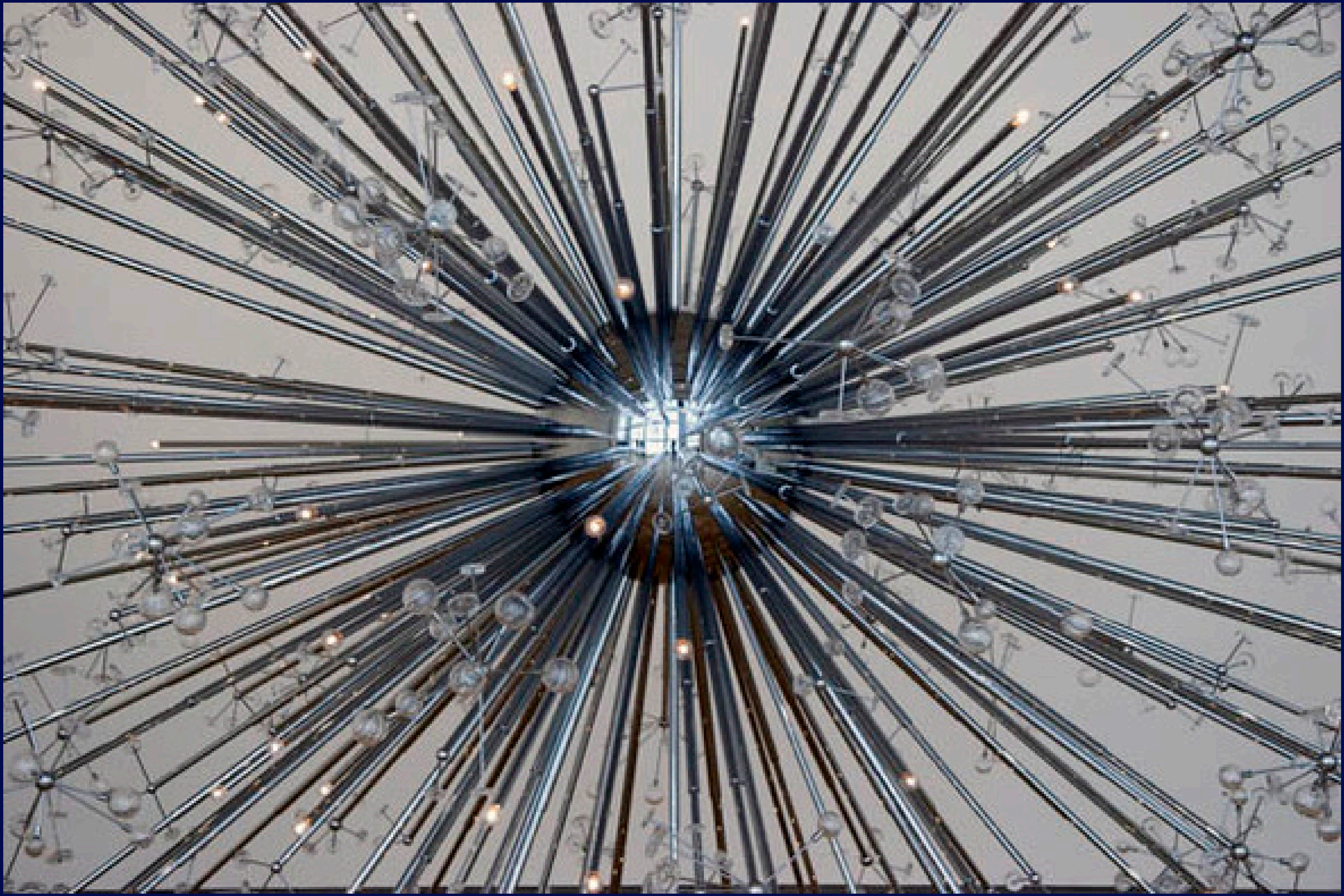


Lecture 2

Three Dimensions of Hydrodynamics in Heavy Ion Collisions

Peter Steinberg
Brookhaven National Laboratory
National Nuclear Physics Summer School, July 2007

just kidding

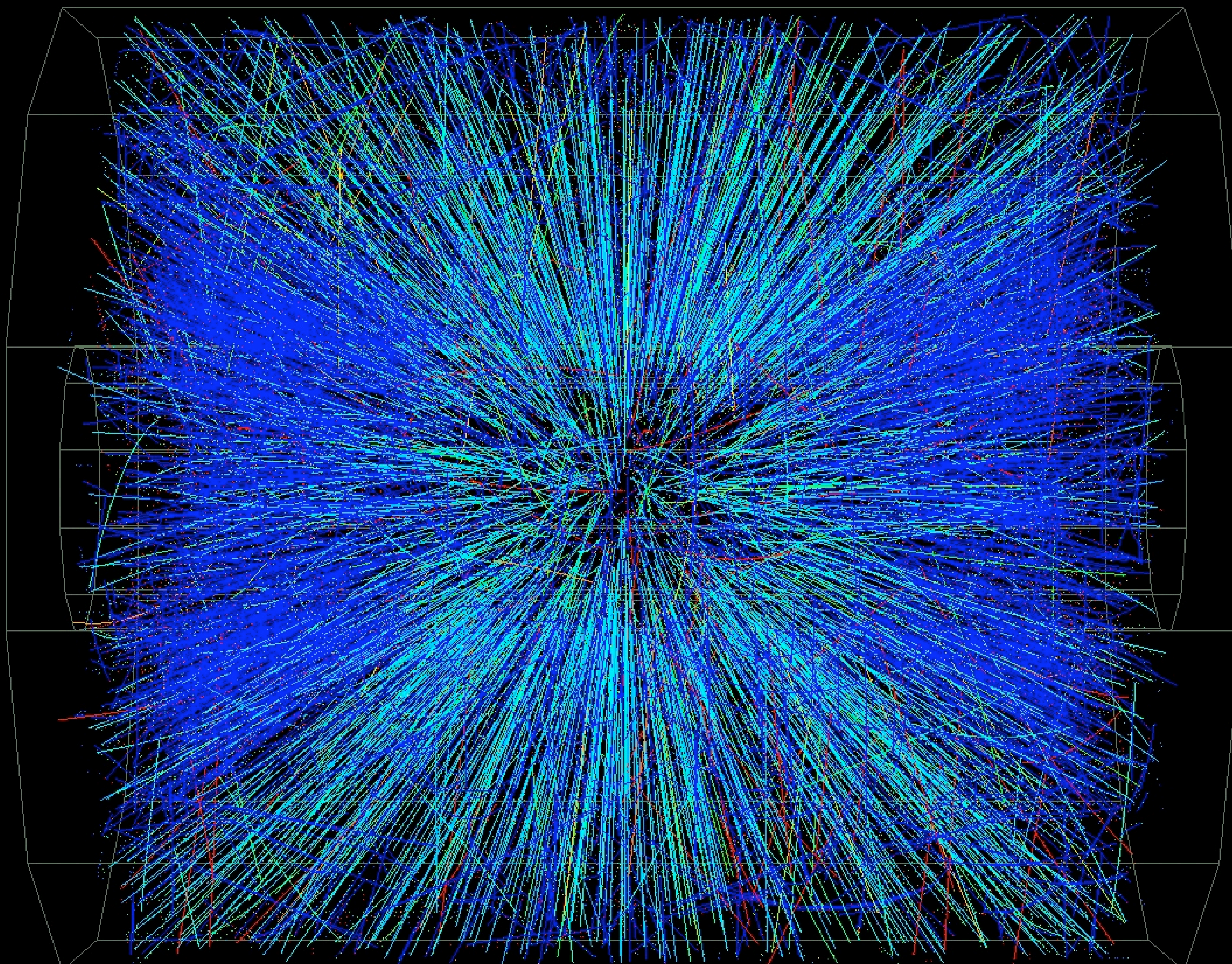
Goal

- **Hydrodynamics has become the "language" of RHIC physics**
- **Essential to be familiar with the <u>concepts</u> and <u>phenomena</u> at LHC**
- **"Three" dimensions of hydro**
  - Initial Conditions - Thermalization & Geometry
  - Hydrodynamic evolution / Equation of state
  - Freezeout to Hadrons
- **The importance of viscosity**
  - Is there an intrinsic scale to the dynamics?

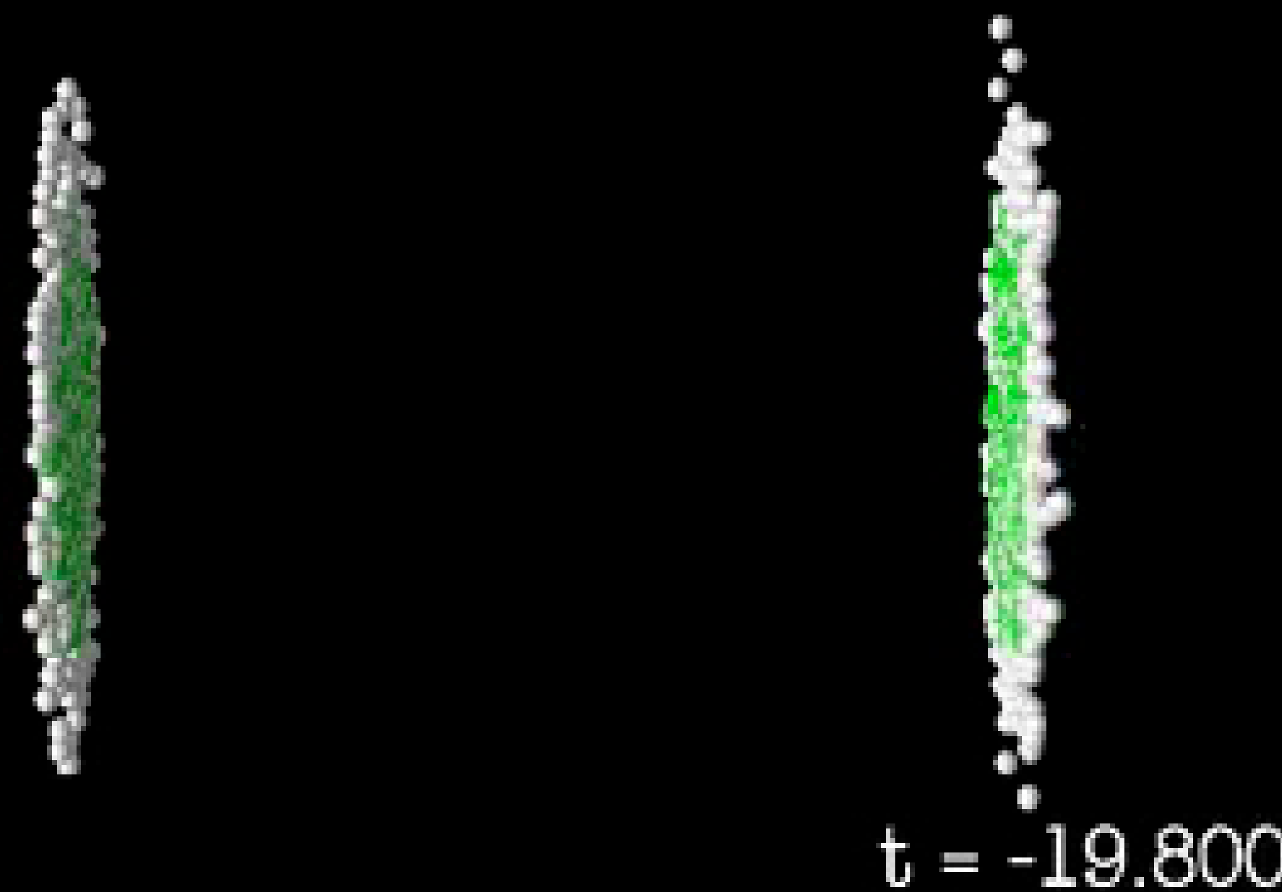
Hope Springs Eternal

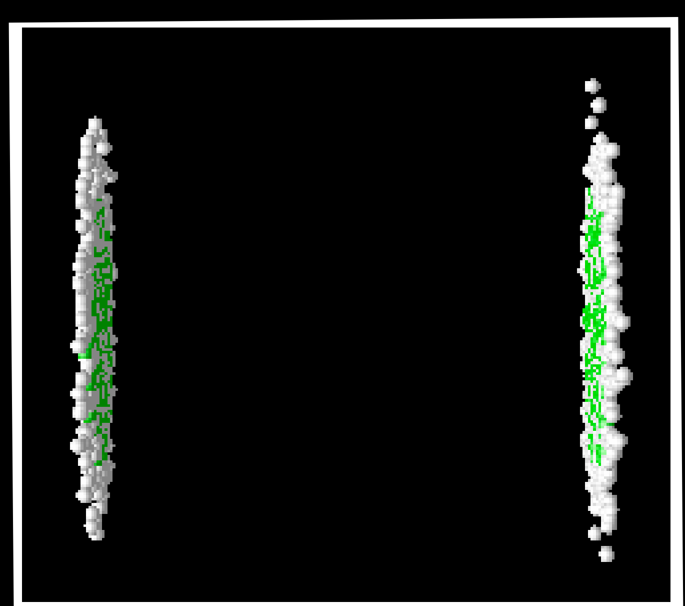
- **This is supposed to be an "easy" talk mathematically**
  - I don't "do" math (but I like to talk about it!)
- **It will also be as <u>conceptually</u> clear as possible**
  - Assume nothing is obvious (it's not!)
  - Ask, and ye shall receive (a "thoughtful" answer from me or someone in the audience!)
- **I want to leave you with a concrete space-time image of the dynamics**

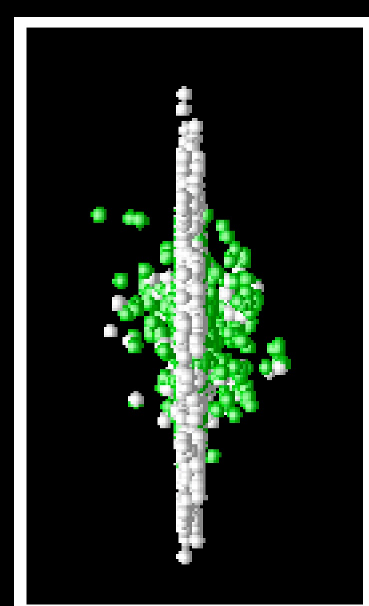
Phenomena @ RHIC

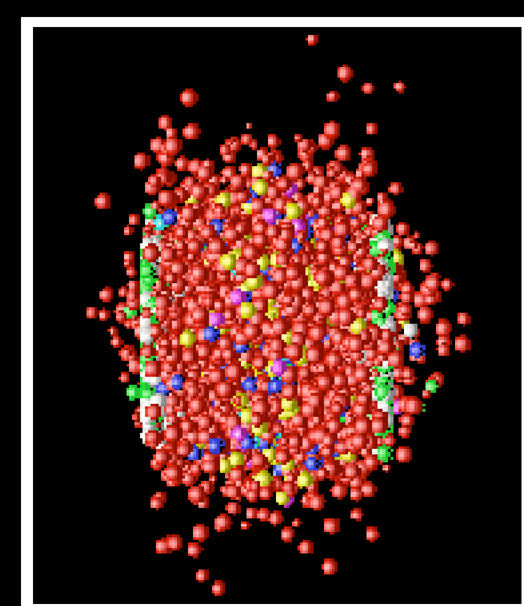


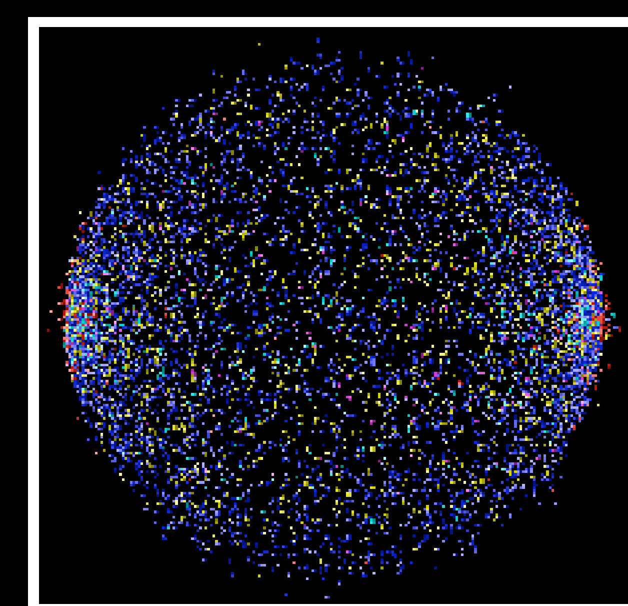
RHIC: The Movie



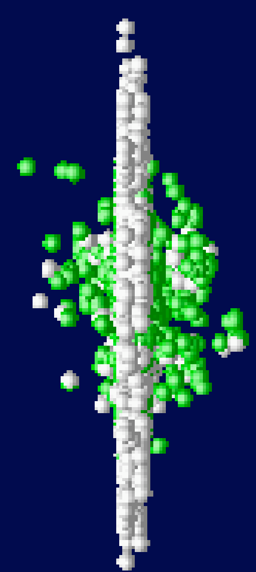

Colliding gold ions


Bang!

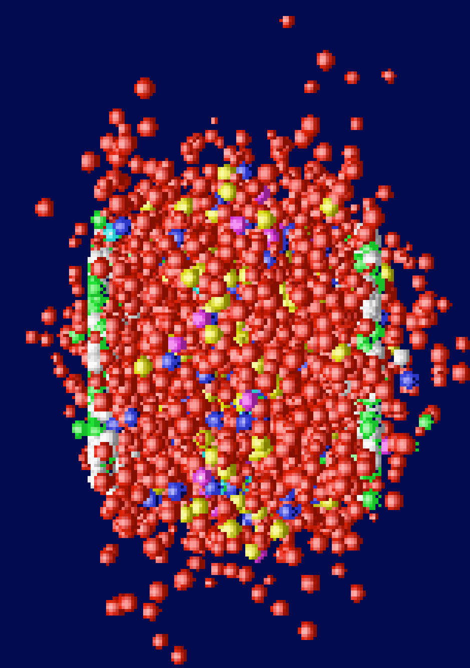

What is this?


Final particles

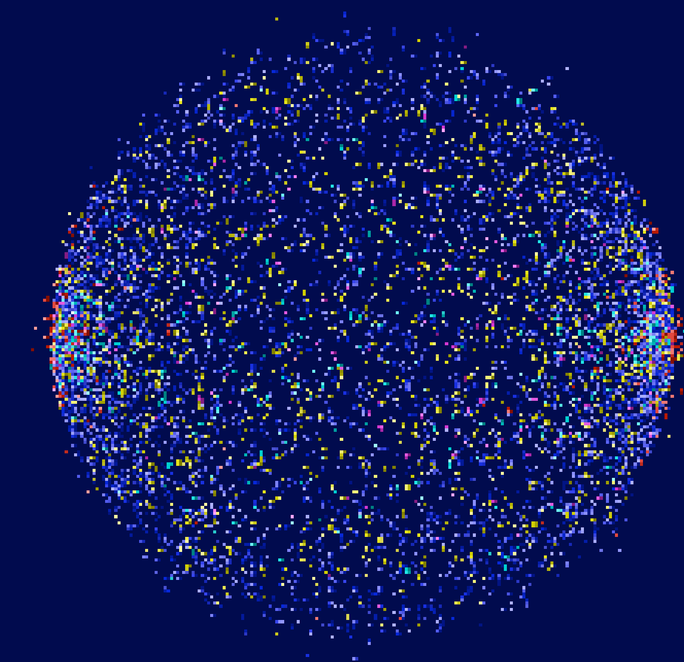
Three Stages



Initial Conditions

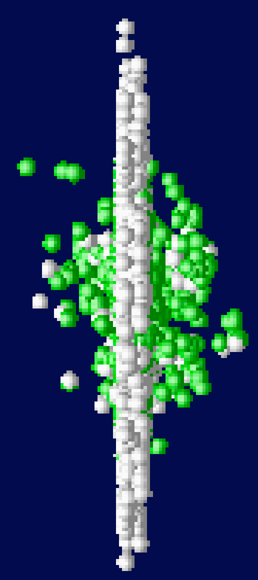


Dynamical Evolution

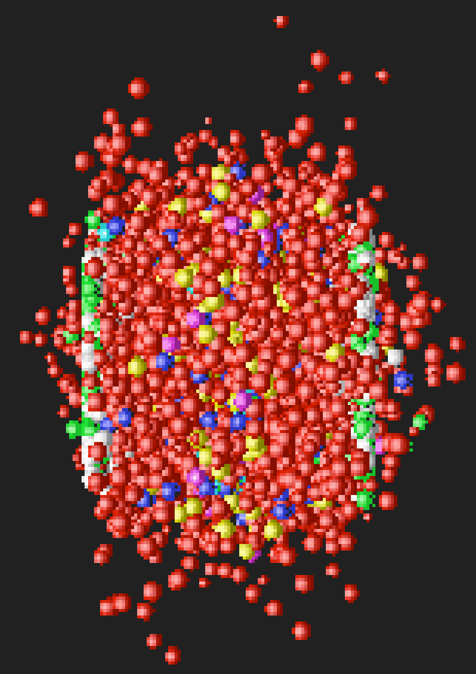


Hadronic Freezeout

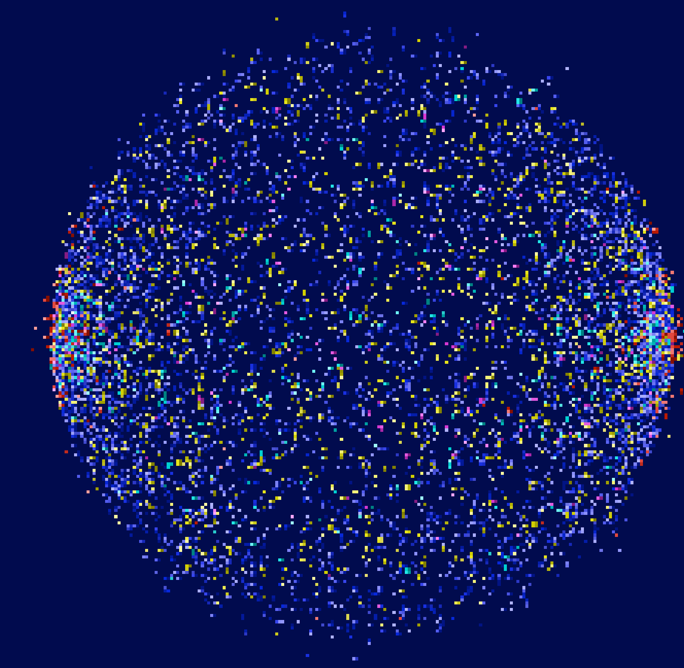
Three Stages



Initial
Conditions

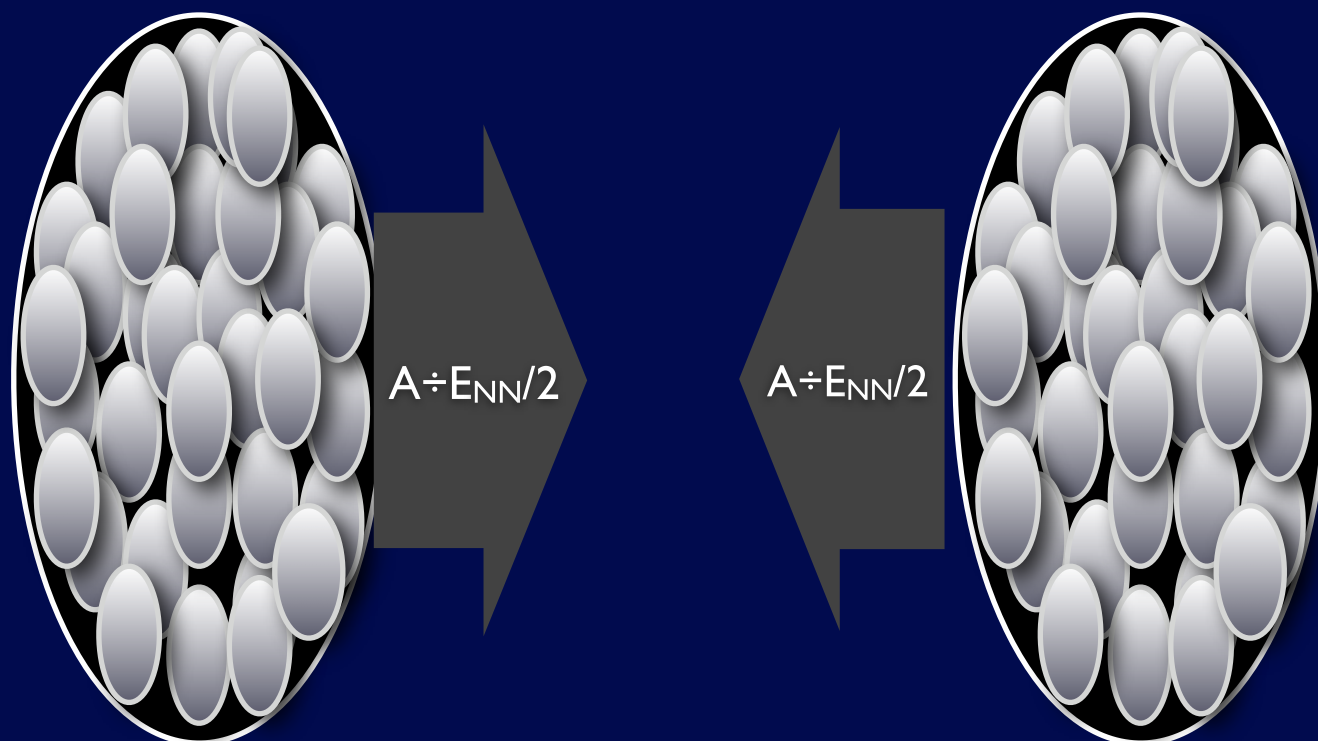


Hydrodynamic
Evolution



Hadronic
Freezeout

Thermalization



In our field, we have to make the imaginative leap from two contracted nuclei (clusters of nucleons) composed of nucleons (clusters of “partons”) transforming into a “fireball” (cluster of ??)

Thermalization

In our field, we have to make the imaginative leap from two contracted nuclei (clusters of nucleons) composed of nucleons (clusters of “partons”) transforming into a “fireball” (cluster of ??)

