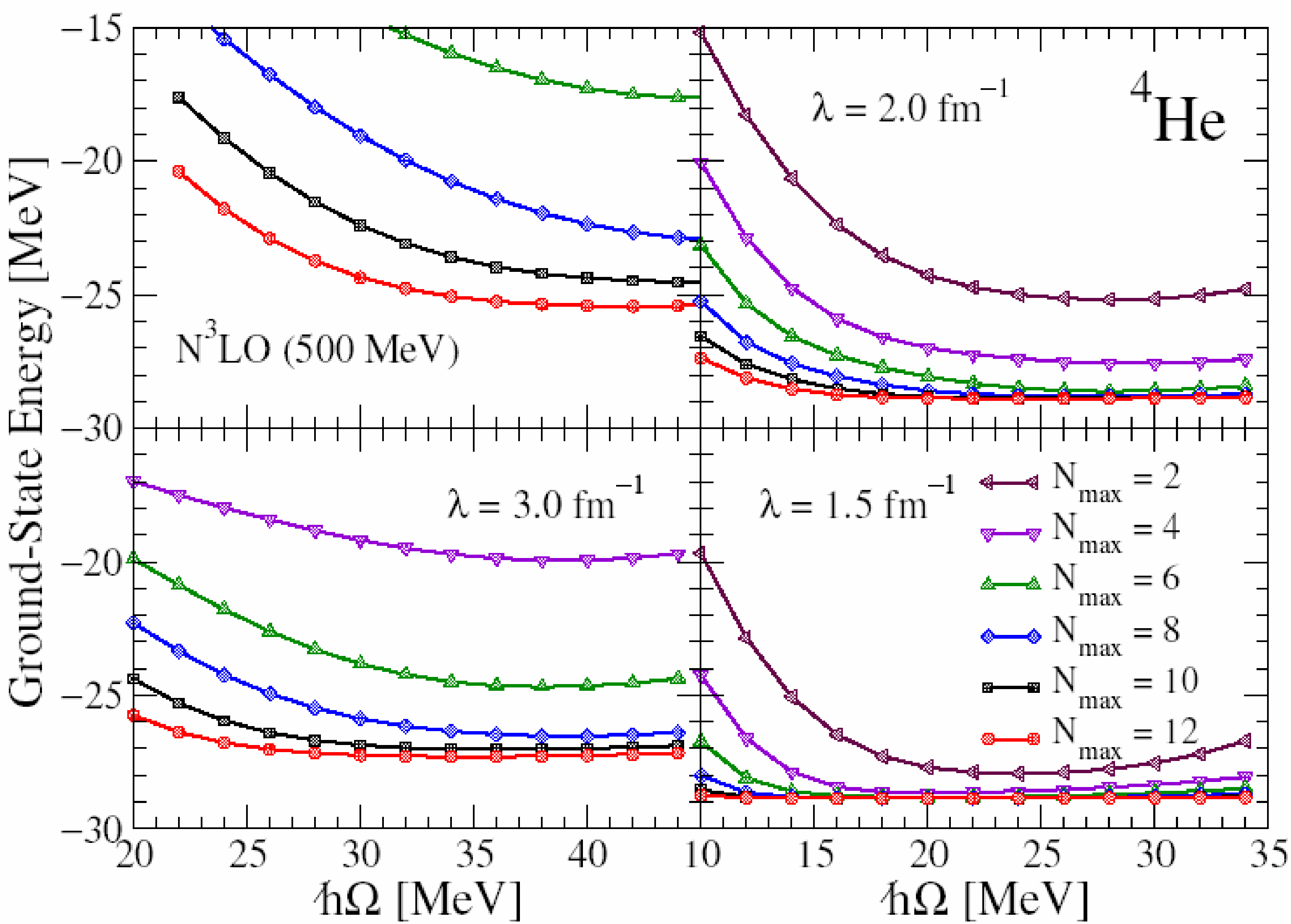
Confirmations

- Numerical verification of Unitarity
- Compared computation of Deuteron Eigenvalues using unevolved potential, and SRG evolved potential -- matches ~ 10^{-5}
- Also compared expectation value of Deuteron for various momentum values
- Verified that evolved U is same as U calculated directly from eigenvectors of H

SRG vs. Non-SRG Computations



SRG vs. Non-SRG Computations



Future Explorations / Applications

- Study evolution of additional operators
  - Unitary – look for factorization
- Look for better transformations – something other than T_{rel}
- Analyze 3-body interactions

The End

See us on the web at: www.physics.ohio-state.edu/~ntg/srg/

Or just Google:

Similarity Renormalization Group, SRG evolve, etc.

(We'll be at the top)

The Similarity Renormalization Group's (SRG) Place in Nuclear Physics

SRG Evolution Equations

SRG Evolution Movies