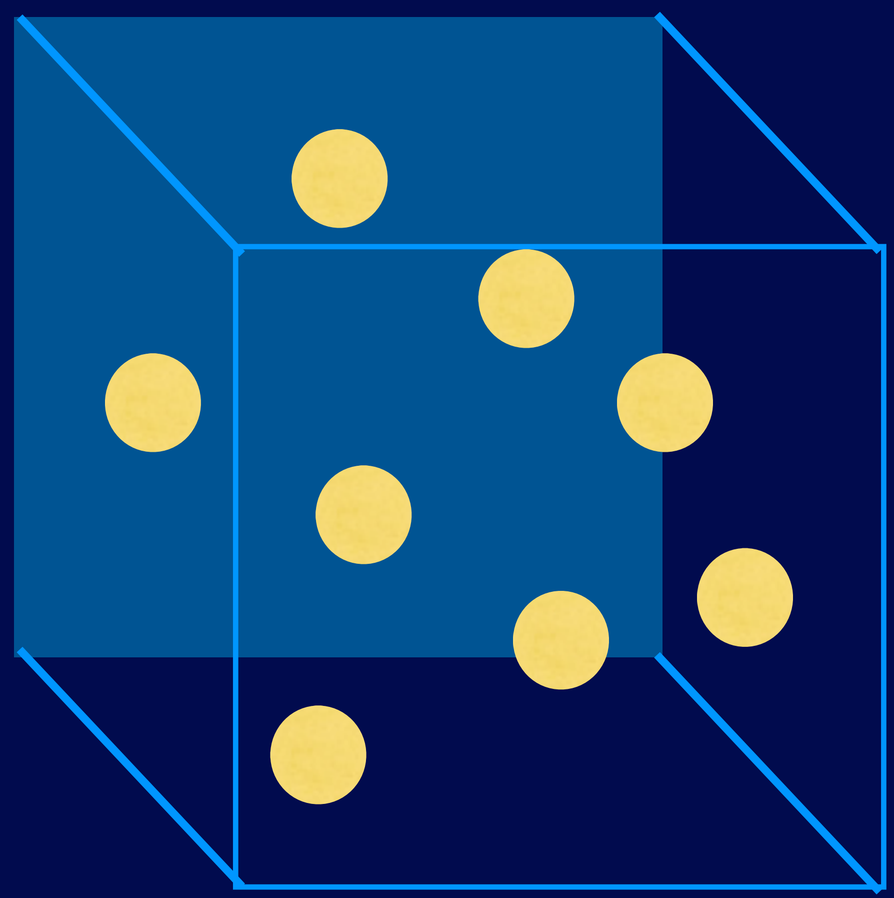
Fluids

All we need to assume is:

1. Infinitessimally small cells are <u>locally</u> thermalized
2. The cells are interacting rapidly with each other

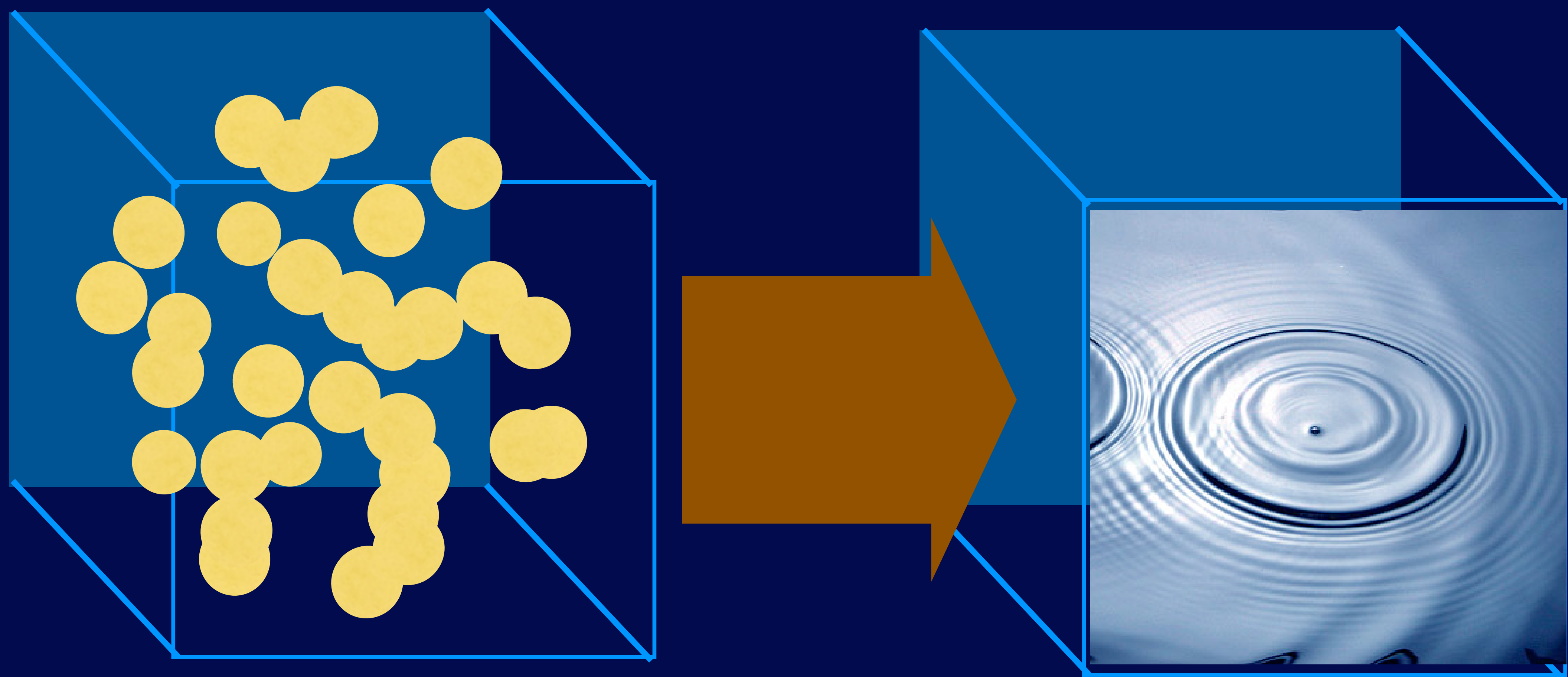
Then we can define a <u>temperature</u>, <u>energy density</u>, and <u>pressure</u>

Thermalization



The usual picture:
particles start with an arbitrary velocity distribution,
but over time “equilibrate” or “thermalize”,
maximizing entropy by choosing the
most probable velocity (Boltzmann) distribution

Continuum Limit



Boltzmann equation:
particle collisions

Fluid:
no particles

Stress Energy Tensor

In continuum limit, we define bulk variables

$$T_{\mu\nu} \equiv (\epsilon + p) u_\mu u_\nu - g_{\mu\nu} p$$

$\epsilon(x_\mu)$ Energy density

$p(x_\mu)$ Pressure

$u_\mu(x_\mu)$ Relativistic velocity

$= \gamma(1, \beta_x, \beta_y, \beta_z)$

$$T_{\mu\nu} = \left\{ \begin{array}{cccc} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{array} \right\}$$

in the local fluid rest frame: pressure is isotropic!

Hydrodynamic Equations

Given all the assumptions above,
ideal hydrodynamics (w/ no baryons) is a coupled set of non-linear differential equations which merely express that energy/momentum is <u>conserved locally</u>

$$\partial_\mu T_{\mu\nu} \equiv \frac{\partial T_{\mu\nu}}{\partial x_\mu} = 0$$

4 equations for 5 functions of x,y,z,t: $(\epsilon, p, \vec{u})$,
since $u_\mu u^\mu = 1$.

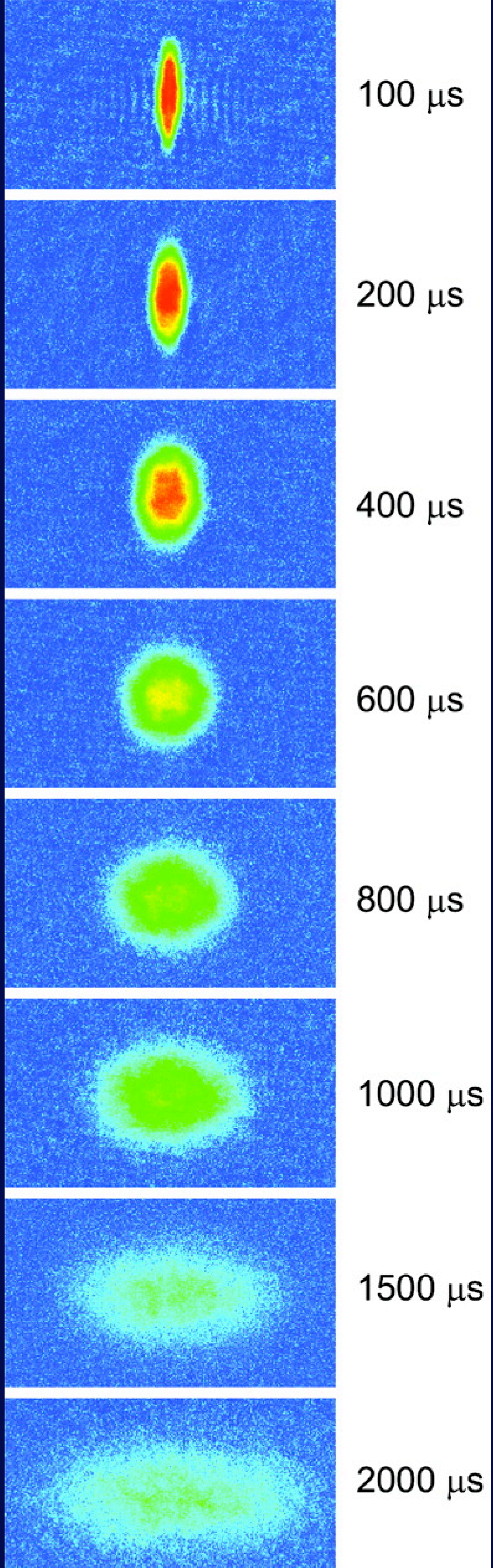
$$T_{\mu\nu} \equiv (\epsilon + p) u_\mu u_\nu - g_{\mu\nu} p$$

$$\partial_\mu T_{\mu\nu} \equiv \frac{\partial T_{\mu\nu}}{\partial x_\mu} = 0$$

Relativistic Euler equation

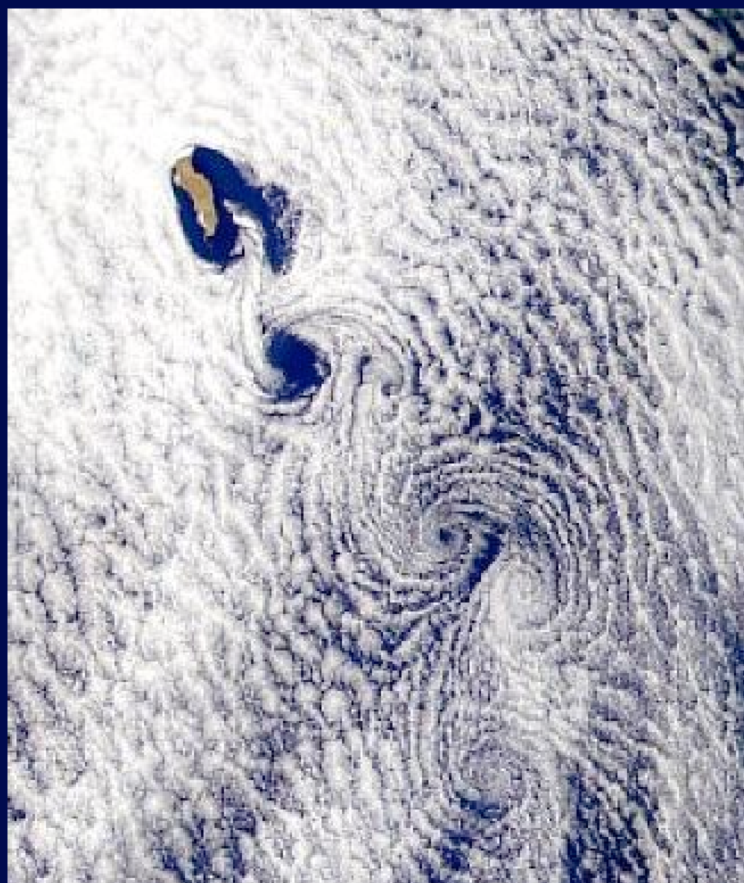
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot ((\rho \mathbf{u}) \mathbf{u}) + \nabla p = 0$$

Pressure gradients drive changes in velocity (i.e. F=ma)

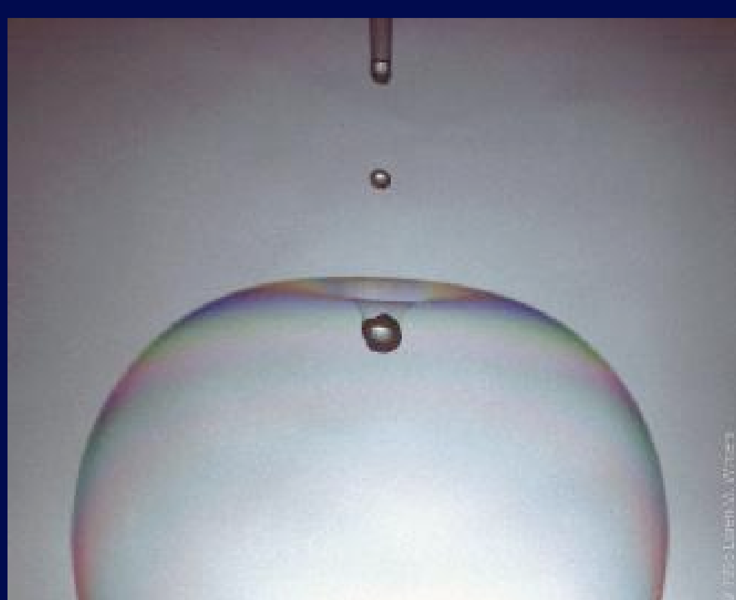


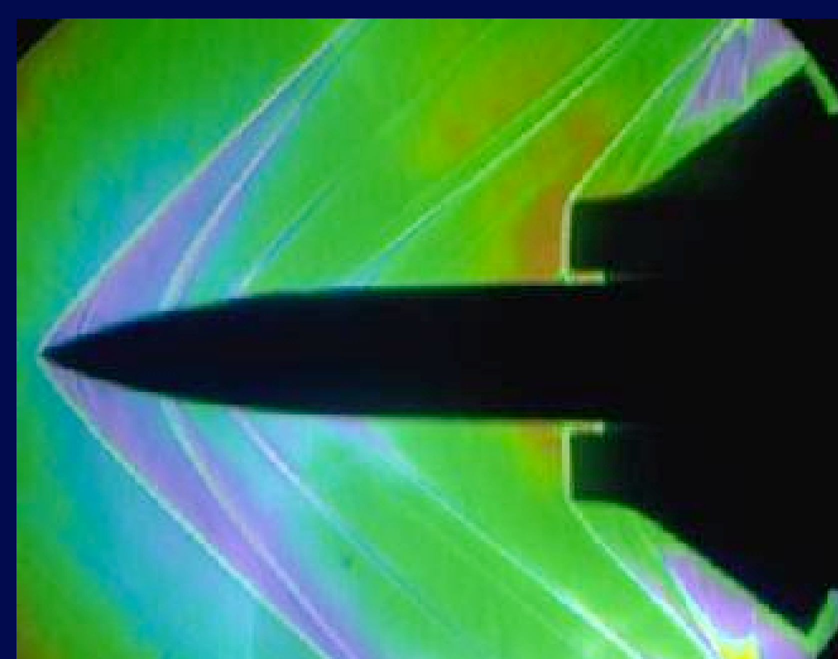
Hydrodynamic expansion of strongly-coupled ^{6}Li ions released from a trap (O'Hara et al 2002)

Hydrodynamic Phenomena



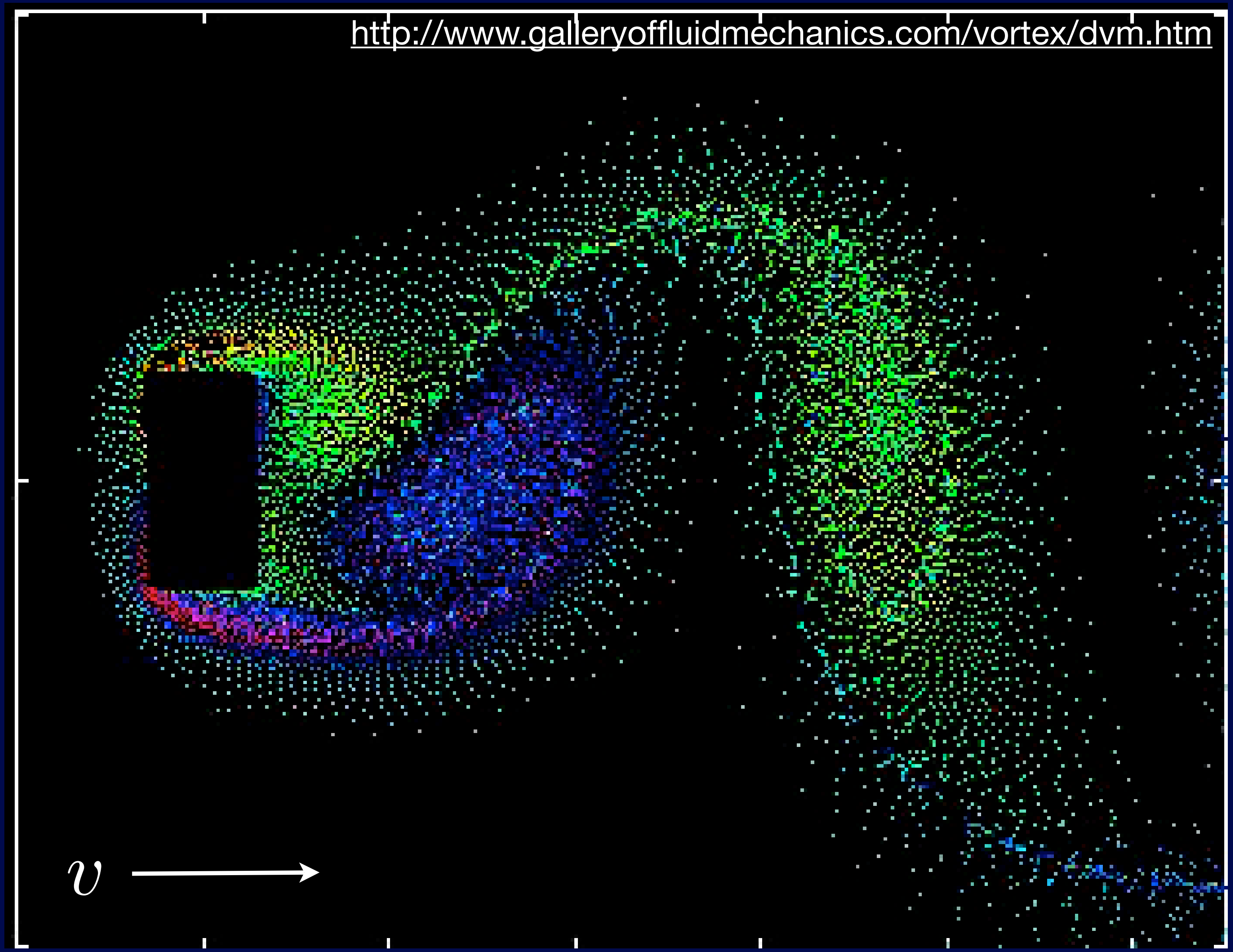
Simple equations of motion
applied to (viscous) fluids:
leads to a wide variety
of phenomena in real world





for more fun: http://www.galleryoffluidmechanics.com/

Vortex Separation



Vortex separation due to viscous flow past a barrier

Equation of State (EOS)

Need one more equation to close the system. Nothing is required by the mathematics, so we can make a choice

The trace of the stress energy tensor is Lorentz invariant: if non-zero it implies a fixed scale in the problem. It also must be positive.

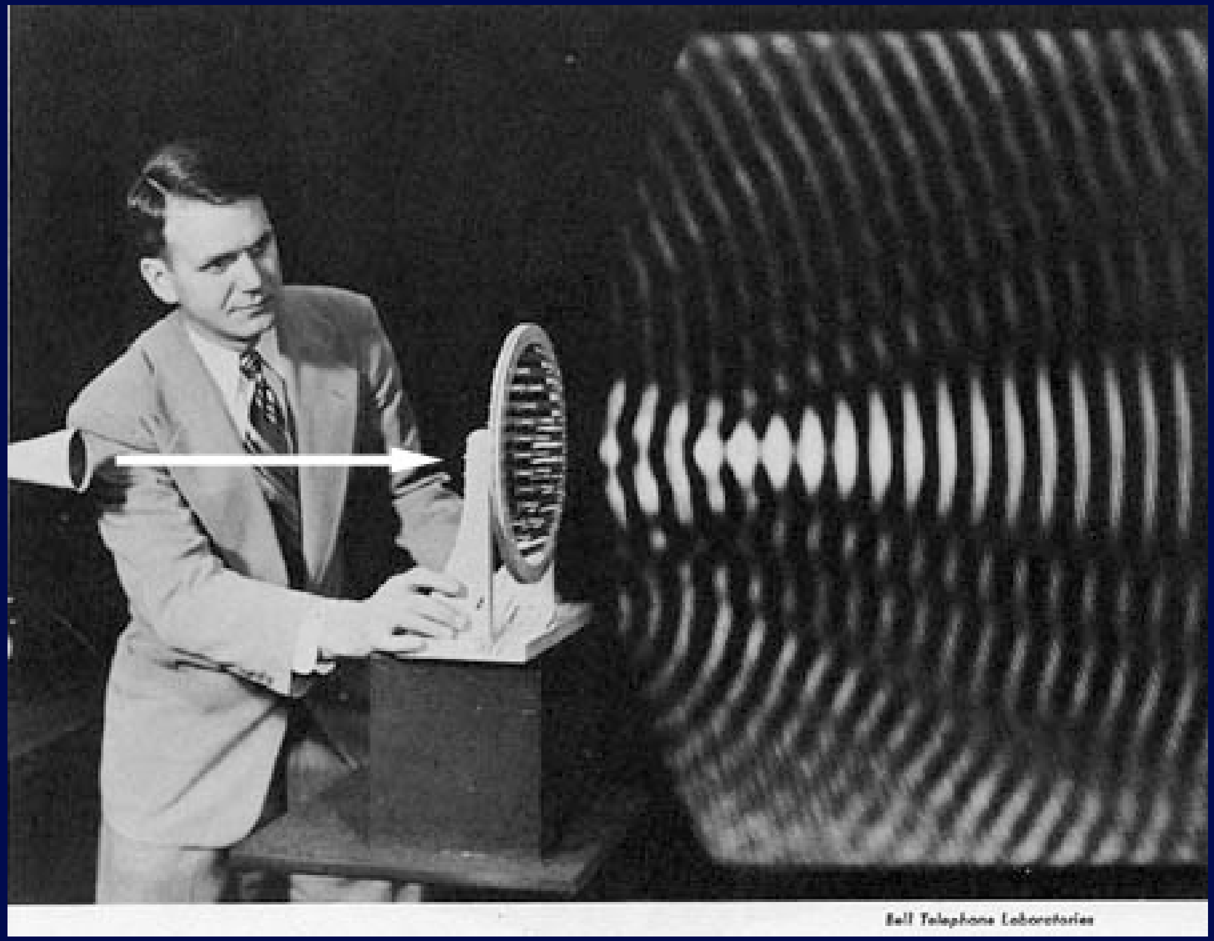
$$T_{\mu\nu} = \left\{ \begin{array}{cccc} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{array} \right\}$$

$$T^{\mu}_{\mu} = \epsilon - 3p \geq 0 \rightarrow p \leq \epsilon/3$$

Speed of Sound

$$c_s^2 = \frac{dp(\epsilon)}{d\epsilon} \leq \frac{1}{3}$$

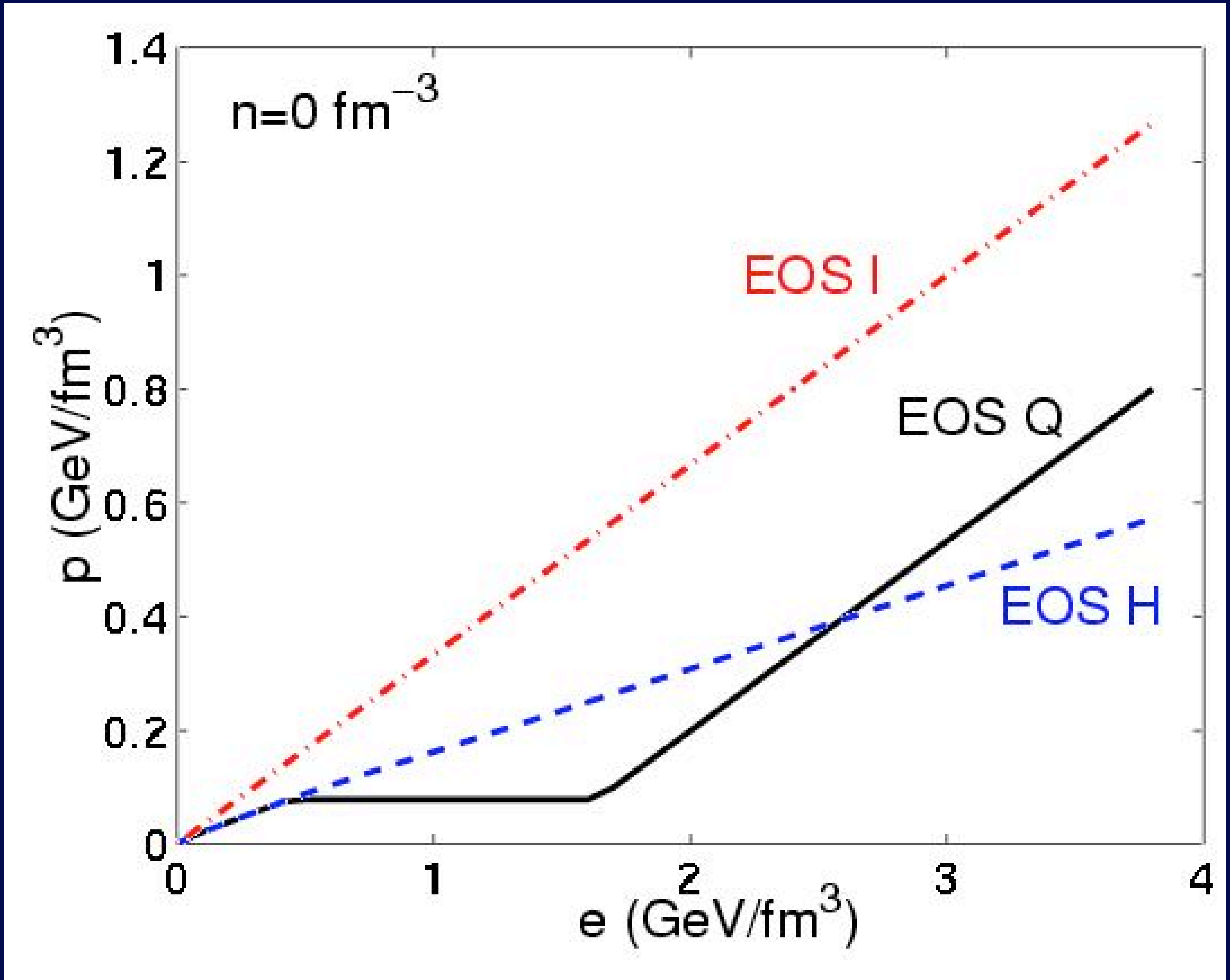
In a compressible fluid, velocity of excitations



Real sound waves disperse as they propagate (another hint about what viscosity does)

The First Book of Sound: A Basic Guide to the Science of Acoustics by David C. Knight, Franklin Watts, Inc. New York (1960). p. 80

Examples



Ideal

"QGP"

Hadronic

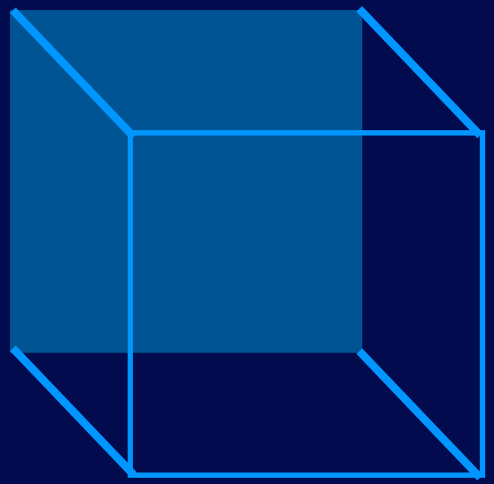
Kolb, PhD Thesis

Scale Invariance

$$T^{\mu}_{\mu} = \epsilon - 3p = 0 \rightarrow p = \frac{\epsilon}{3} \qquad c_s^2 = \frac{1}{3}$$

This is a special case:

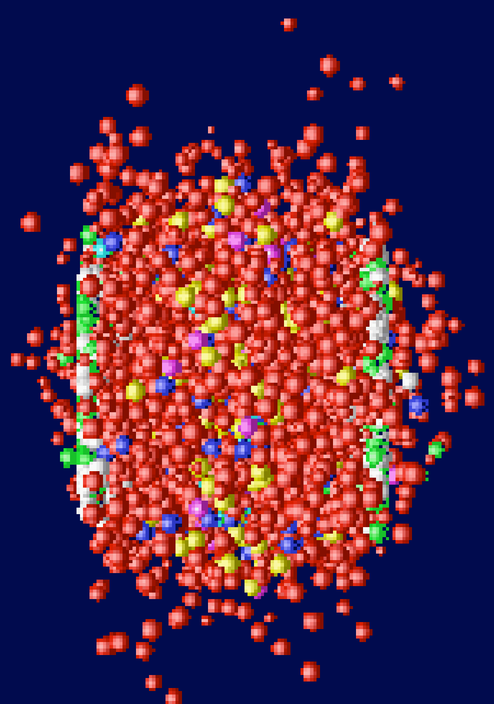
1. No intrinsic scale in hydrodynamics
2. Speed of sound is determined by the number of spatial dimensions!



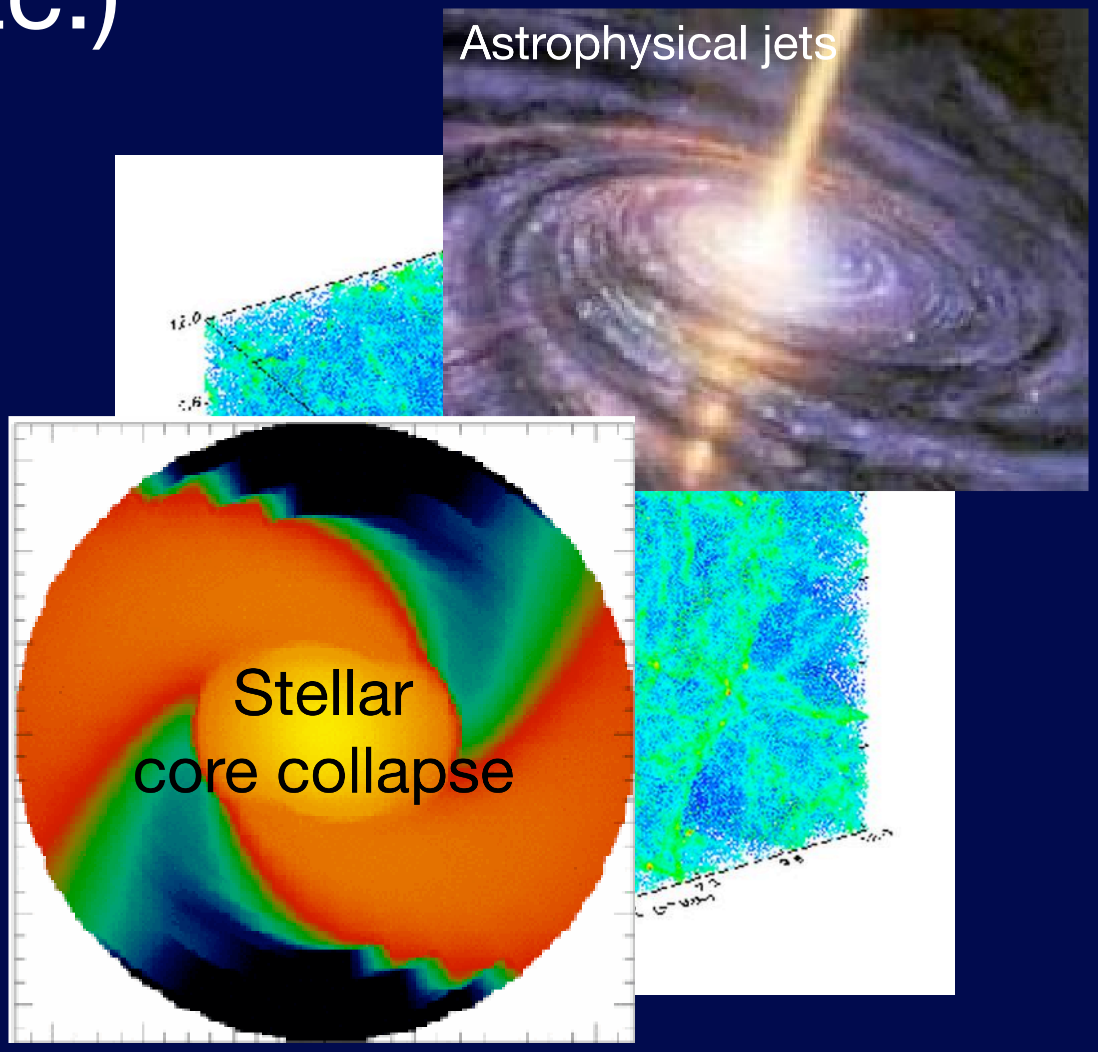
Often called the “ideal gas” EOS.
This seems misleading: it is used for non-interacting (e.g. E&M blackbody) <u>and</u> strongly-interacting systems

Scale Invariance, cont

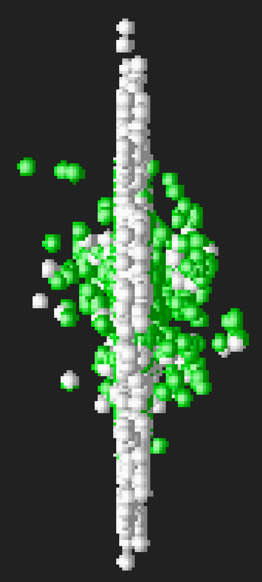
The entire universe is often modeled as
an ideal fluid with same equations as us!
(of course GR determines geometry etc.,
dynamics controlled by baryonic & dark matter/
energy, etc.)



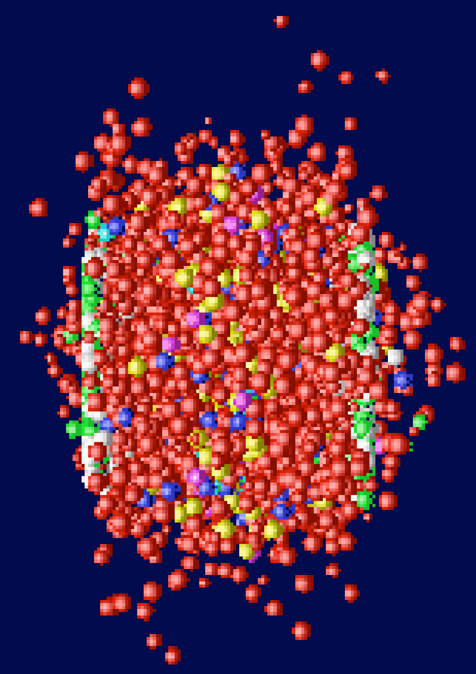
$$\partial^\mu T_{\mu\nu} = 0$$



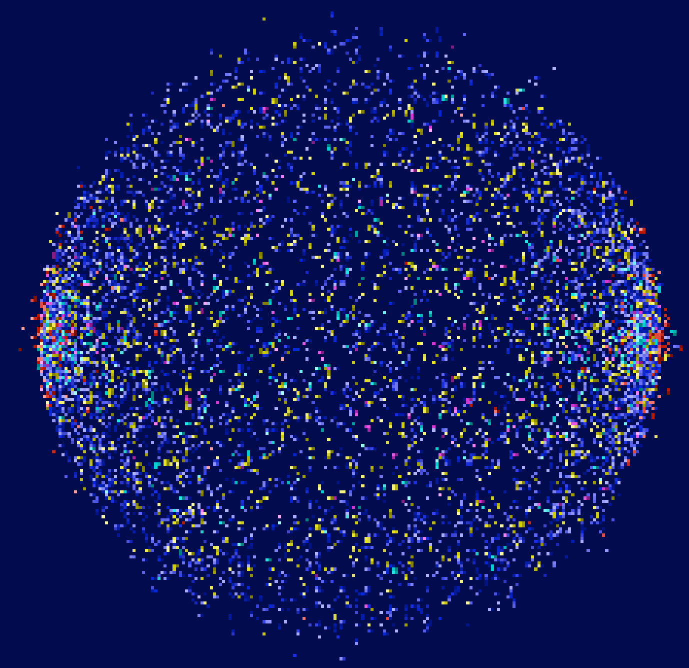
Three Stages



Initial Conditions

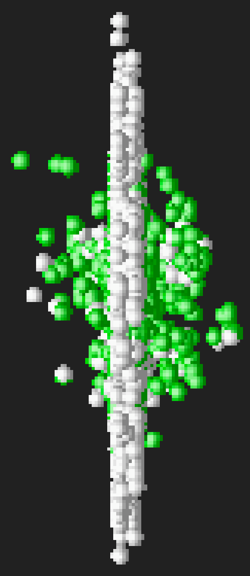


Hydrodynamic Evolution



Hadronic Freezeout

Three Stages

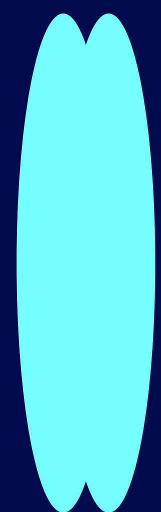


Initial
Conditions

Space-time profile of
energy or entropy
with a space-time-dependent
velocity distribution

Initial Conditions

Landau

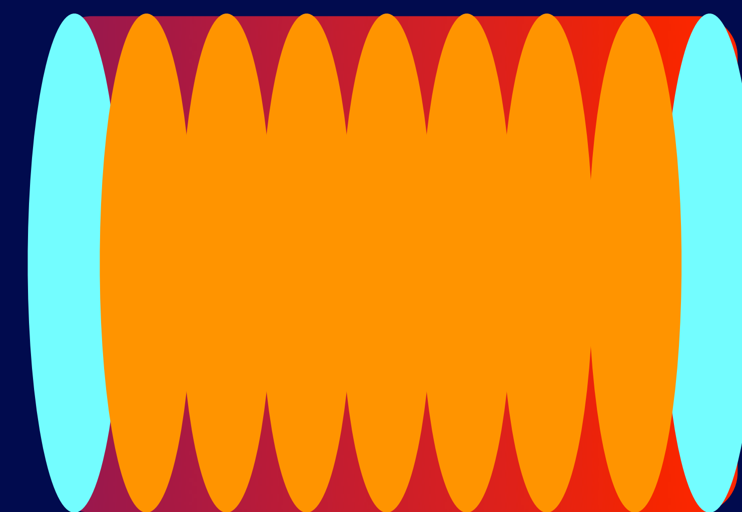


Total stopping, immediate thermalization & longitudinal **3D** re-expansion

$$\tau_0 \sim \frac{1}{\sqrt{s}} fm/c$$

Tomorrow

Bjorken



Partial stopping, "boost-invariant" **2D** dynamics

$$\tau_0 \sim 1 fm/c$$

Today

Bjorken Initial Conditions

Choose one "slice" in rapidity space

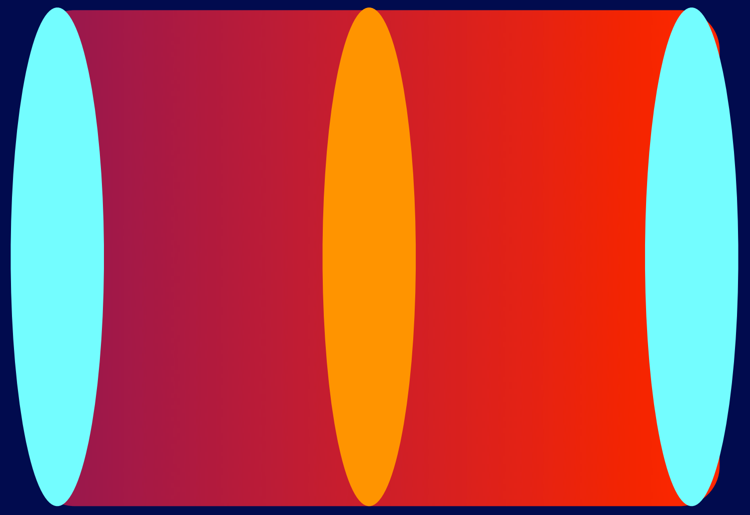
Choose a consistent set of parameters:

1. initial <u>proper</u> time (τ_0)
2. energy OR entropy density constant in rapidity

$$\epsilon_0(\tau_0, y) = \epsilon(\tau_0)$$

Assume that system expands in a boost invariant way for <u>entire</u> evolution

Bjorken



$$v = z/t$$

$$\partial^\mu T_{\mu\nu} = 0 \rightarrow \frac{d\epsilon}{d\tau} = -\frac{\epsilon + p}{\tau}$$

$$\frac{\epsilon(\tau)}{\epsilon(\tau_0)} = \left(\frac{\tau_0}{\tau}\right)^{4/3}$$

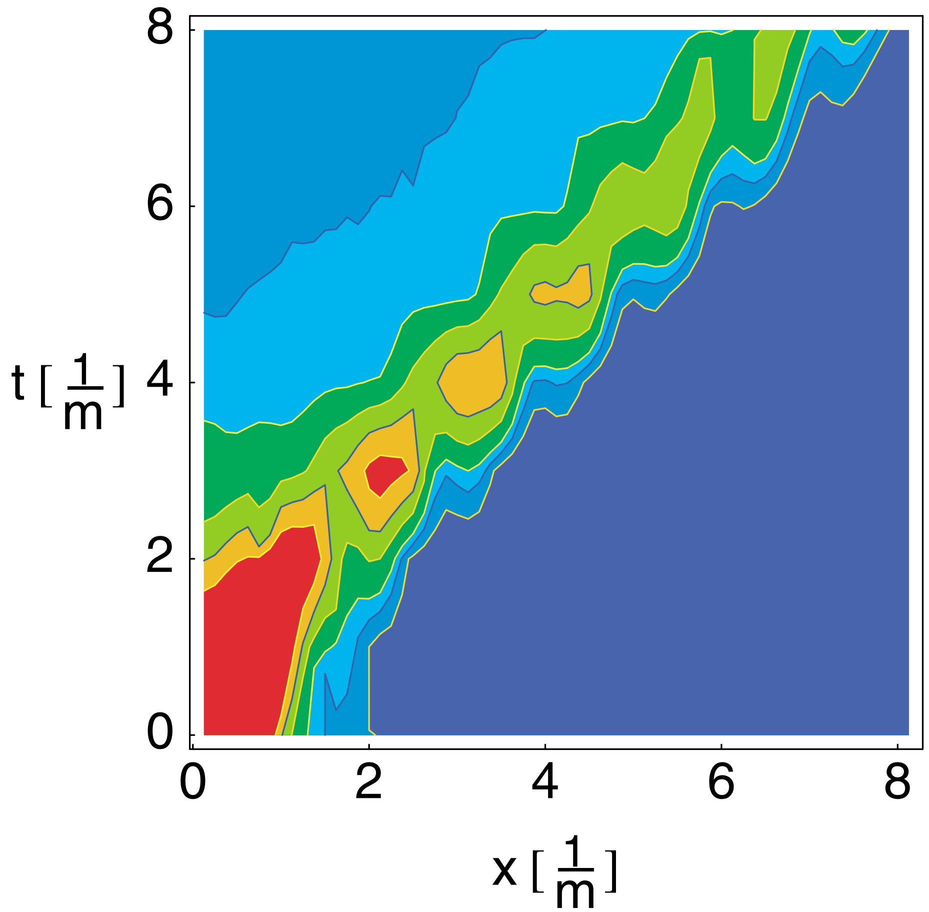
thermalization time is an evolution scale!

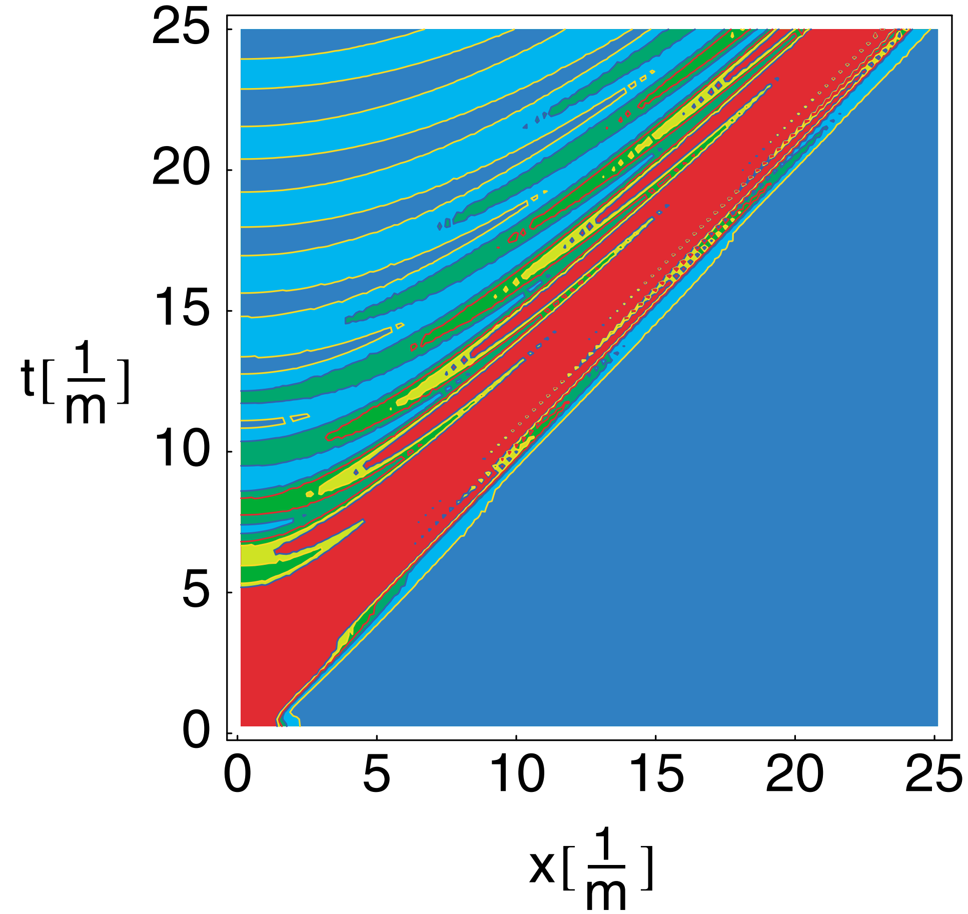
Bjorken flow in ϕ^4 Field Theory

VOLUME 89, NUMBER 11 PHYSICAL REVIEW LETTERS 9 SEPTEMBER 2002

Relativistic Hydrodynamic Scaling from the Dynamics of Quantum Field Theory

Luís M. A. Bettencourt,[1] Fred Cooper,[2] and Karen Pao[3]





Collision of two packets (“leading particles”)

Decay of a stationary “lump”

Density vs. Time

- **The time of full thermalization (τ_0) controls the initial energy density**
  - Lowering the thermalization time increases the initial density
  - Increasing density increases the initial temperature

$$\epsilon \propto n_{DOF} T^4$$

- **Heinz/Kolb found in their fits (with Bjorken expansion) to data that** $\tau_0 T_0 \sim 1$
- **Important points:**
  - There is no <u>single</u> "temperature", but a "temperature history" (i.e. hydro gives a space-time history)
  - The thermalization time is a Very Important <u>Choice</u>

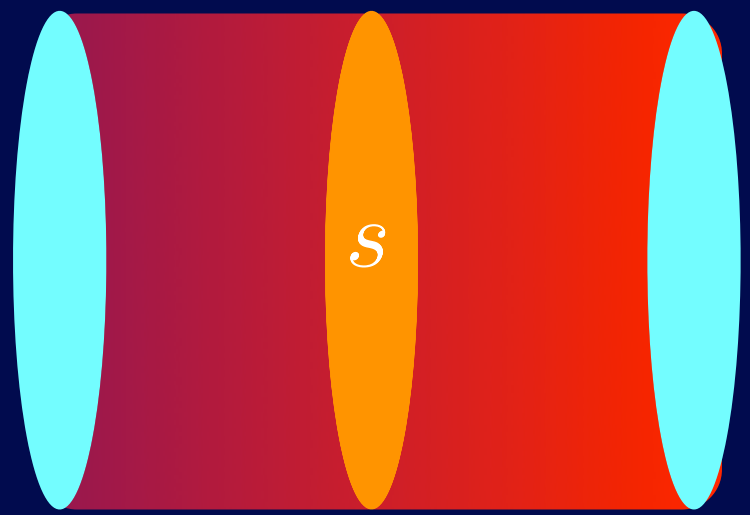
Energy & Entropy

$$TS = E + pV \to s = \frac{\epsilon + p}{T}$$ entropy density

$$s_\mu \equiv s u_\mu$$ entropy current

$$\partial^\mu T_{\mu\nu} = 0 \to \partial^\mu s_\mu = 0$$

Total entropy is conserved, in ideal hydrodynamics



So choose **entropy density** and **τ_0** to reproduce multiplicity data

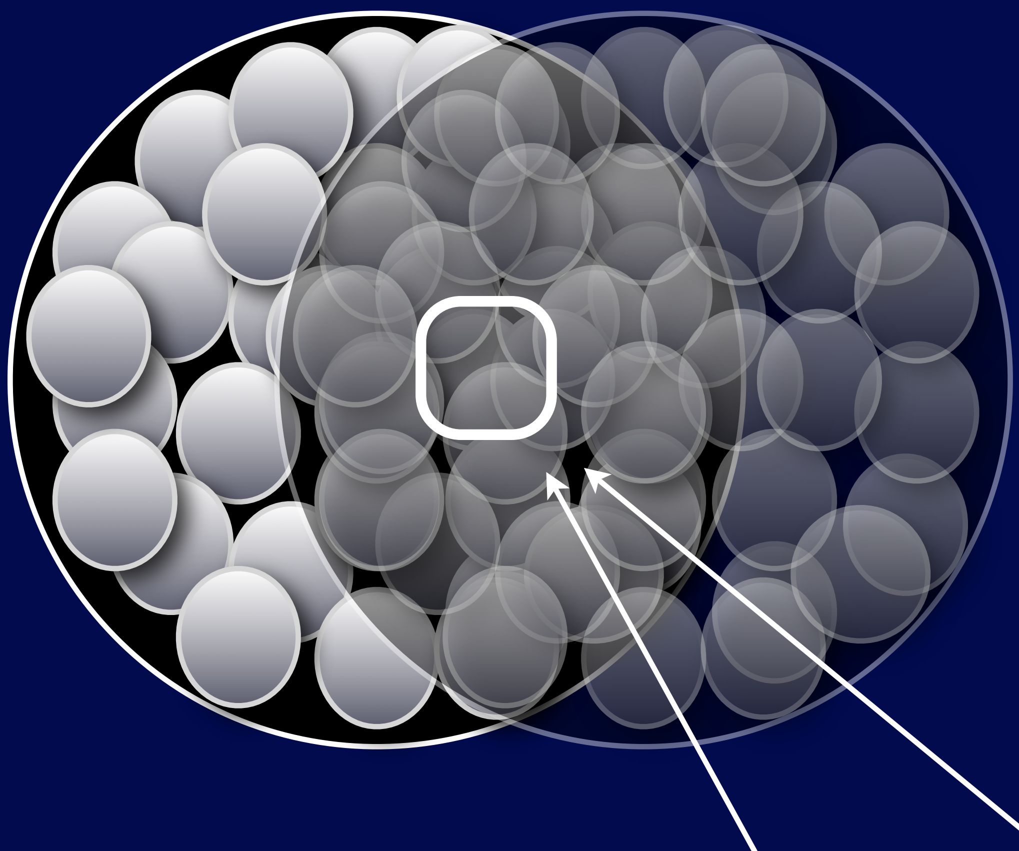
$$N \propto S = \int_V dV\, s$$

Bjorken hydro does not generate rapidity distributions (cf. Landau). It **preserves** them (and is asymptotic to most solutions)

Spatial Distributions

We “know” that nuclei are “made” of nucleons (QM many body problem makes this a non-trivial thing to say...)

“Glauber” calculations treat them as smooth densities in order to calculated participant and collision densities n_{part} and n_{coll}

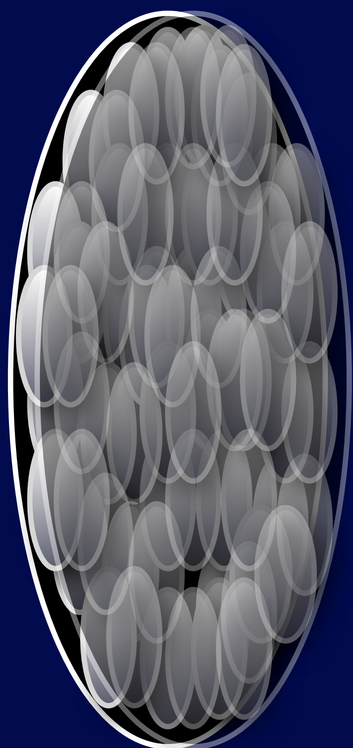


$$\frac{d^3 s}{d\eta d^2\vec{x}} = n_{pp}\left\{(1-x)\frac{n_{part}(\vec{x})}{2} + x n_{coll}(\vec{x})\right\}$$

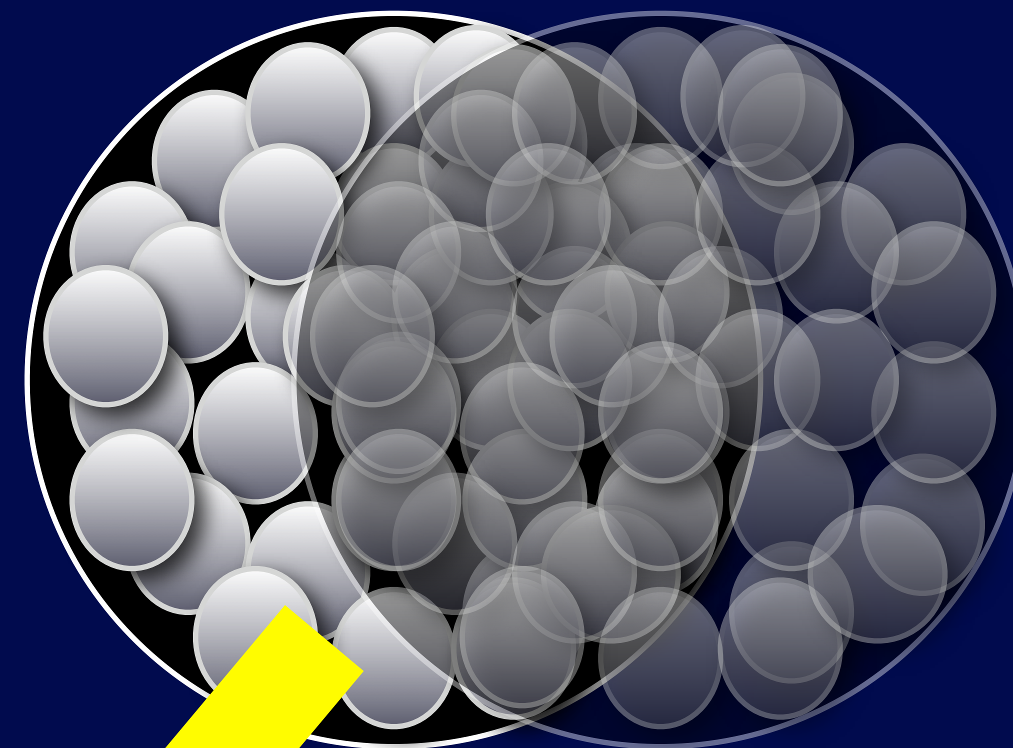
Participant density

Collision density

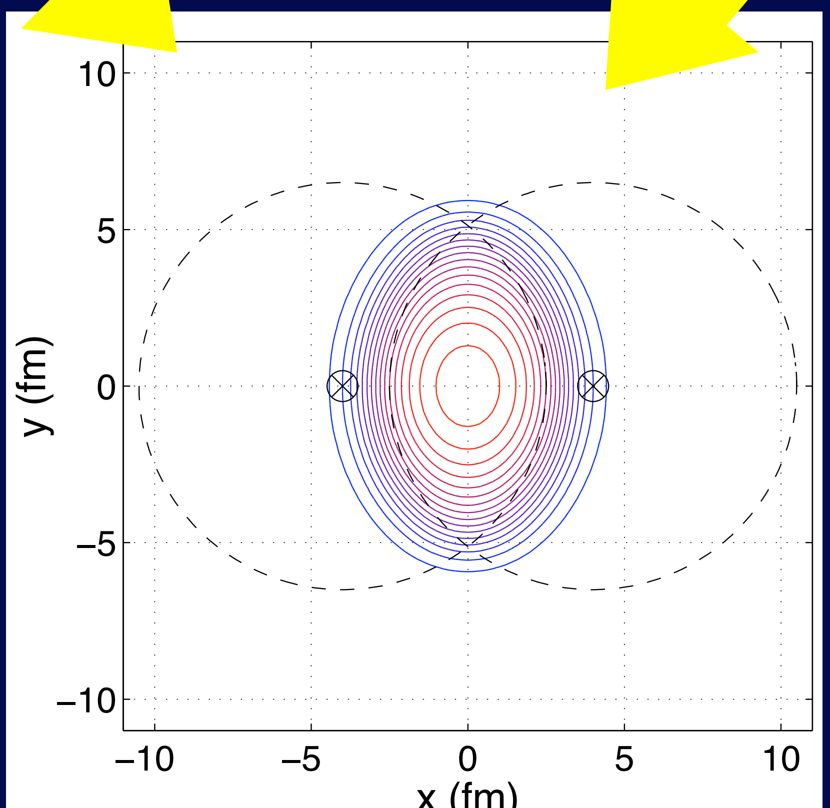
Spatial Distributions



Longitudinal



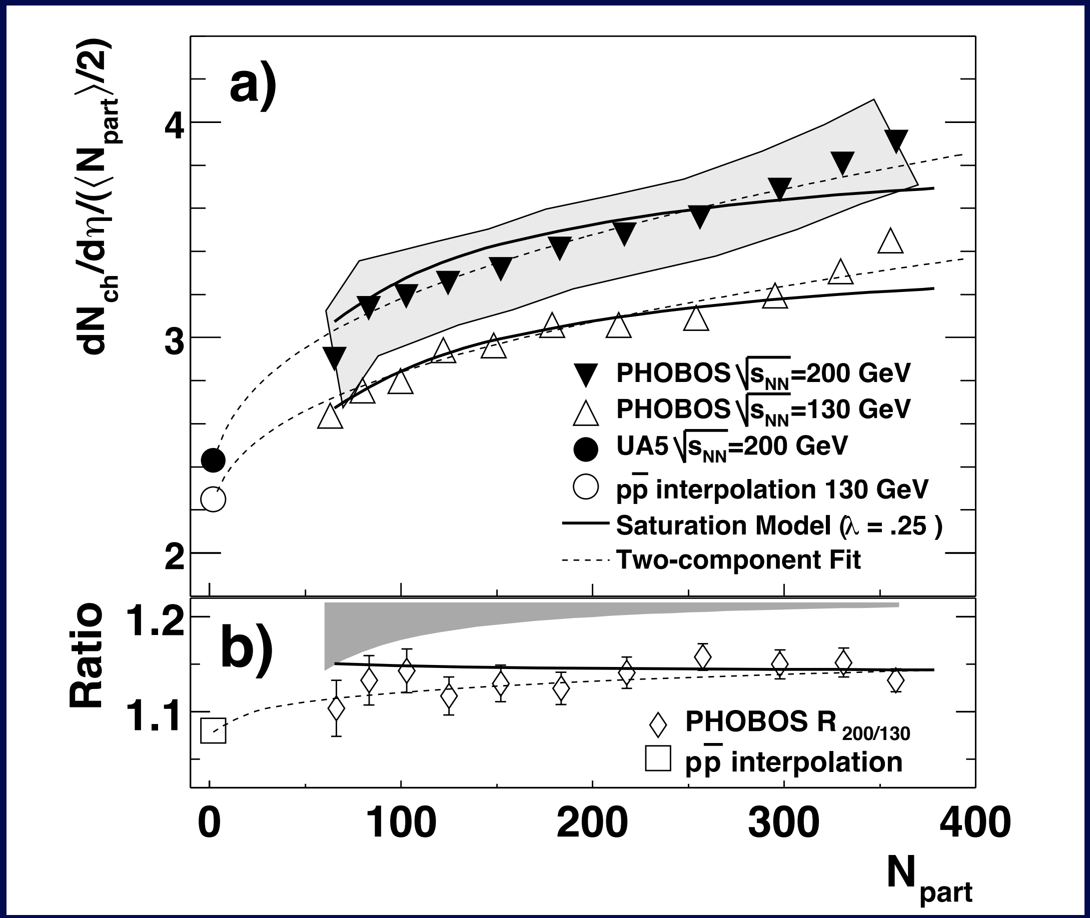
Transverse



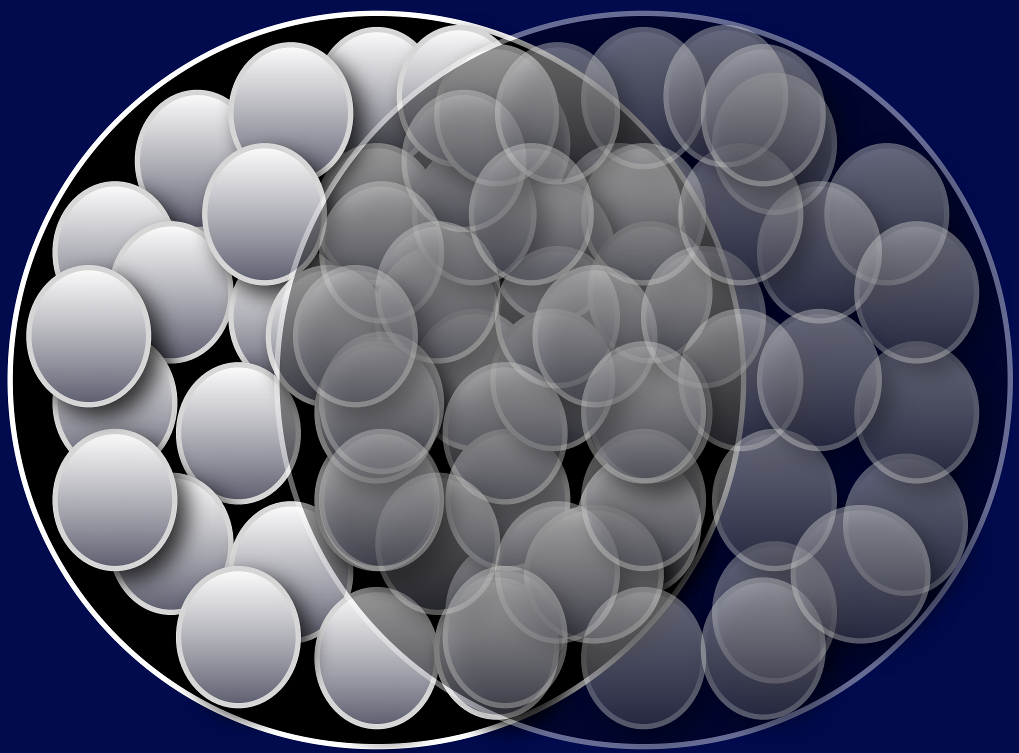
Initial density profile is a convolution of longitudinal & transverse

P. Kolb, PhD Thesis

Energy/Entropy Density

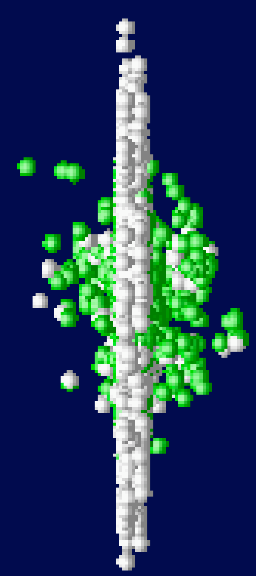


PHOBOS (2002)



Hydrodynamic calculations use two-component picture, tune on central events, and test tuning on experimental data (e.g. PHOBOS)

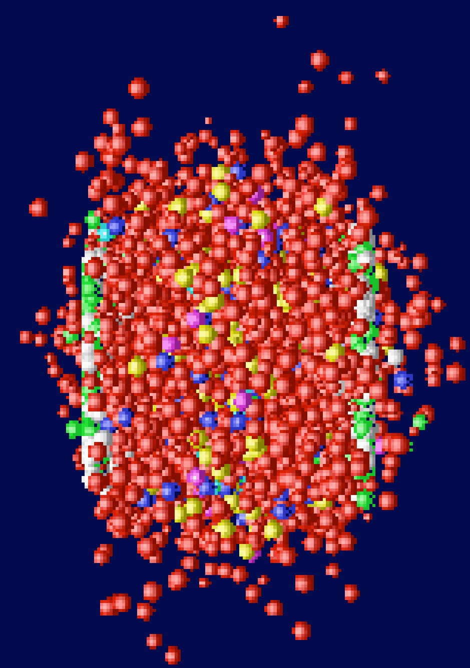
Three Stages



Initial Conditions

$$\frac{ds}{d\eta}(\vec{x})$$

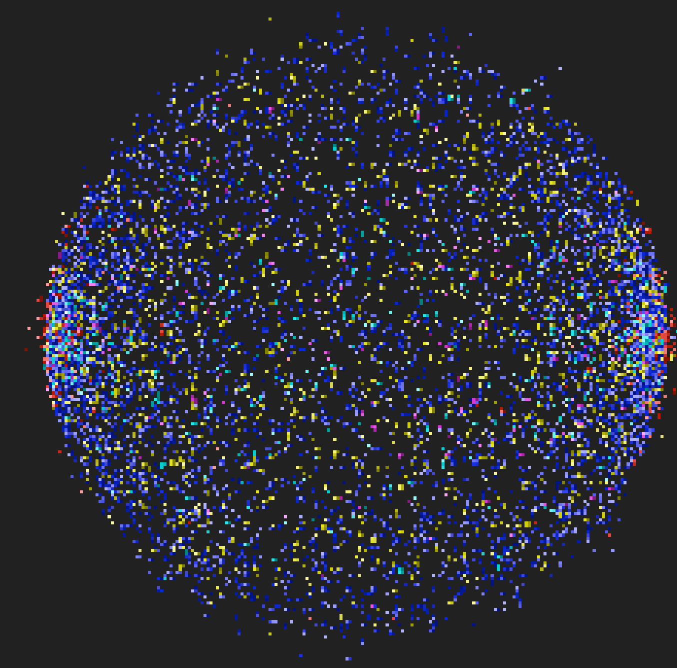
Glauber + data



Hydrodynamic Evolution

$$\partial^\mu T_{\mu\nu} = 0$$

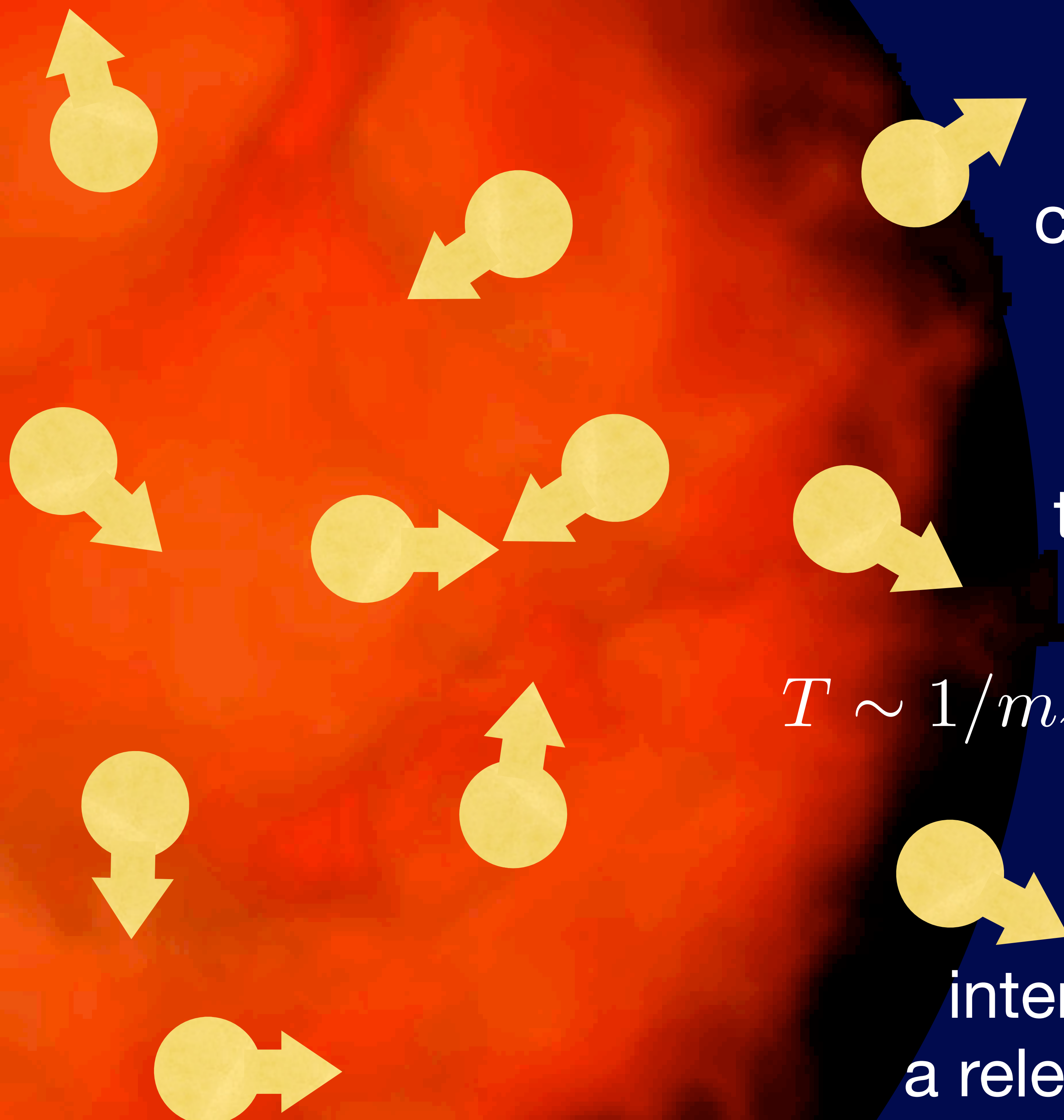
hydro “codes” (e.g. SHASTA)



Hadronic Freezeout

?

It's Cold Out There



Eventually the system cools via hydro expansion

Whatever carries the momentum, we have to presume it carries quark quantum #'s found in final state

$T \sim 1/m_\pi$

When the typical interparticle distance exceed a relevant Compton wavelength (~$1/m_\pi$ ~ 1fm for a pion) interactions "freeze-out"

Cooper-Frye Formula

Define a “hypersurface” in space-time (3D) where the temperature (energy/entropy) falls below a “critical” value

Let the system at that surface be a “fireball” which decays isotropically in its own rest frame

$$E\frac{d^3N}{d\vec{\mathbf{p}}} = \int_\sigma f(x,p)p^\mu d\sigma_\mu$$

$T \sim 1/m_\pi$

# of fireballs normal to surface

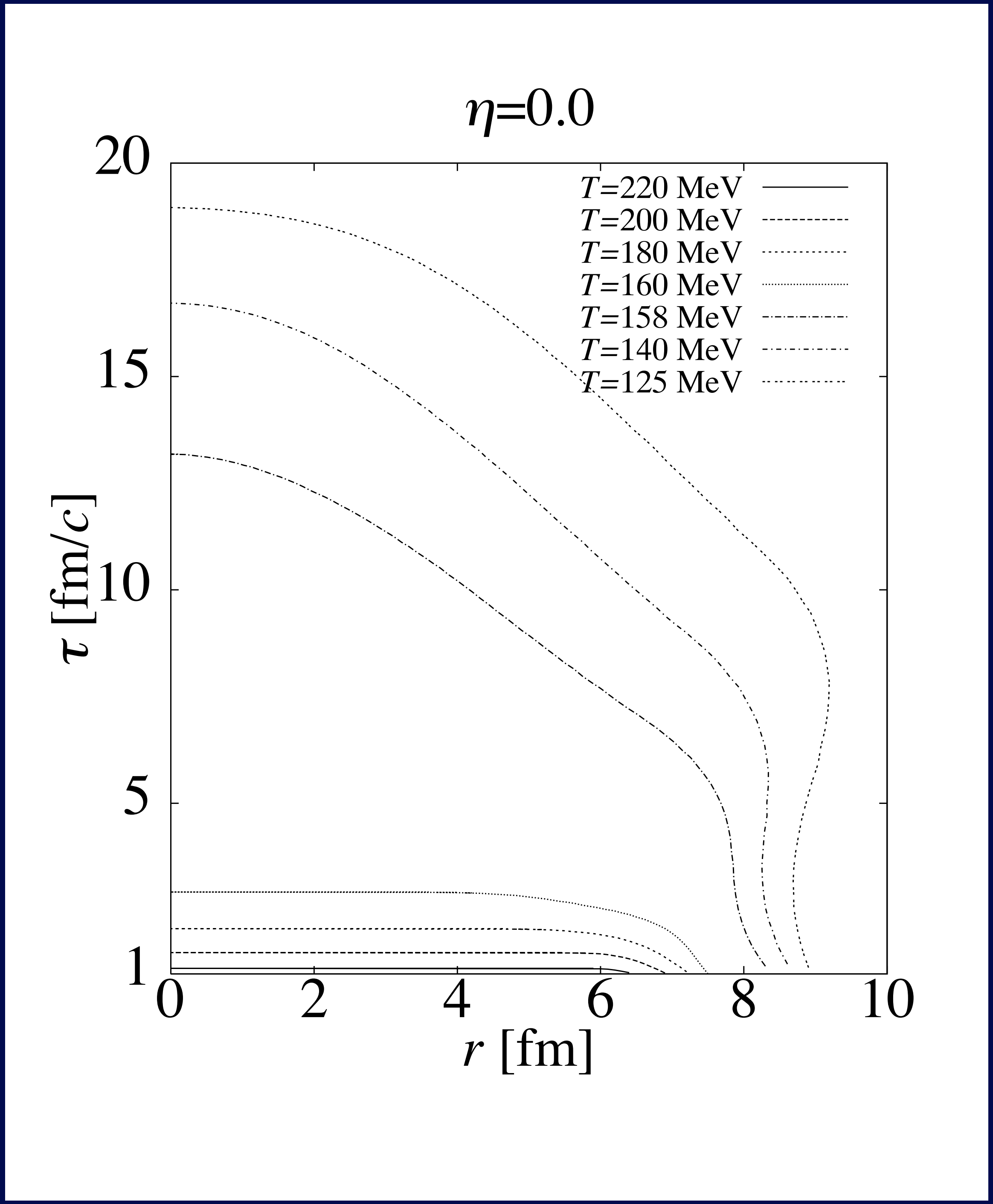
Fireball sits in flow field

$$f(x,p) = \frac{g}{(2\pi)^3}\frac{1}{\exp(p^\mu u_\mu/kT) - 1}$$

Fireball Bose distribution at freezeout temperature (controlled by QCD mass spectrum...)

$$E(x) = p^\mu u_\mu = \gamma(\vec{\beta})E - \gamma(\vec{\beta})\vec{\beta}\cdot\vec{p}$$

Cooper-Frye Formula



Hirano et al (2001)

Contours are result of numerical calculations

Unique consequence of initial conditions
+
equation of state

Choose freezeout

Variety of quarks, angular momentum, parity, etc. gives exponential rise in number of states!

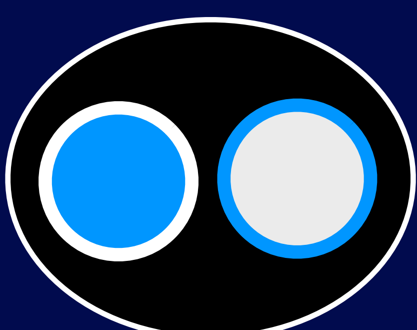
Σ

Λ Ω

p Δ Ξ

n

Baryon
(3 q or q̄)

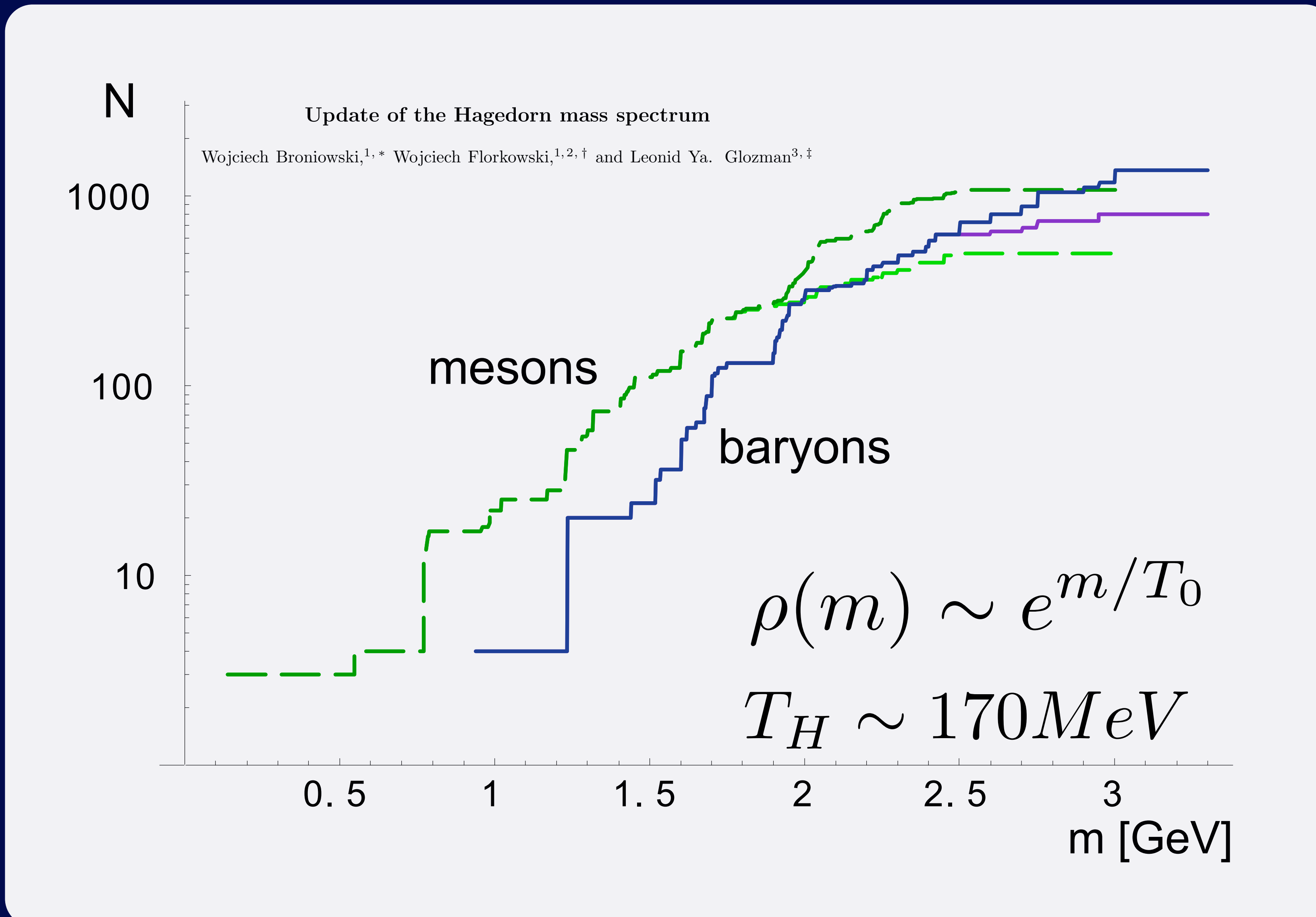


ω ρ

K π ϕ

ρ

Meson
(1 q & q̄)



Broniowski et al (2004)

$$\rho(m) \sim m^a e^{m/T_0} \rightarrow Z = \int \rho(m) e^{-m/T} \rightarrow \infty (T \geq T_0)$$

Freezeout "happens" when all hadron states available

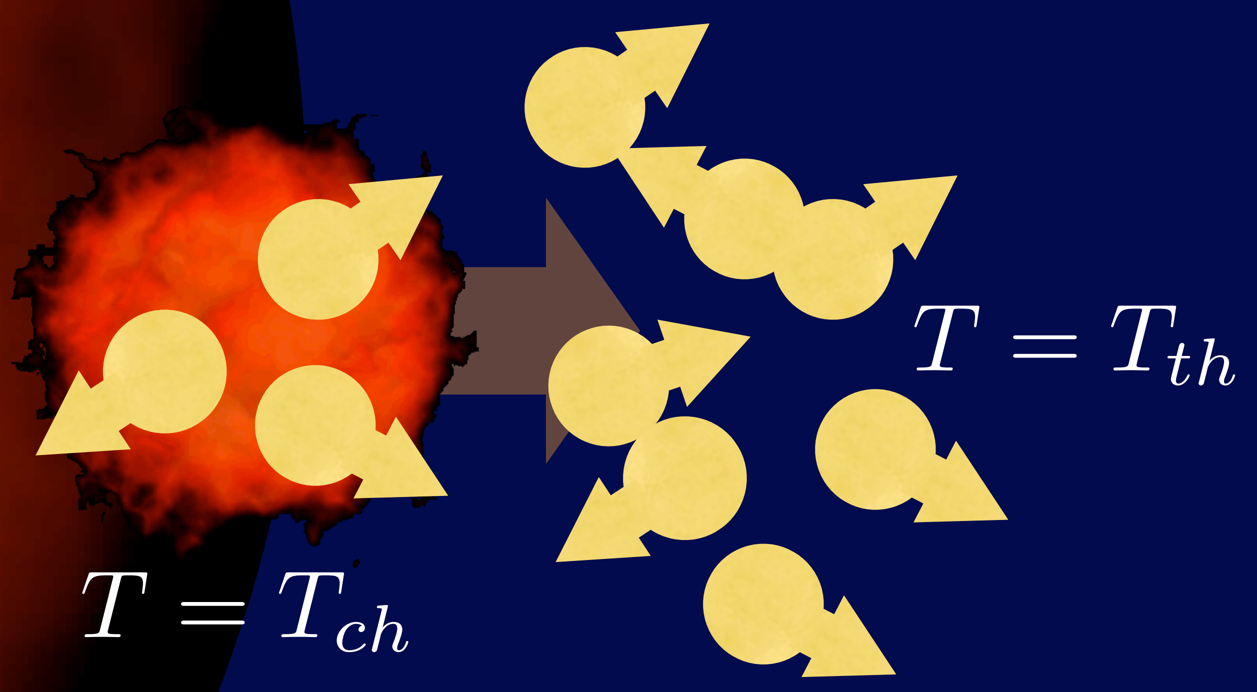
Content of Cooper-Frye

- **First law of hydro:**
  - Before freezeout, there are **no** particles
- **Cooper-Frye is a way to "fix" this**
  - A "hack" to translate space-time-velocity distribution into particles via a local "thermal model"
  - All mass dependencies happen at hypersurface
- **This is true of all hydro calculations we use**
  - Csernai et al are trying more complicated freeze-out schemes, which I won't discuss

Chemical vs. Thermal

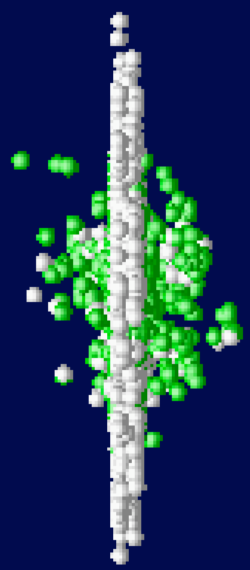
Hadronic chemistry is set by Hagedorn temperature

Some assume interacting hadron gas after freezeout (e.g. Heinz/Kolb, Shuryak et al, Hirano, etc.)

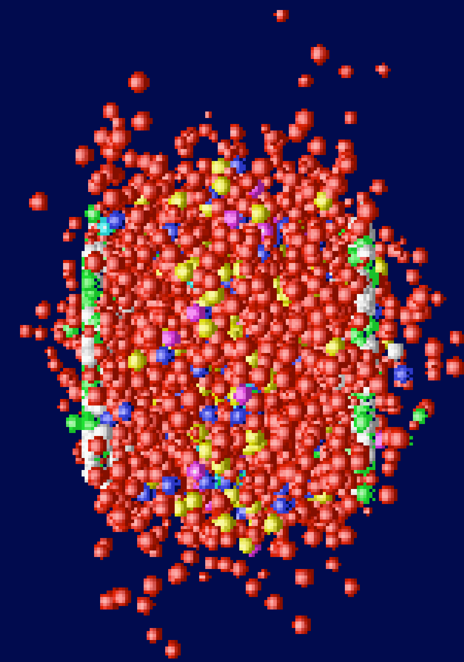


Relative amount of each species fixed at T_{ch}, but system cools to $T<T_{ch}$

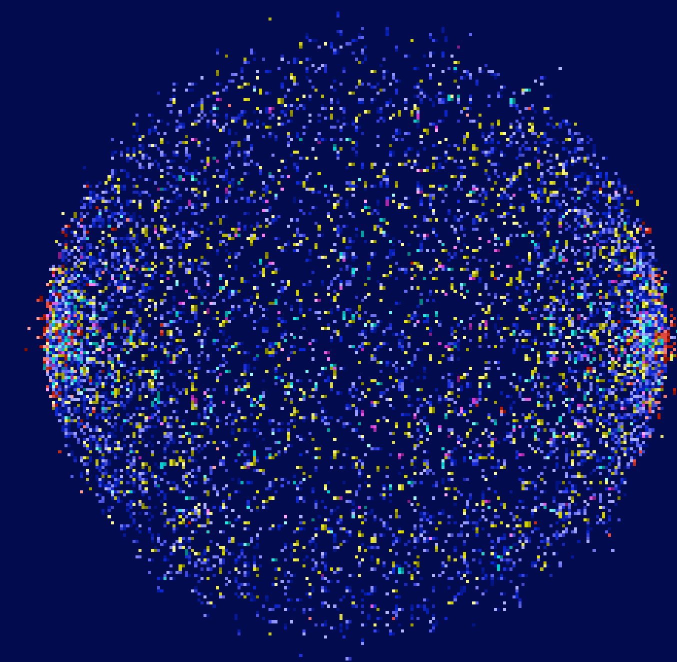
The Movie, Backwards



Initial Conditions



Hydrodynamic Evolution

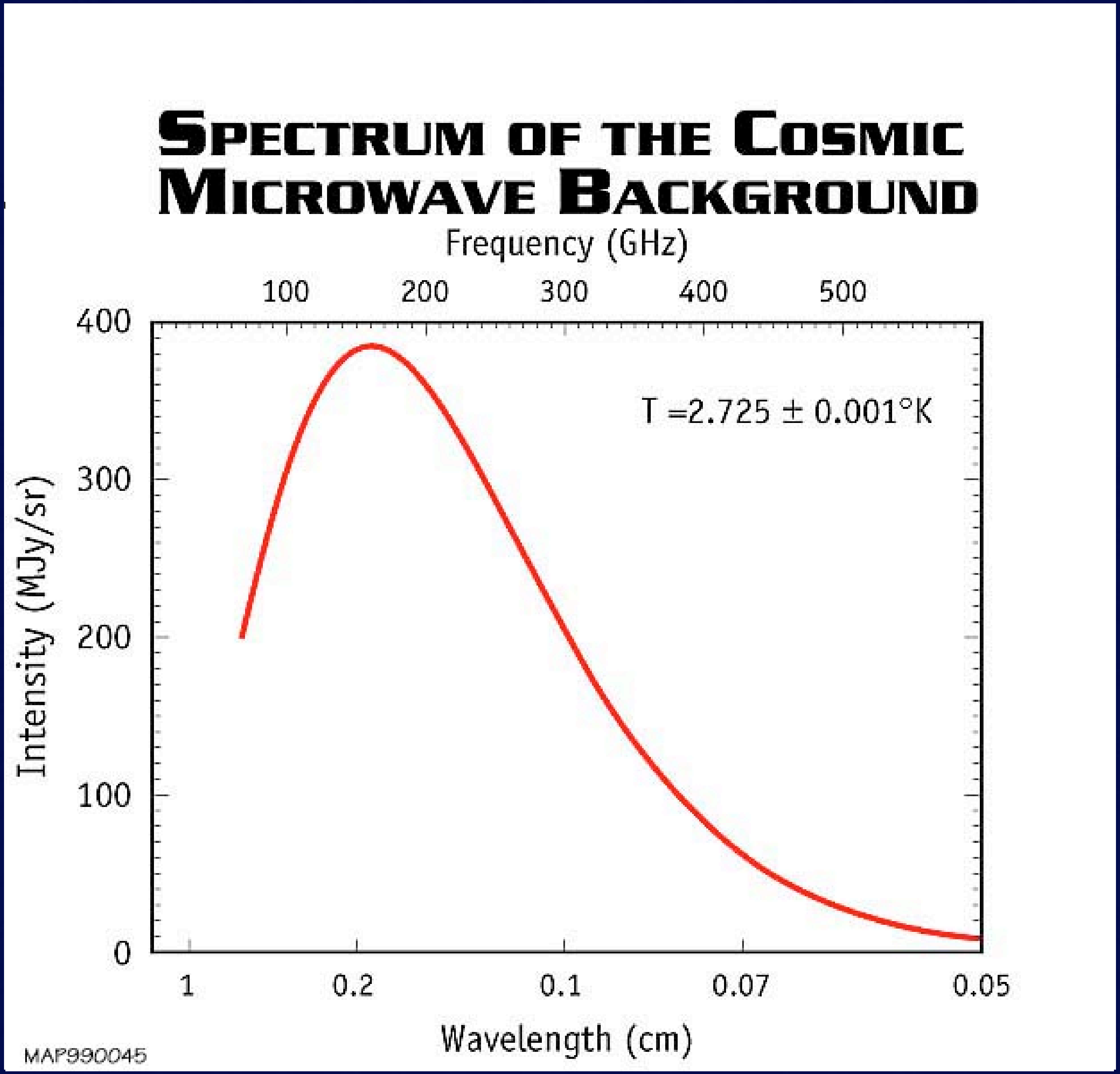


Hadronic Freezeout

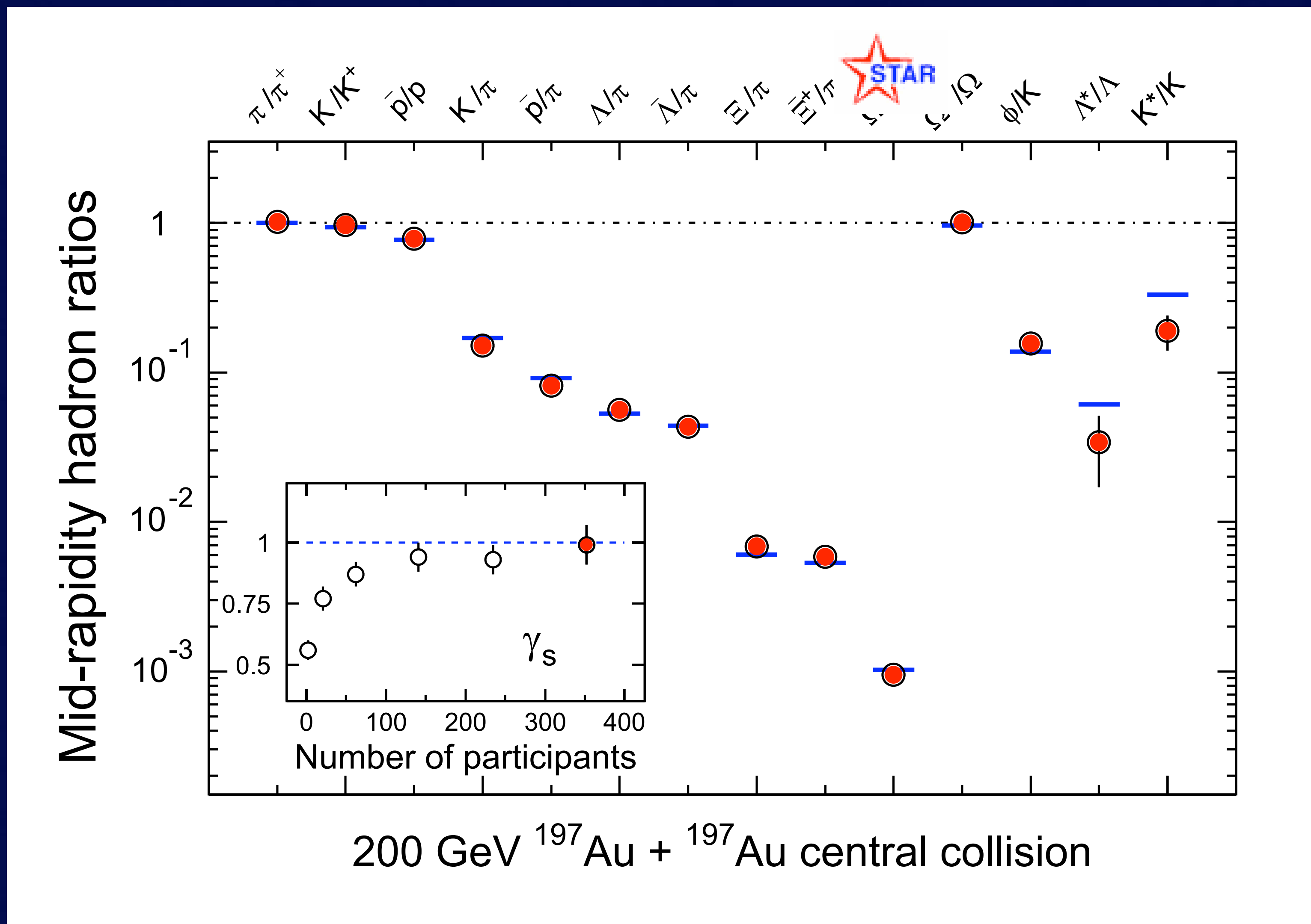
Time →

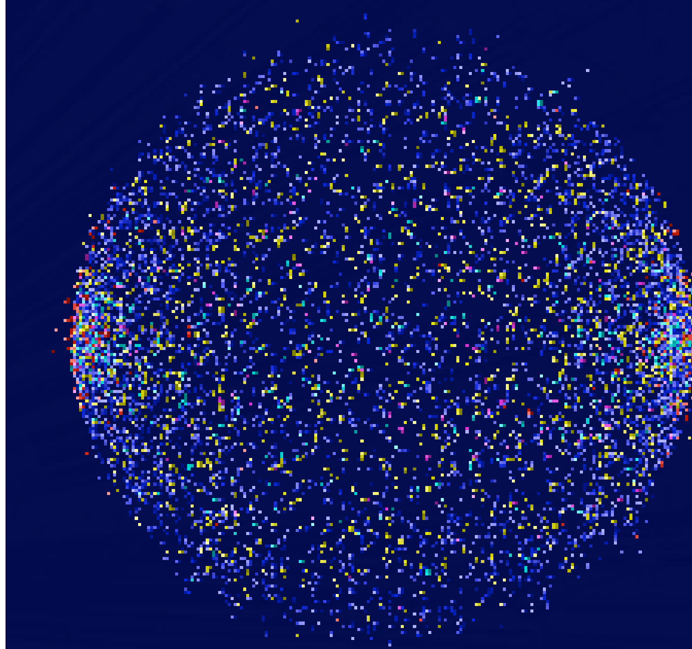
← Experimental Discovery

Strong "Blackbody"



Strong Blackbody





T	Chemical freezeout temperature
μ_B	Baryochemical potential (more matter than antimatter)

$$N_i \propto V \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(\sqrt{p^2+m^2}-\mu_B)/T} \pm 1}$$

k_BT=177 MeV~2x10^{12} ^{o}K

All hadrons are available with thermal abundances @ freezeout

Radial Flow

146 J. D. BJORKEN (1983)

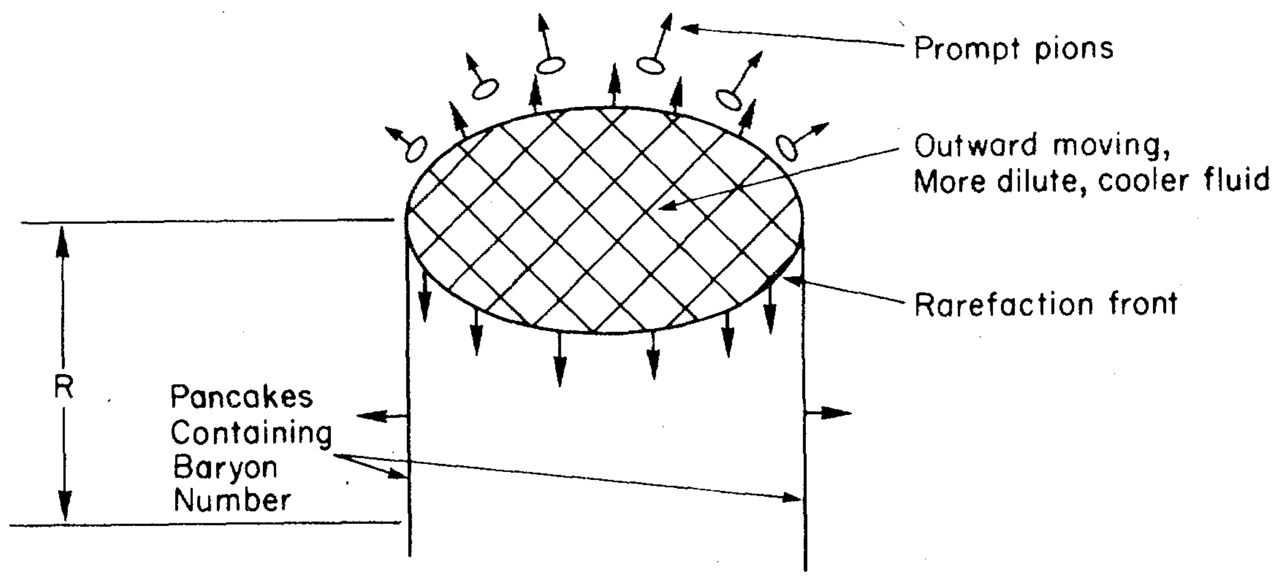
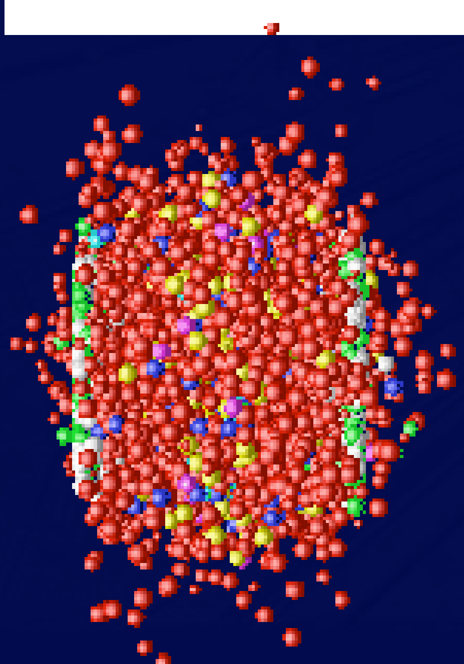
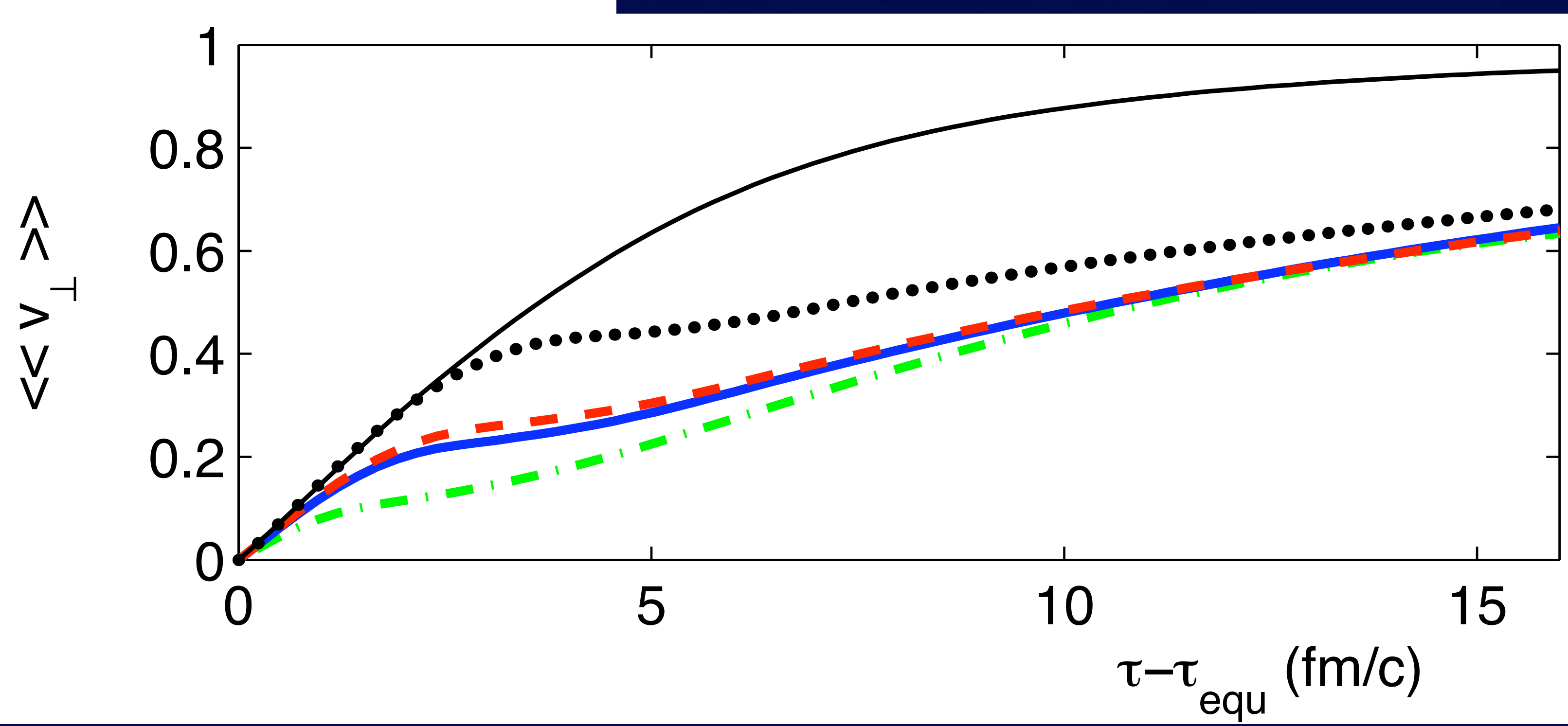


FIG. 4. Geometry of fluid expansion near the edge of the nuclei.

Radial flow starts from edge, builds up slowly, $t \sim O(R/c_s)$

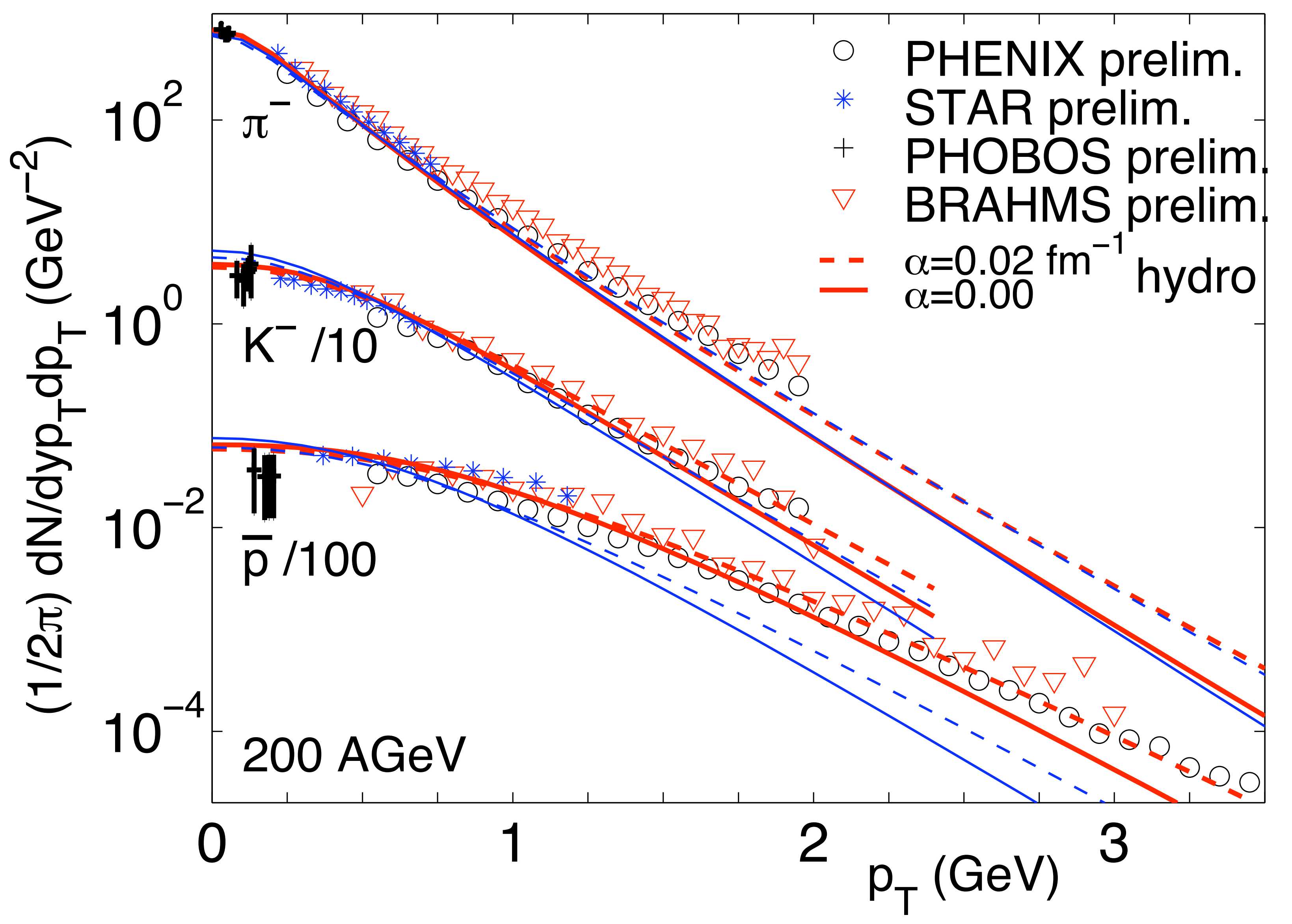


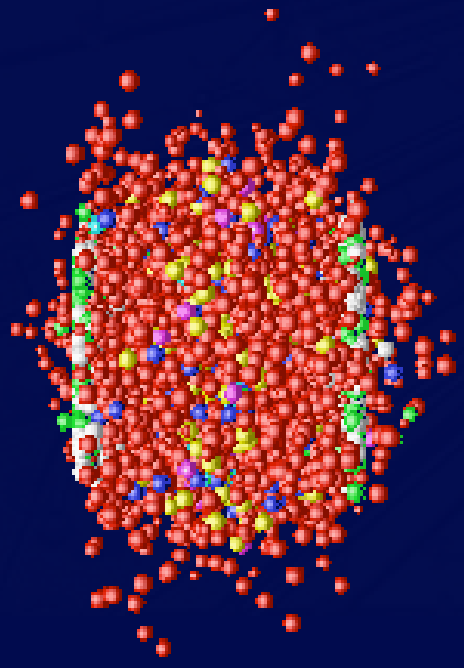


Kolb, PhD Thesis

Radial Flow

Kolb & Rapp (2003)





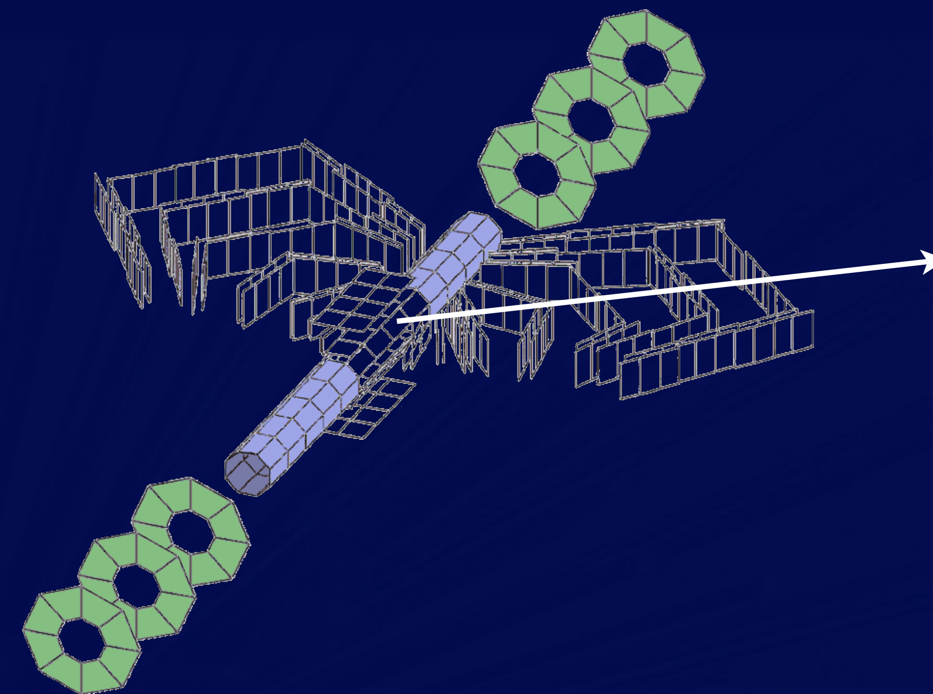
Build up of radial flow needed to describe bulk of data down to very low p_T

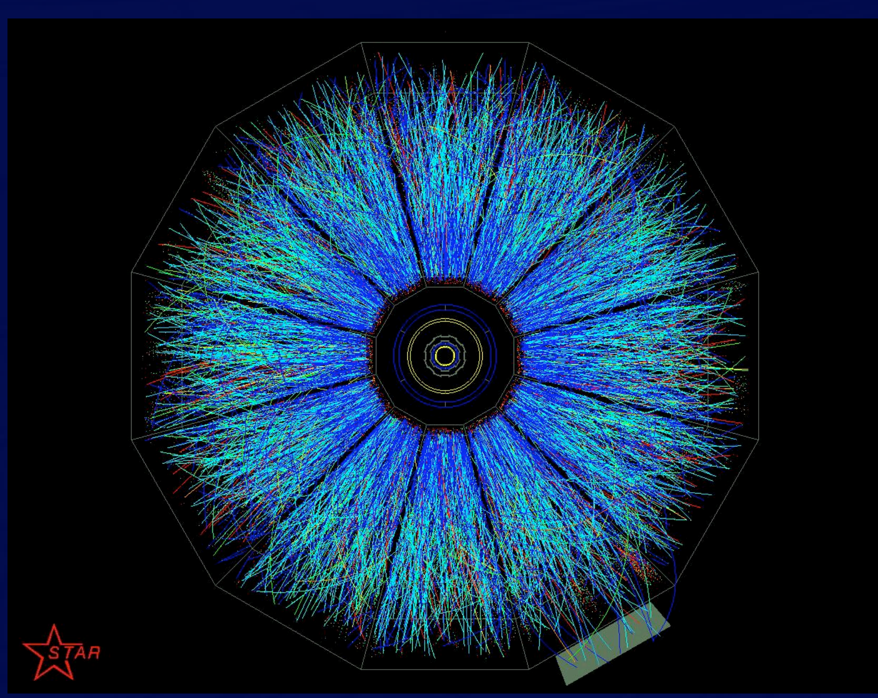
Estimating Reaction Plane

$$e_x = \sum_i cos(2\phi_i)$$

$$e_y = \sum_i sin(2\phi_i)$$

$$\Psi_{EP} = \tan^{-1}\left\{\frac{e_y}{e_x}\right\}$$





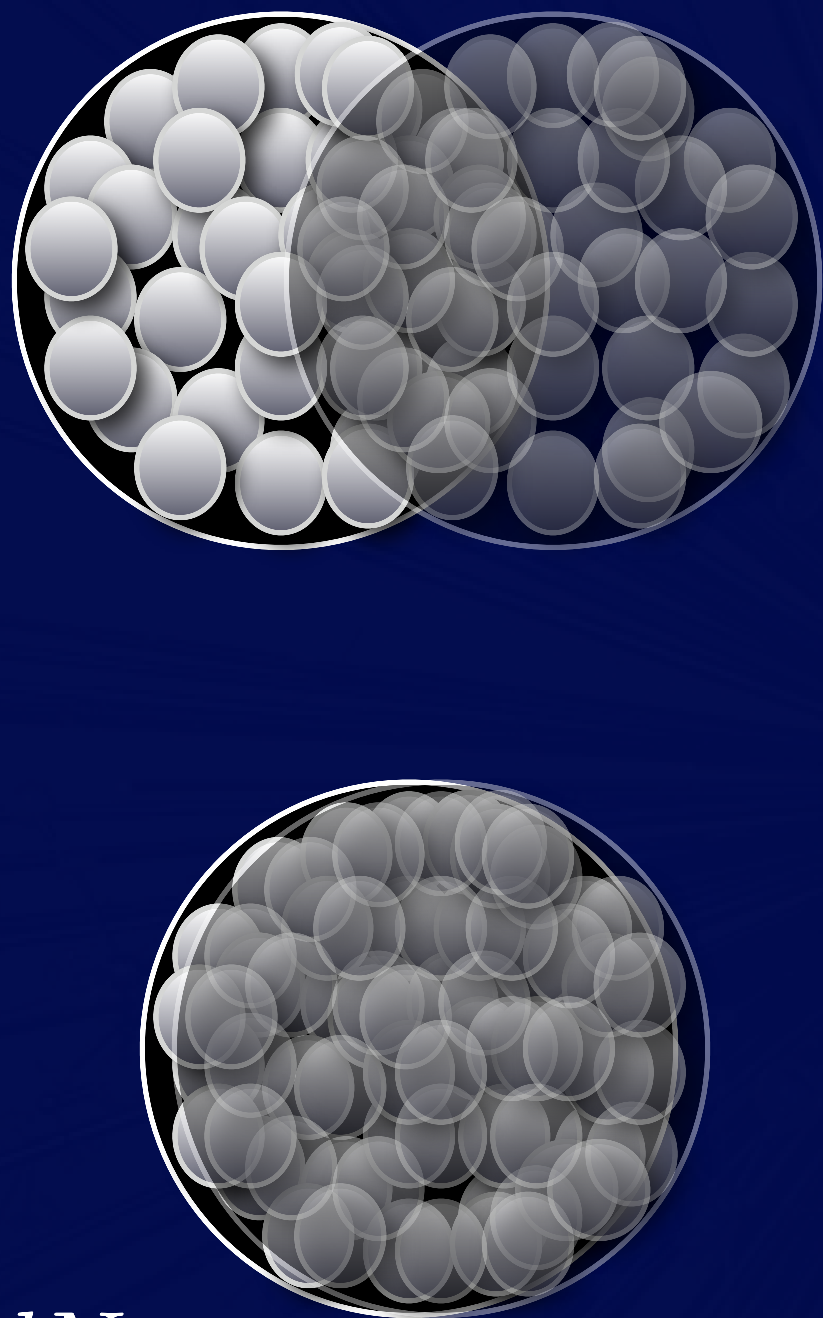
Resolution can be estimated
by comparing subevents separated in
(and away from measured track
to avoid autocorrelation!)η

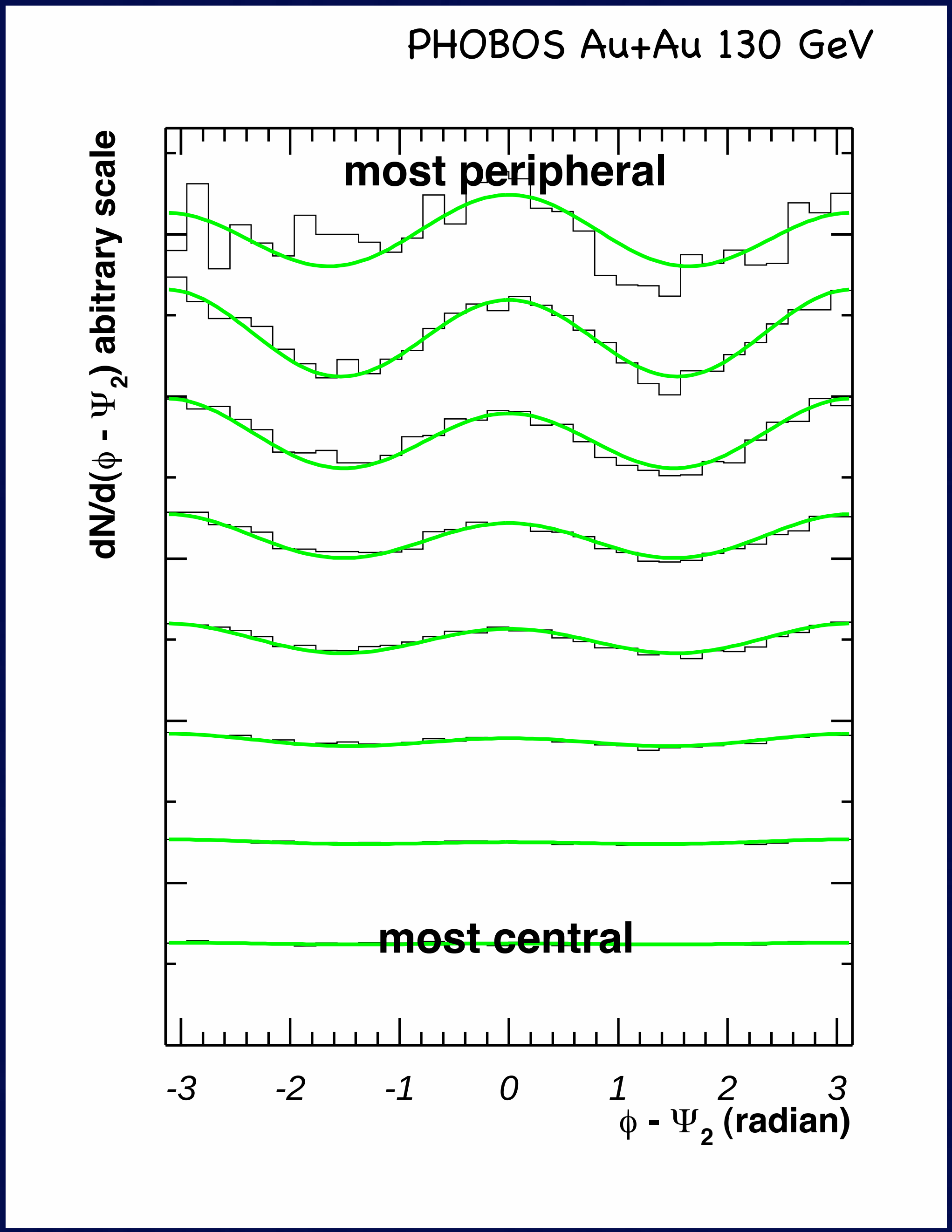
estimator

$$v_2 = \frac{\langle \cos(2[\phi_i - \Psi_R]) \rangle}{\sqrt{\langle \cos(2[\Psi_P - \Psi_N]) \rangle}}$$

resolution

v_2 @ RHIC





$$\frac{1}{N}\frac{dN}{d\phi} = 1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos(2[\phi - \Phi_R]) + \ldots$$

Nicholas already taught us how to find event plane...

Elliptic Flow in Hydro

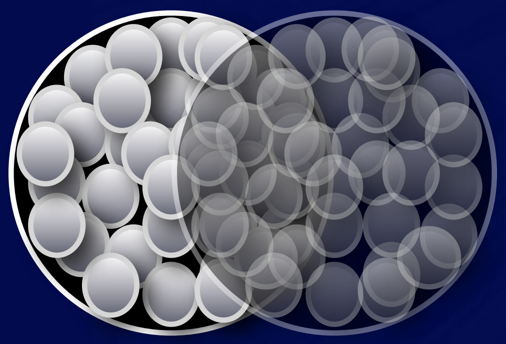
spatial "eccentricity"

$$\epsilon_x = \frac{\langle Y^2 \rangle - \langle X^2 \rangle}{\langle Y^2 \rangle + \langle X^2 \rangle}$$

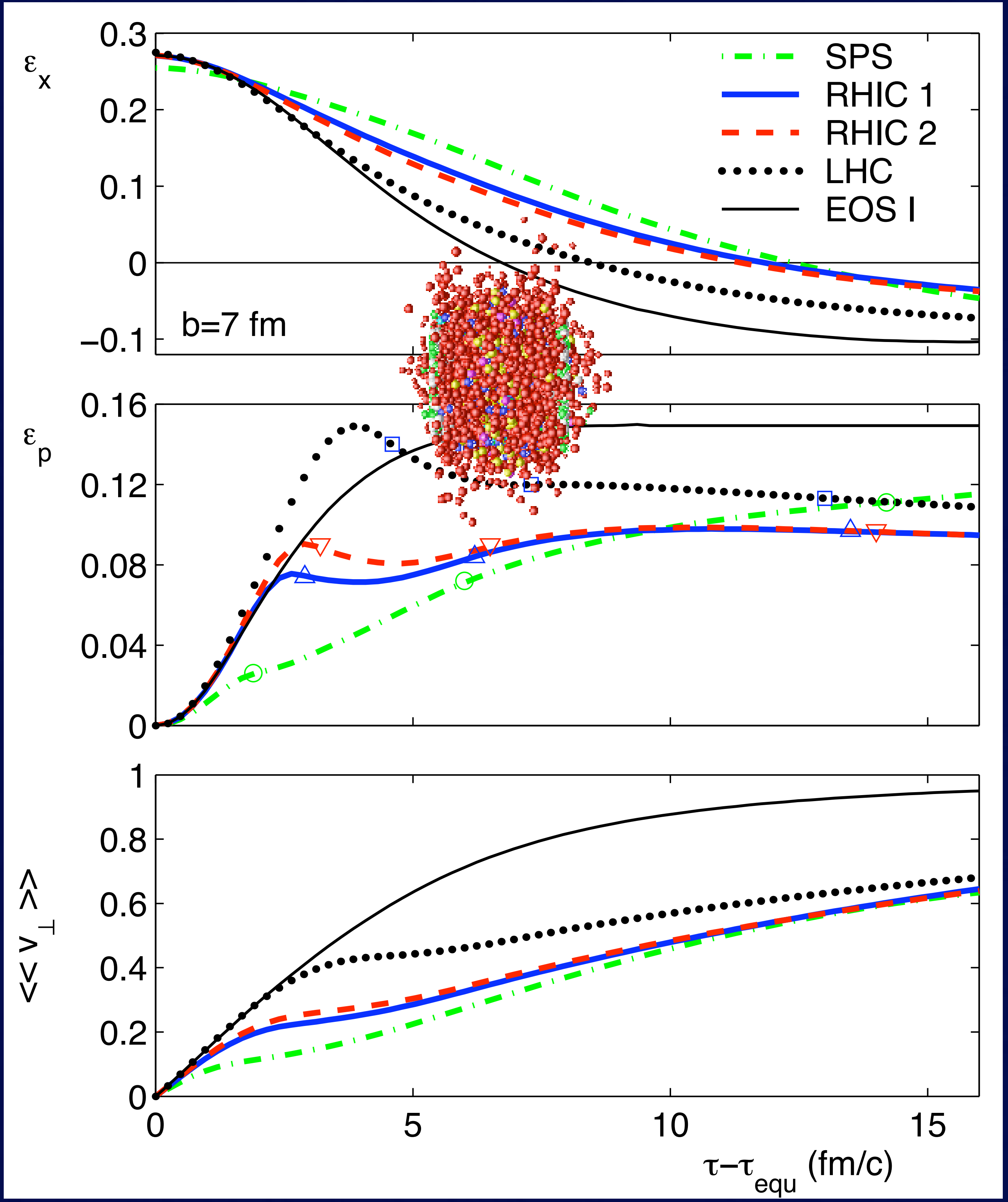
momentum "eccentricity"

$$\epsilon_p = \frac{\langle p_y^2 \rangle - \langle p_x^2 \rangle}{\langle p_y^2 \rangle + \langle p_x^2 \rangle}$$

Elliptic flow also builds up on t~O(R/c_s)



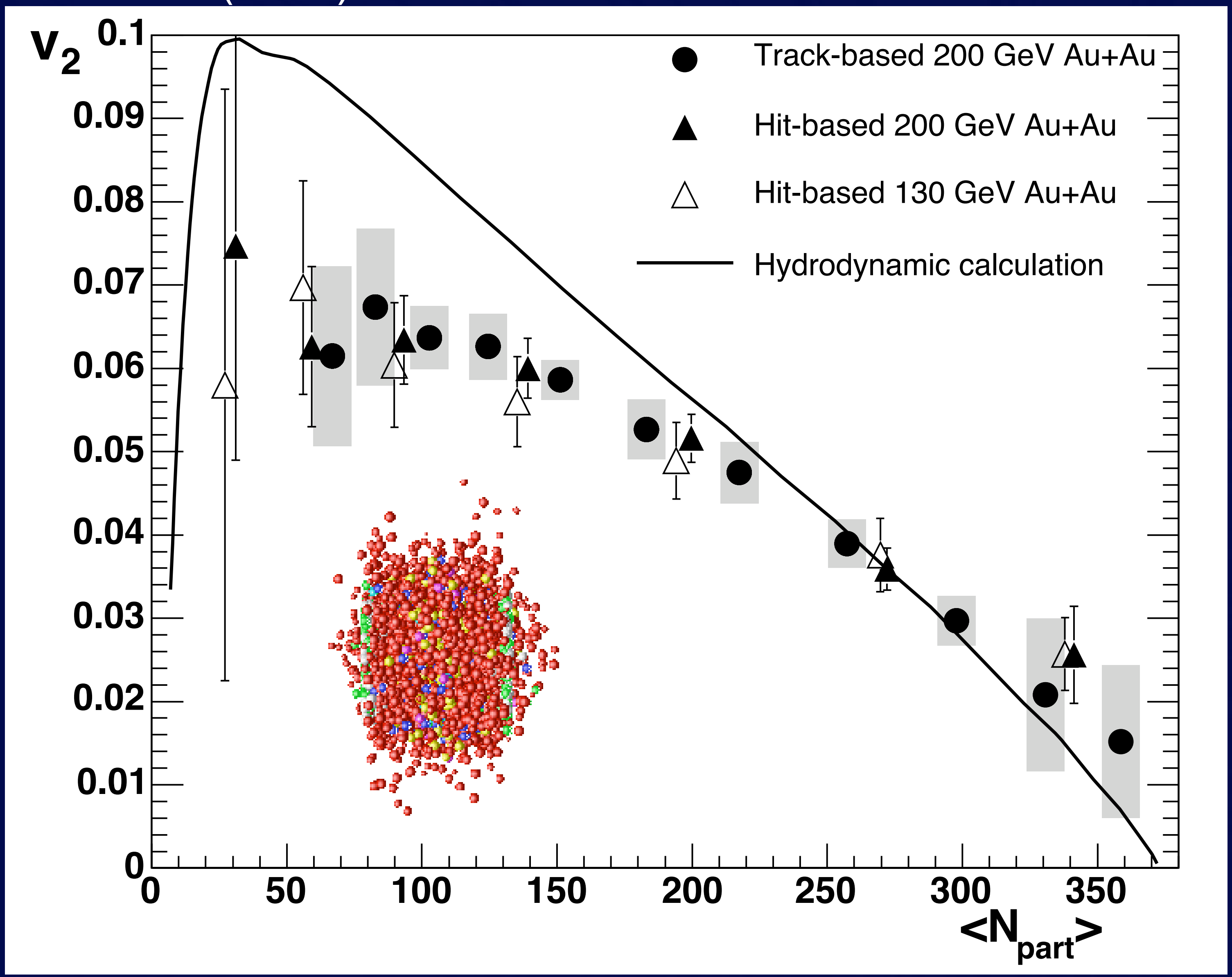
$$v_2 \propto \epsilon_x$$

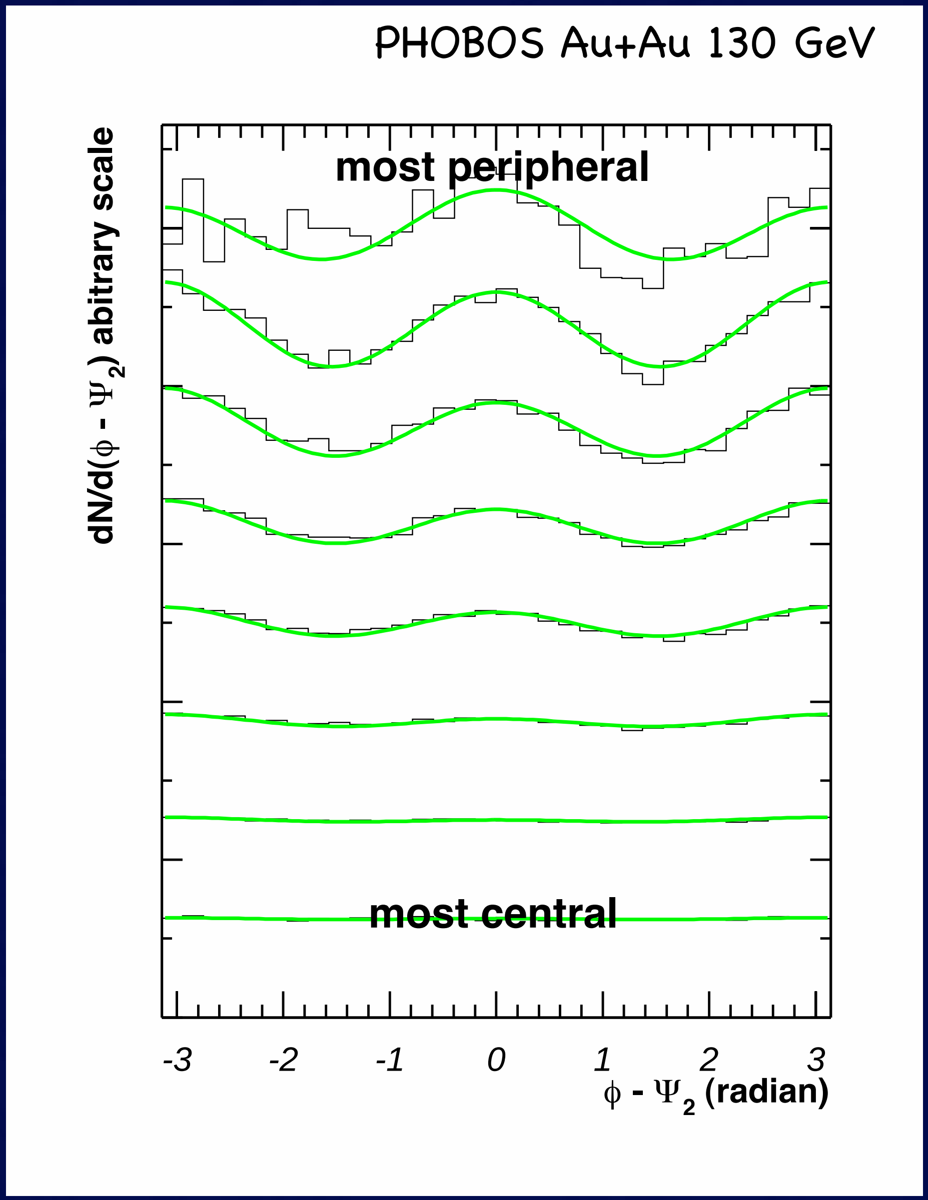


Kolb, PhD Thesis

Hydro @ RHIC

PHOBOS (2004)





hydro scales

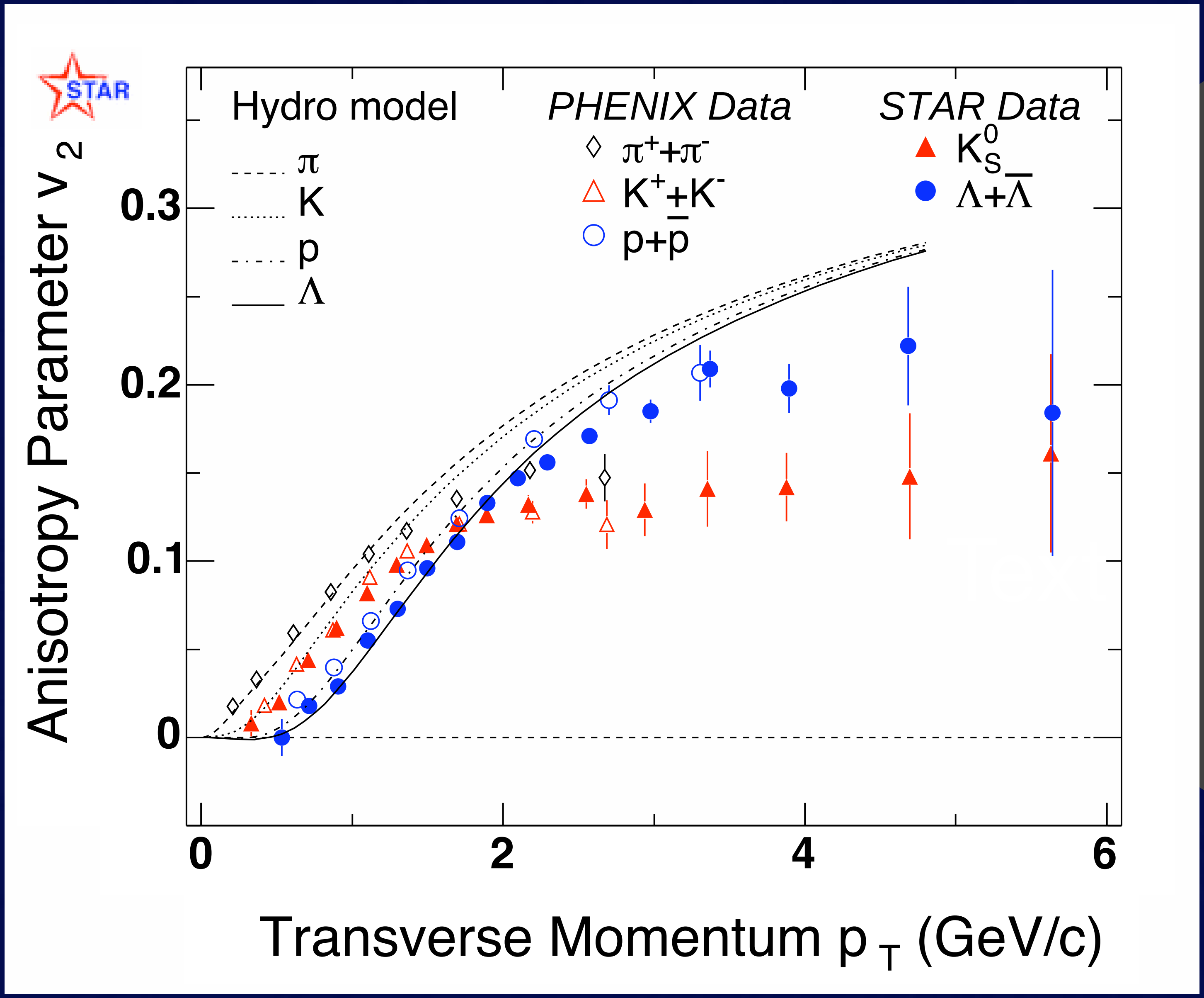
$$\tau_0 \sim 0.6 fm/c$$
$$\epsilon \sim 30\ GeV/fm^3$$

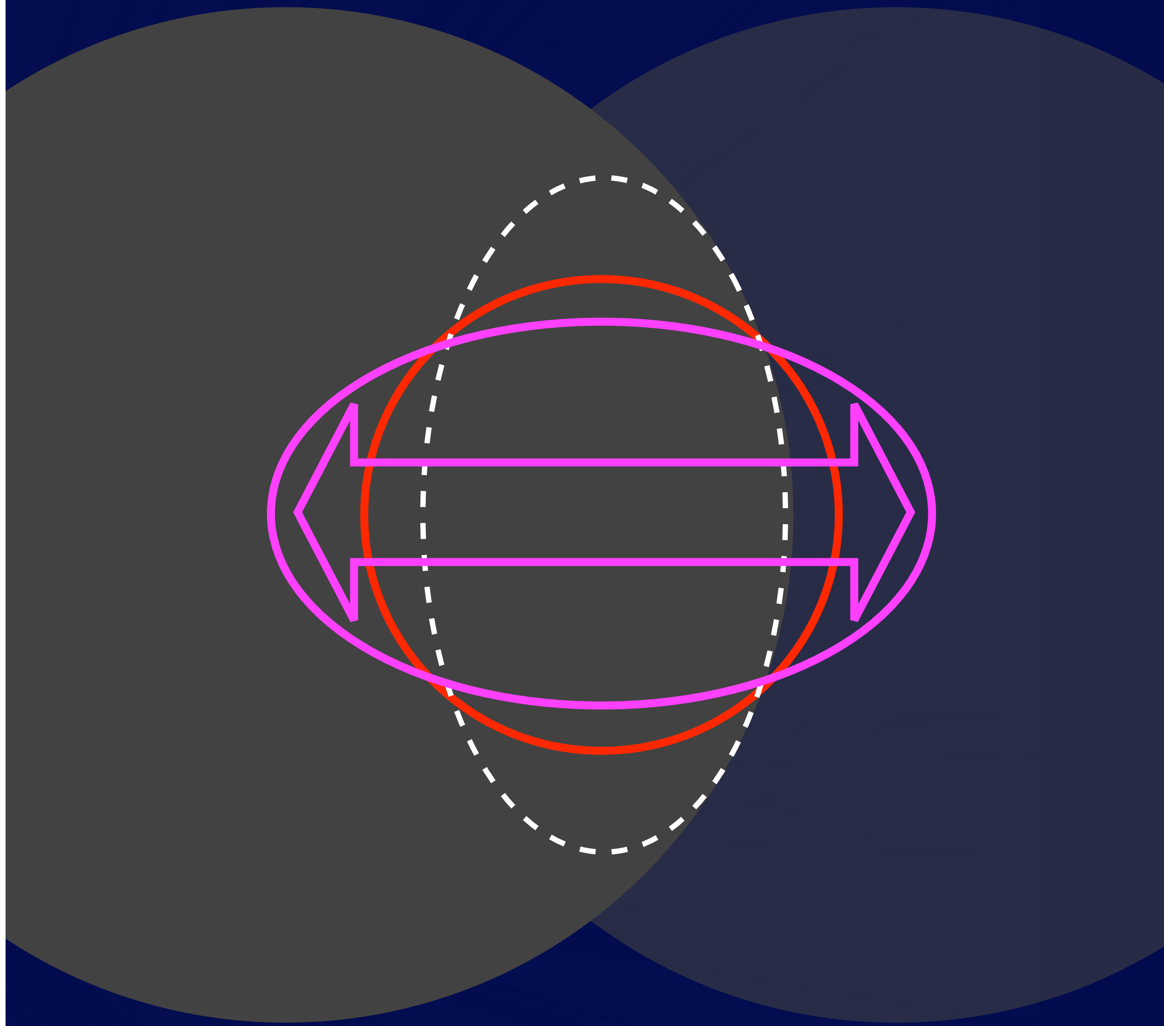
$\Longleftrightarrow$

$$\tau_0 \sim 1\ fm/c$$
$$\epsilon \sim 500\ MeV/fm^3$$

hadronic scales

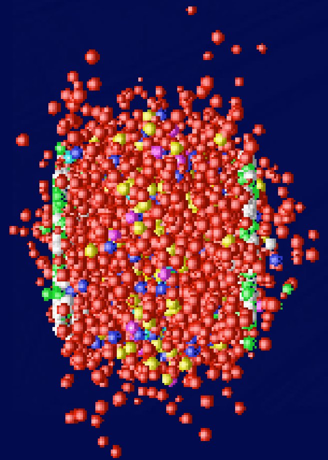
“Fine Structure”





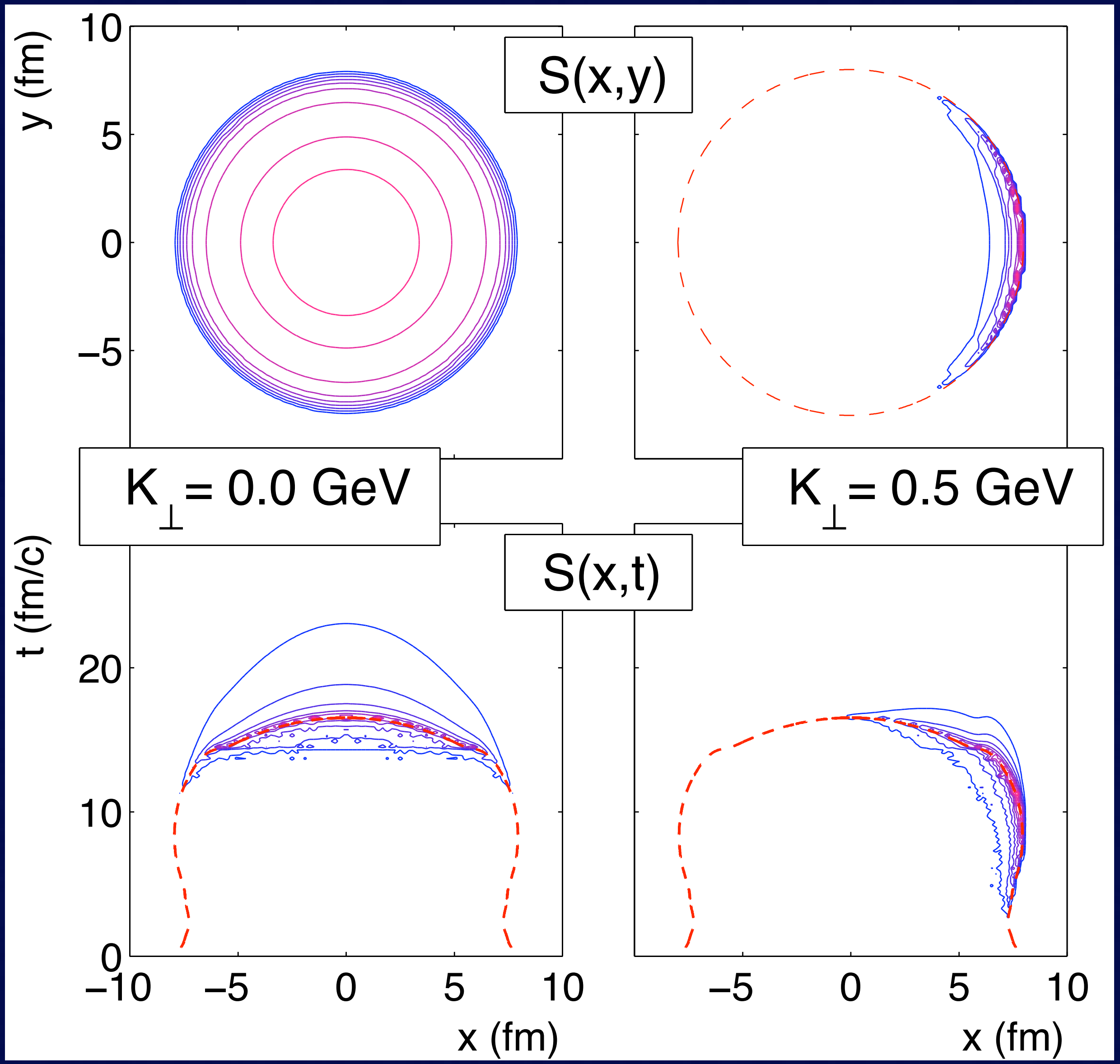
Low energy particles are ~isotropically emitted

Higher energy particles “feel” the geometry



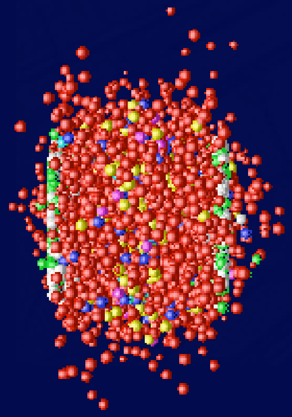
Space-Time Image

Low momentum particles are emitted isotropically from full source



P. Kolb, PhD Thesis

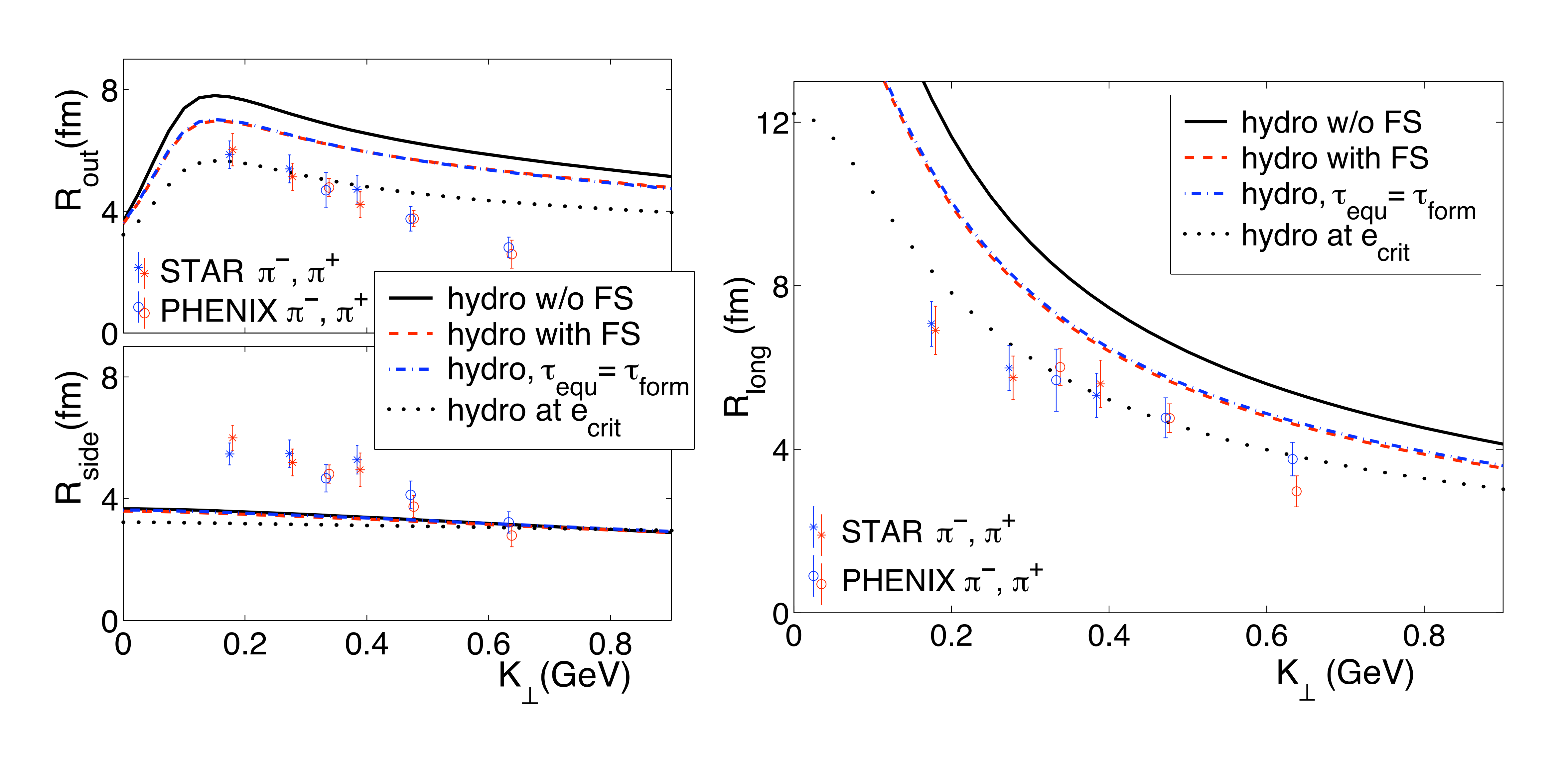
High momentum particles are emitted from a compressed volume near system edge



Study this with HBT correlations

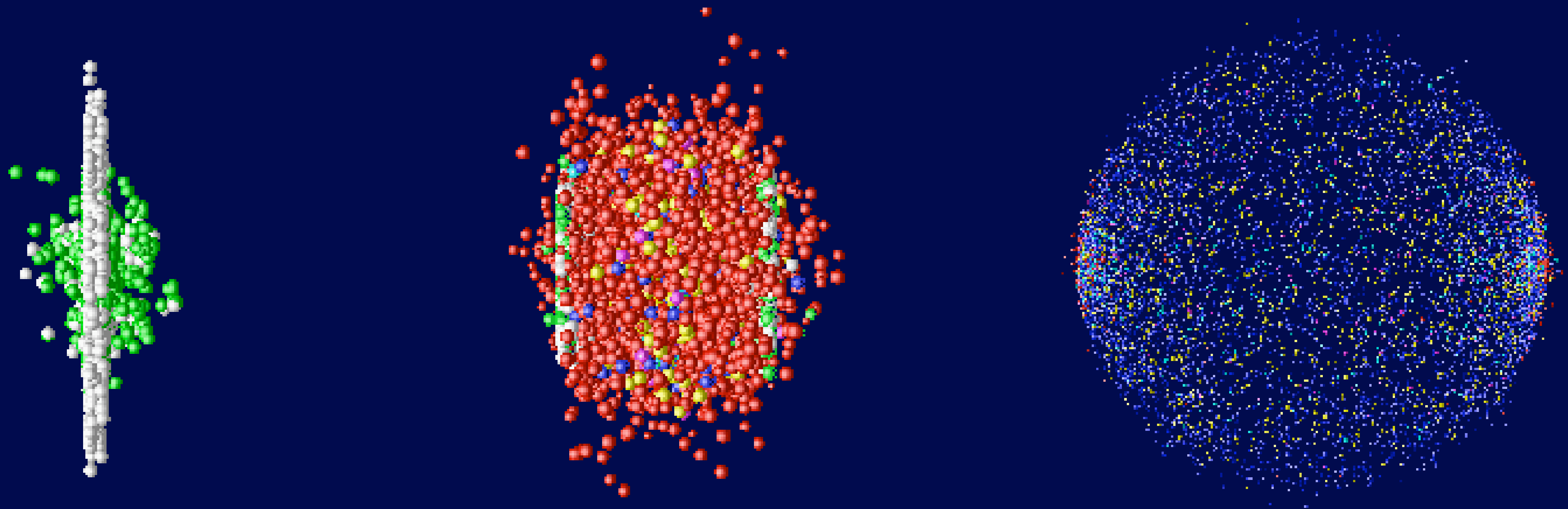
Hydro vs. HBT

P. Kolb, PhD Thesis



Qualitative trend seen in data,
totally missed by 2+1D hydro...

The Movie, Backwards



Initial Conditions

Hydrodynamic Evolution

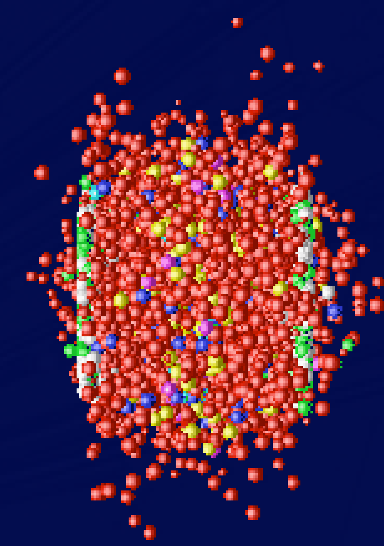
Hadronic Freezeout

Time →

← Experimental Discovery

What have we seen?

thermalized, collective matter that is...



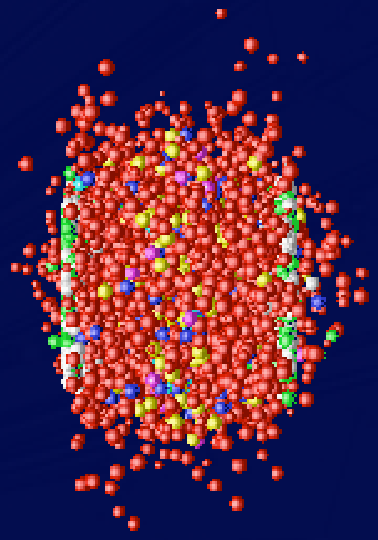
Hotter (>10^{12} °K)
Denser (>30 GeV/fm^3)
Smaller (~6 fm)
Forms faster ($\tau_0<1$ fm/c)

than other known liquids

and perfect?

Do we know that it has zero viscosity?

Does it have attractive interactions characteristic of liquids?

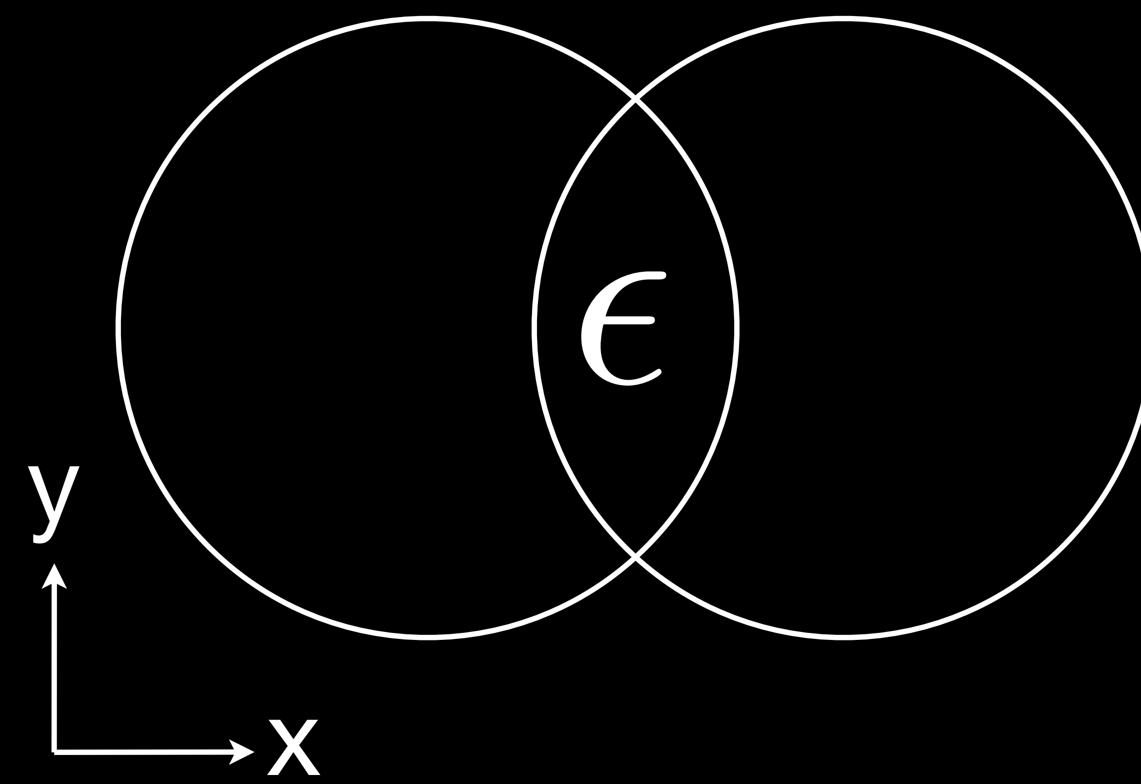


Perfect Liquid?

Eccentricity

Overlap zone where matter thermalizes has a particular "shape" vs. impact parameter

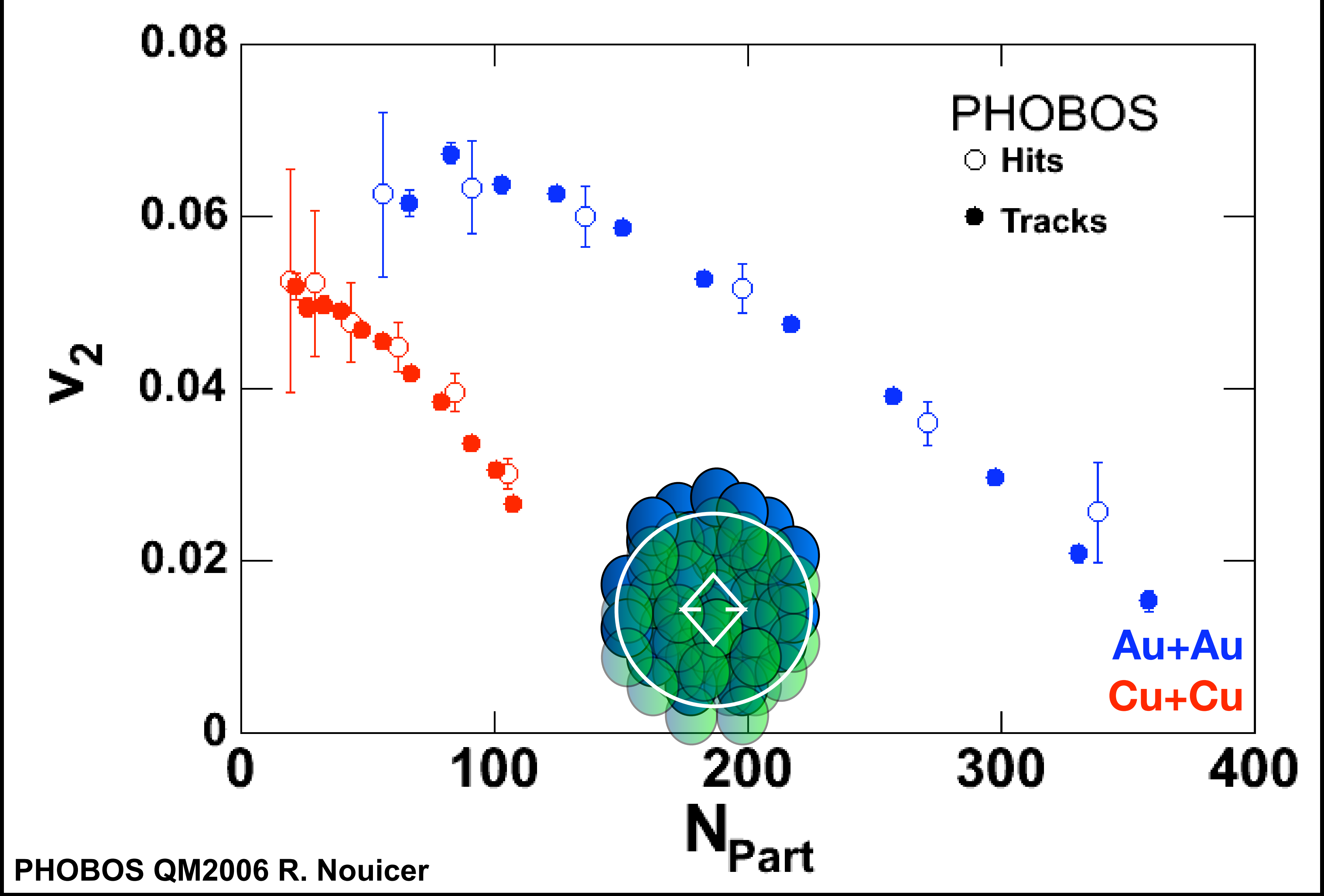
$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$



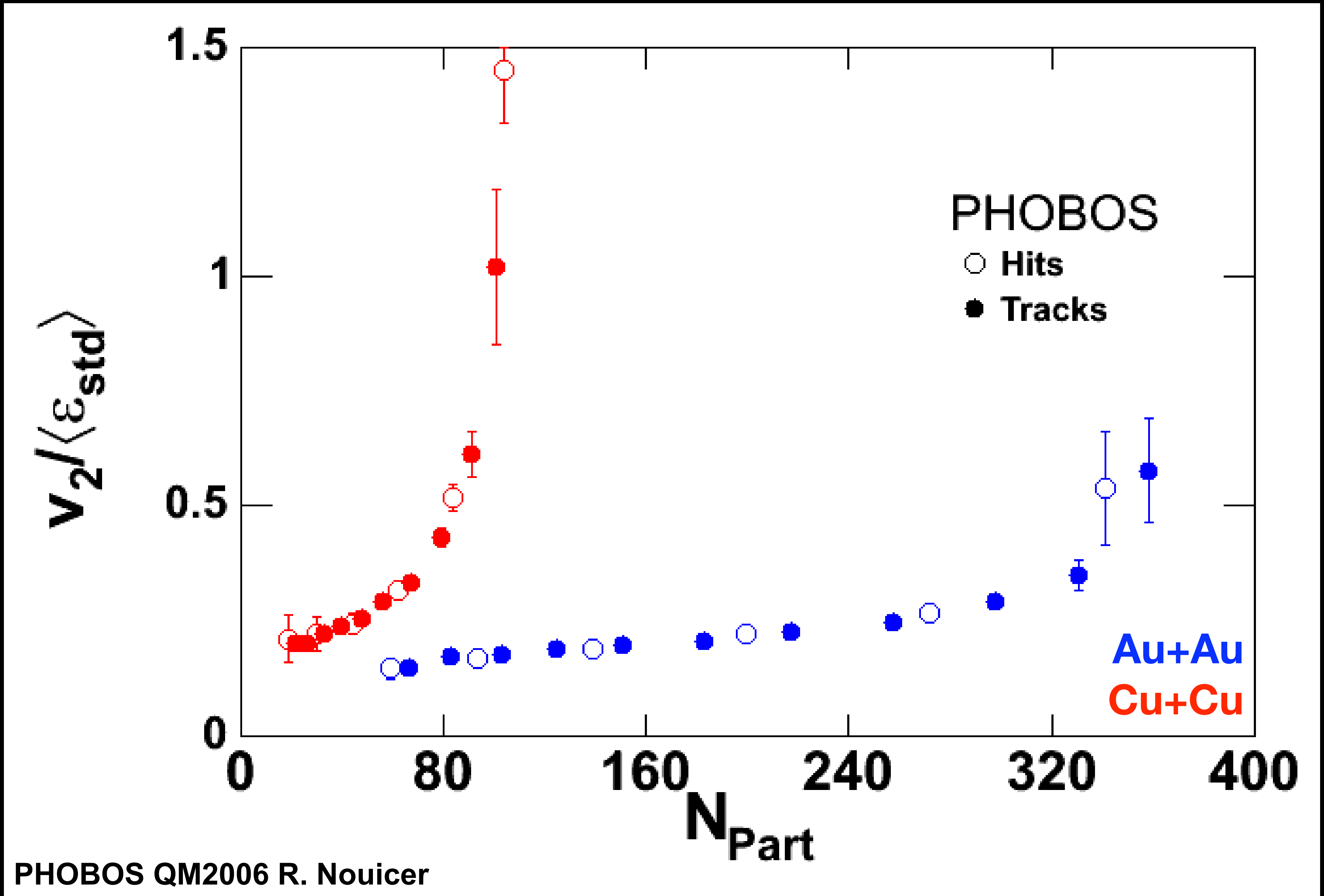
Generically, hydro predicts complete transfer of spatial anisotropy into momentum anisotropy!

$$v_2 \propto \epsilon$$

Does v_2 follow ϵ?



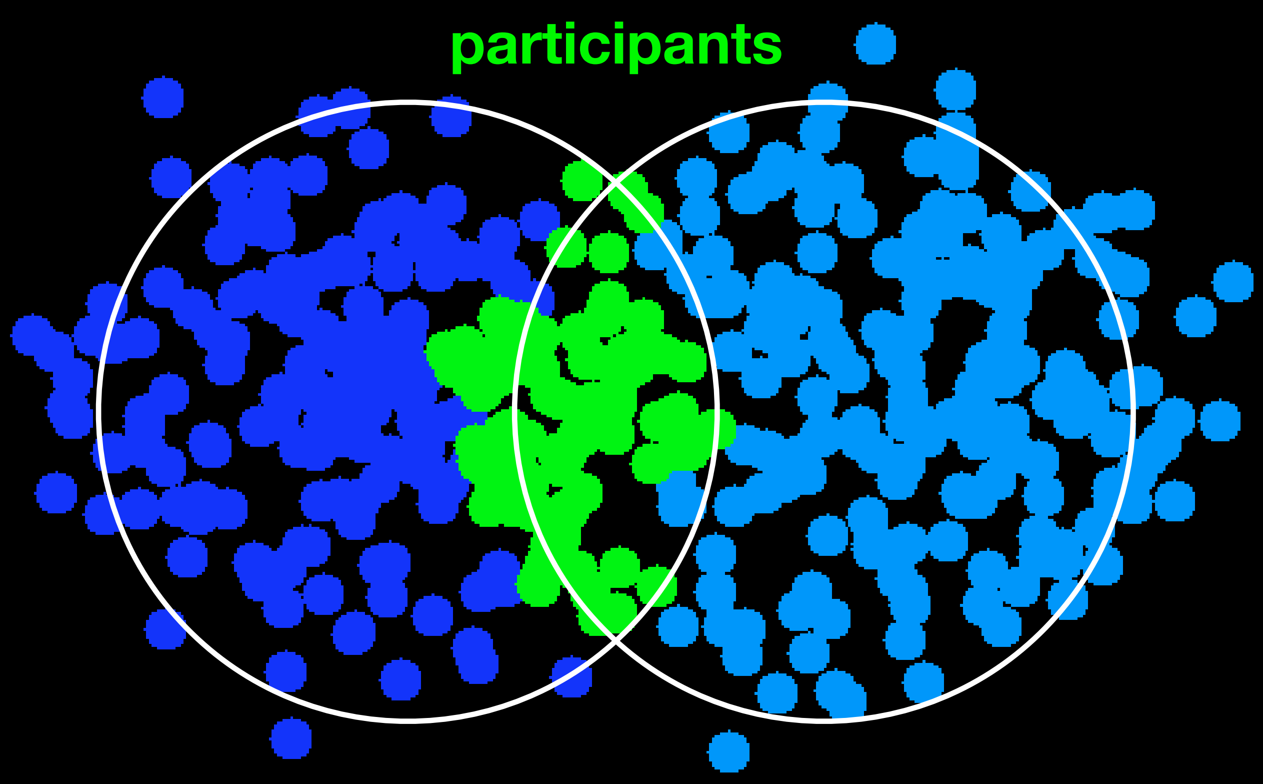
Something wrong...



Eccentricity Fluctuations

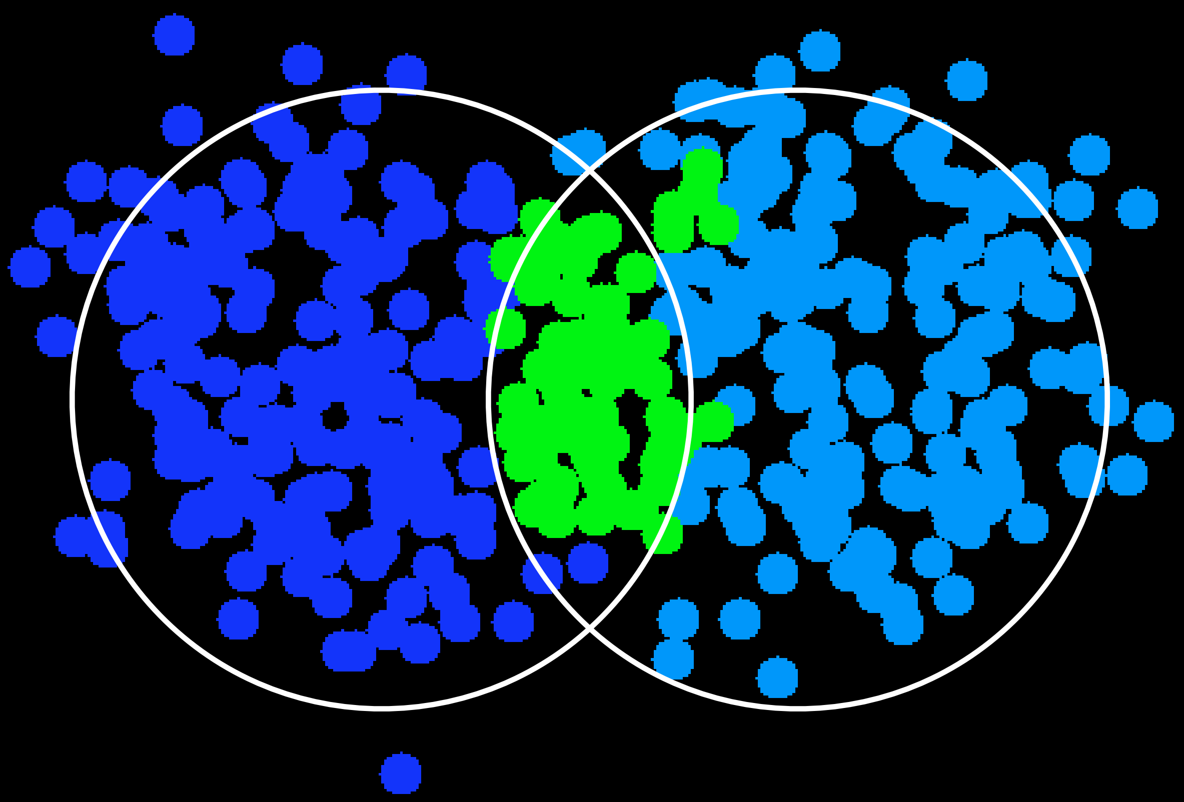
Smooth nuclei

Discrete Nucleons ("Glauber Monte Carlo" approach)

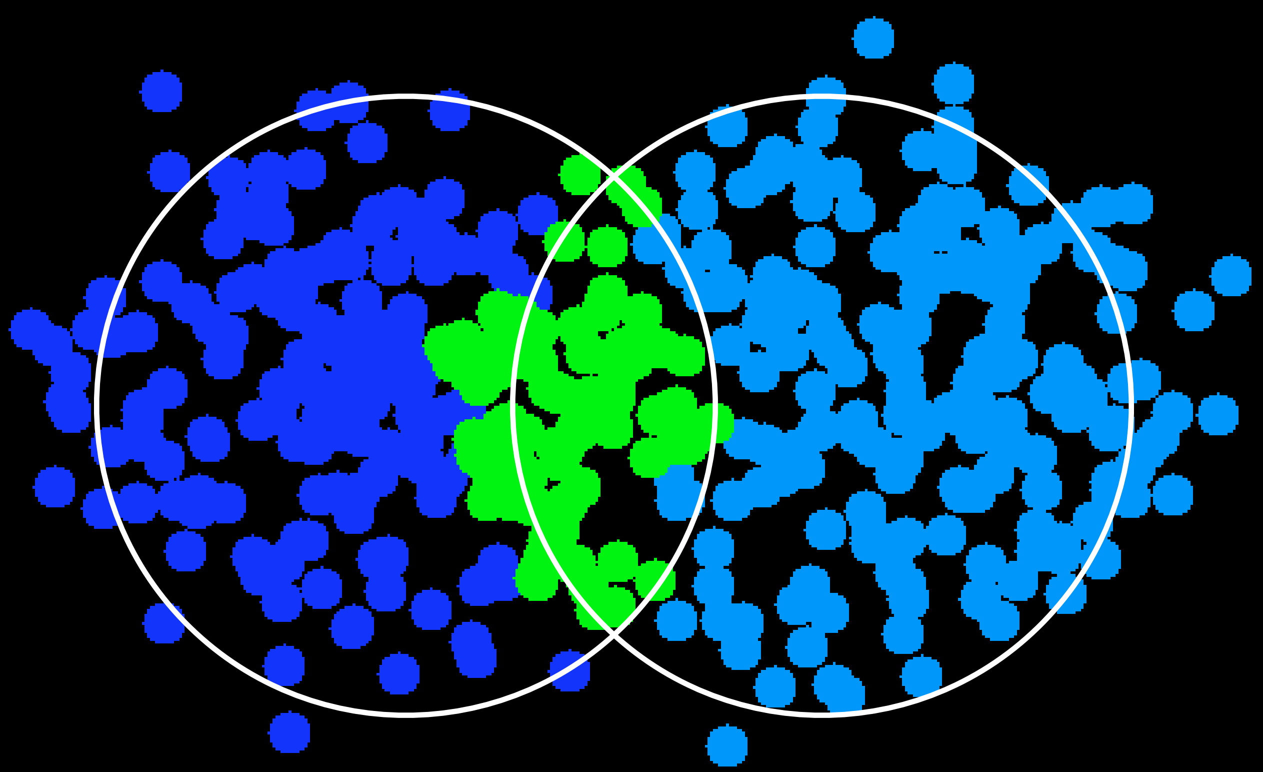


We know nuclei are made of nucleons,
Why "insist" that an <u>average</u> density
matters for flow measurements?

Au+Au



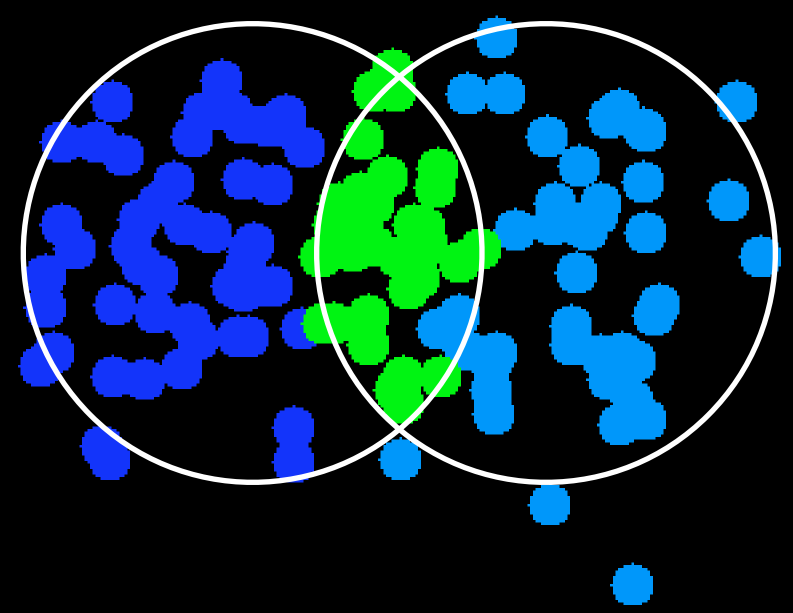
Au+Au



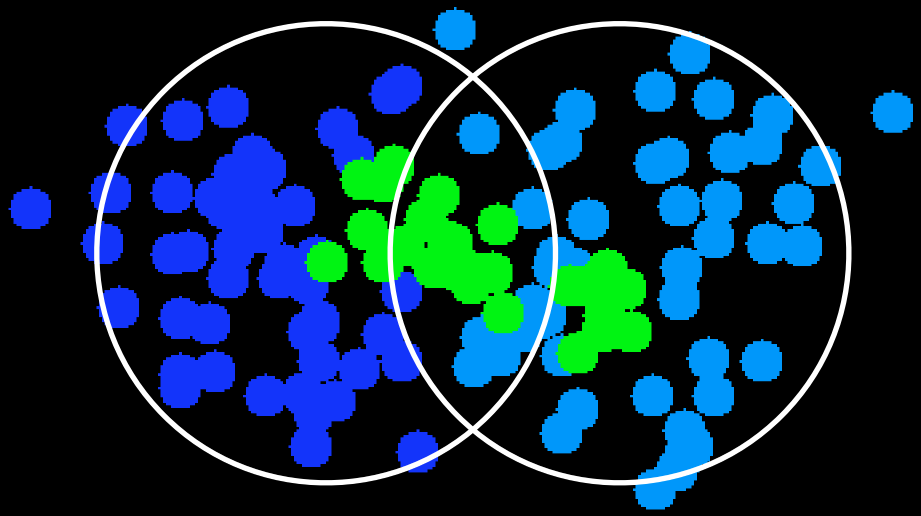
Participants trace out overlap zone, but include

1. Fluctuations (finite number per event)
2. Correlations (it takes two to tango...)

Cu+Cu

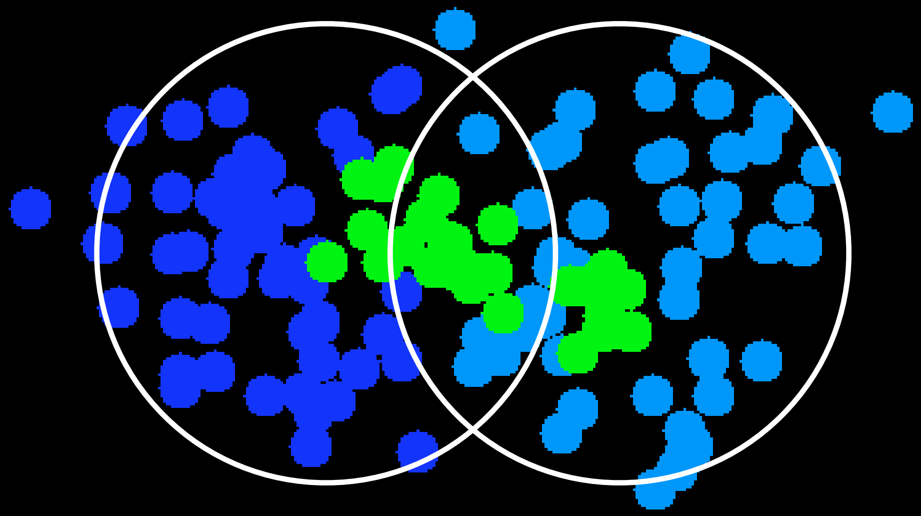


Cu+Cu



Fluctuations can seriously deviate from nominal overlap zone for small numbers of nucleons

Cu+Cu

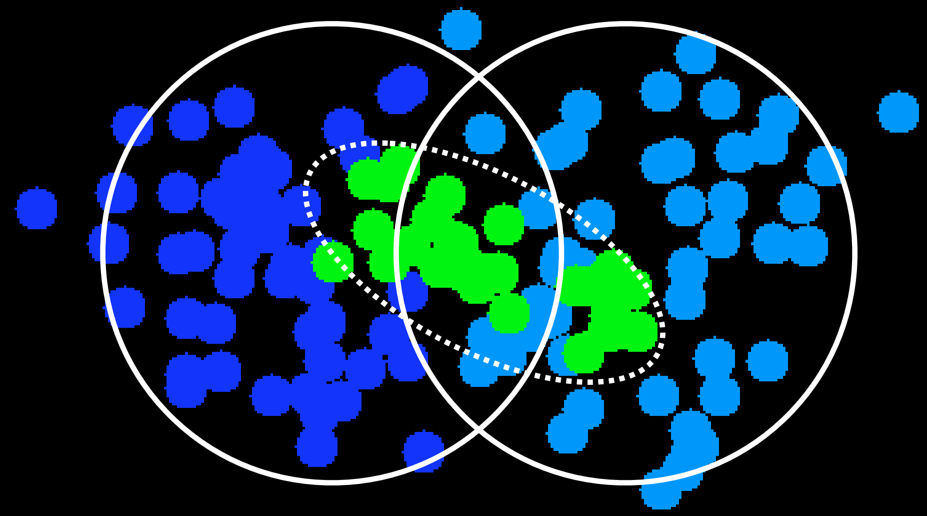


$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

“Standard eccentricity”

Cu+Cu

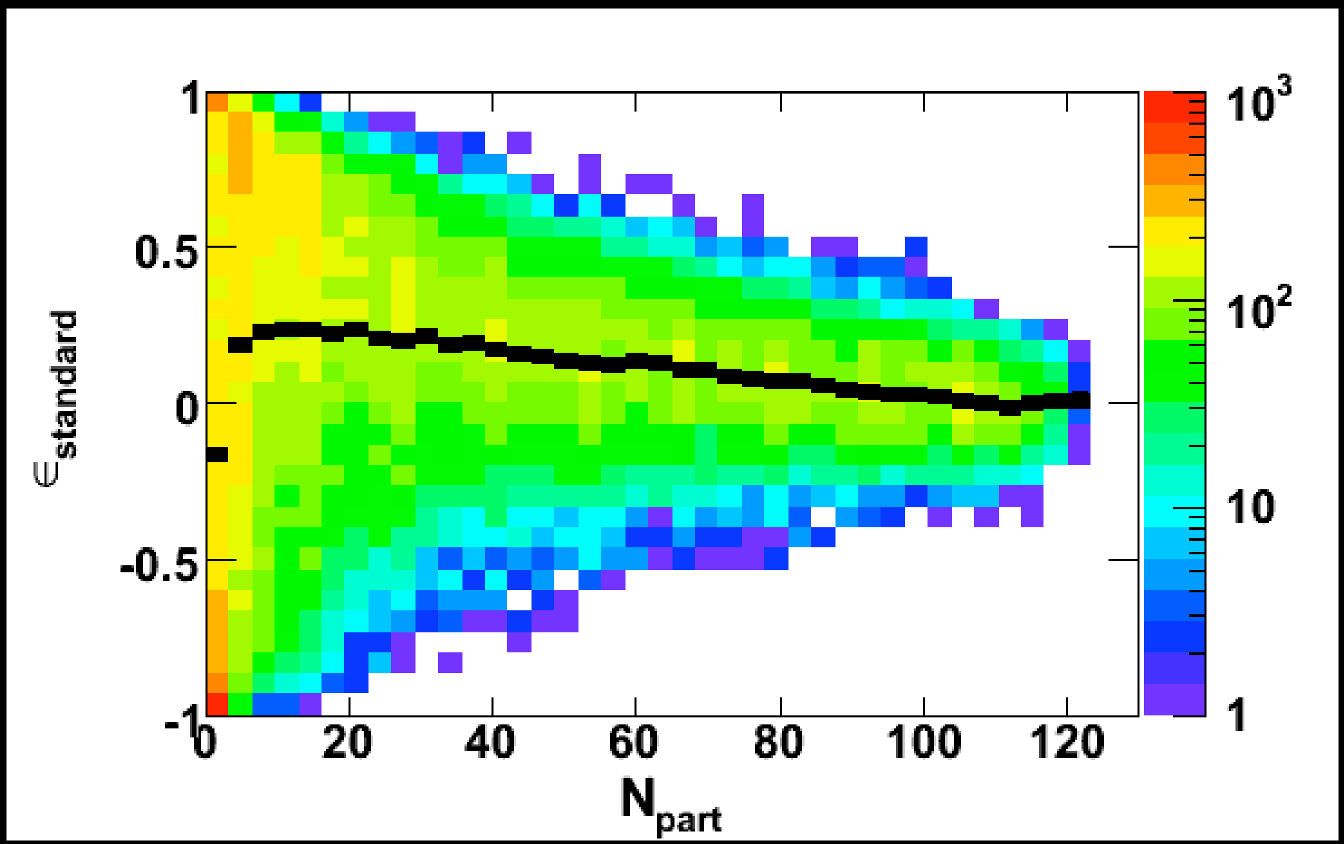
Principal axes make sense if v_2 depends on shape of produced matter, not the reaction plane



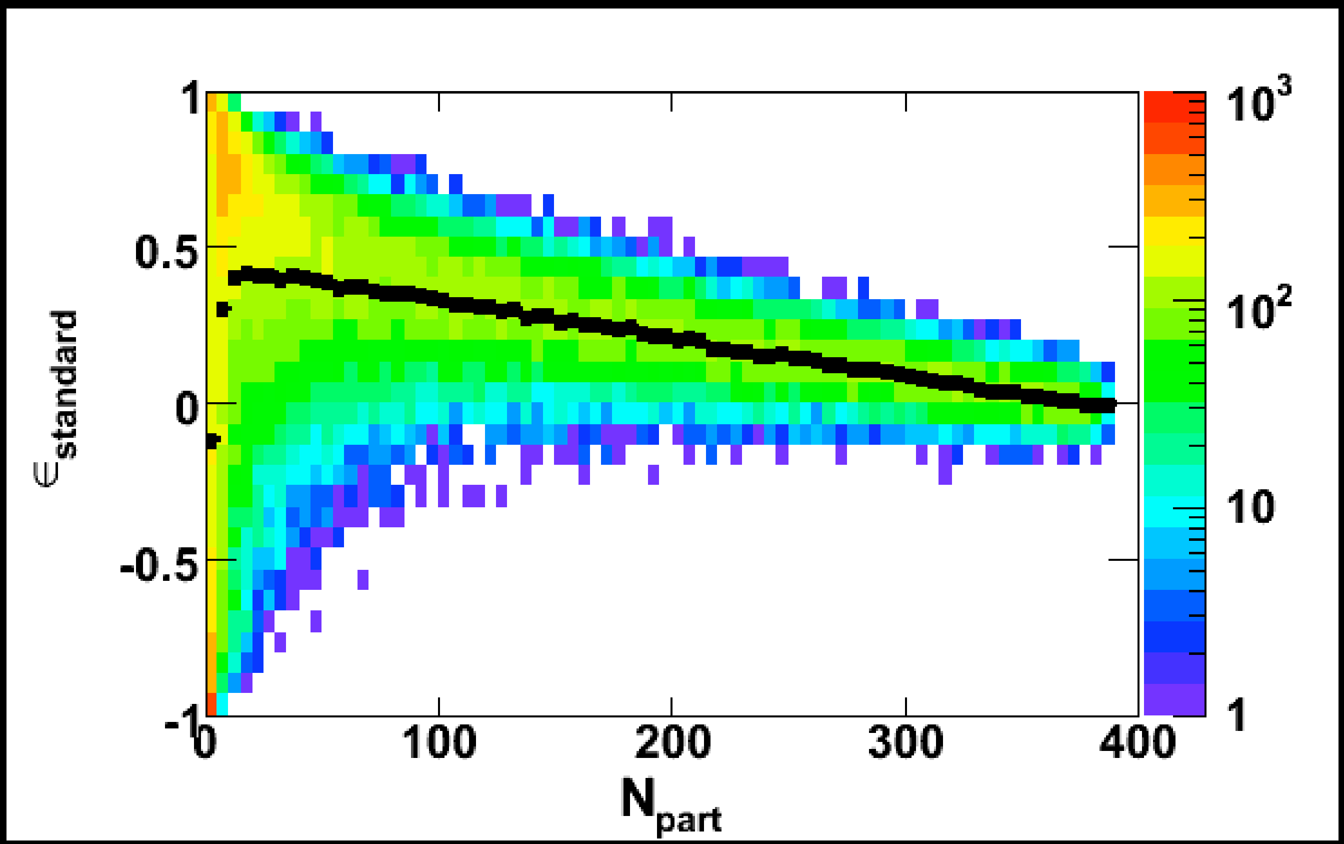
$$\epsilon_{part} = \frac{\sigma_y'^2 - \sigma_x'^2}{\sigma_y'^2 + \sigma_x'^2} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy}^2)^2}}{\sigma_y^2 + \sigma_x^2}$$

“Participant eccentricity”

Standard Eccentricity

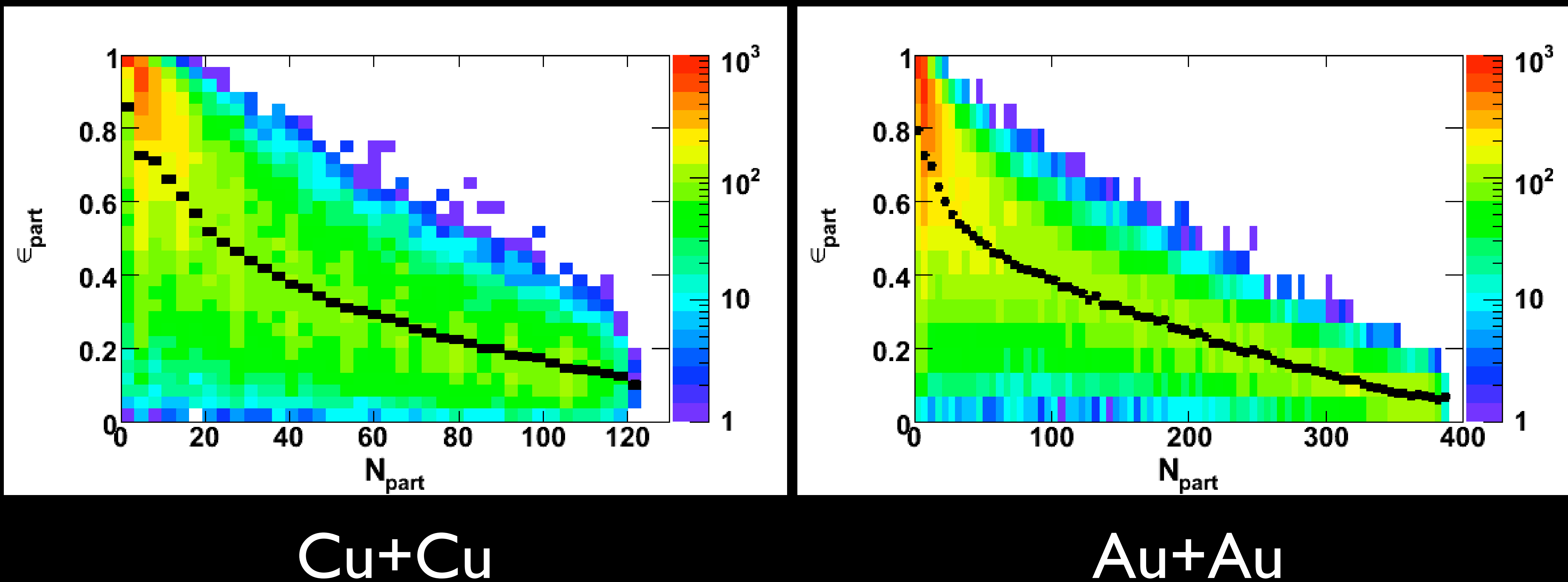


Cu+Cu

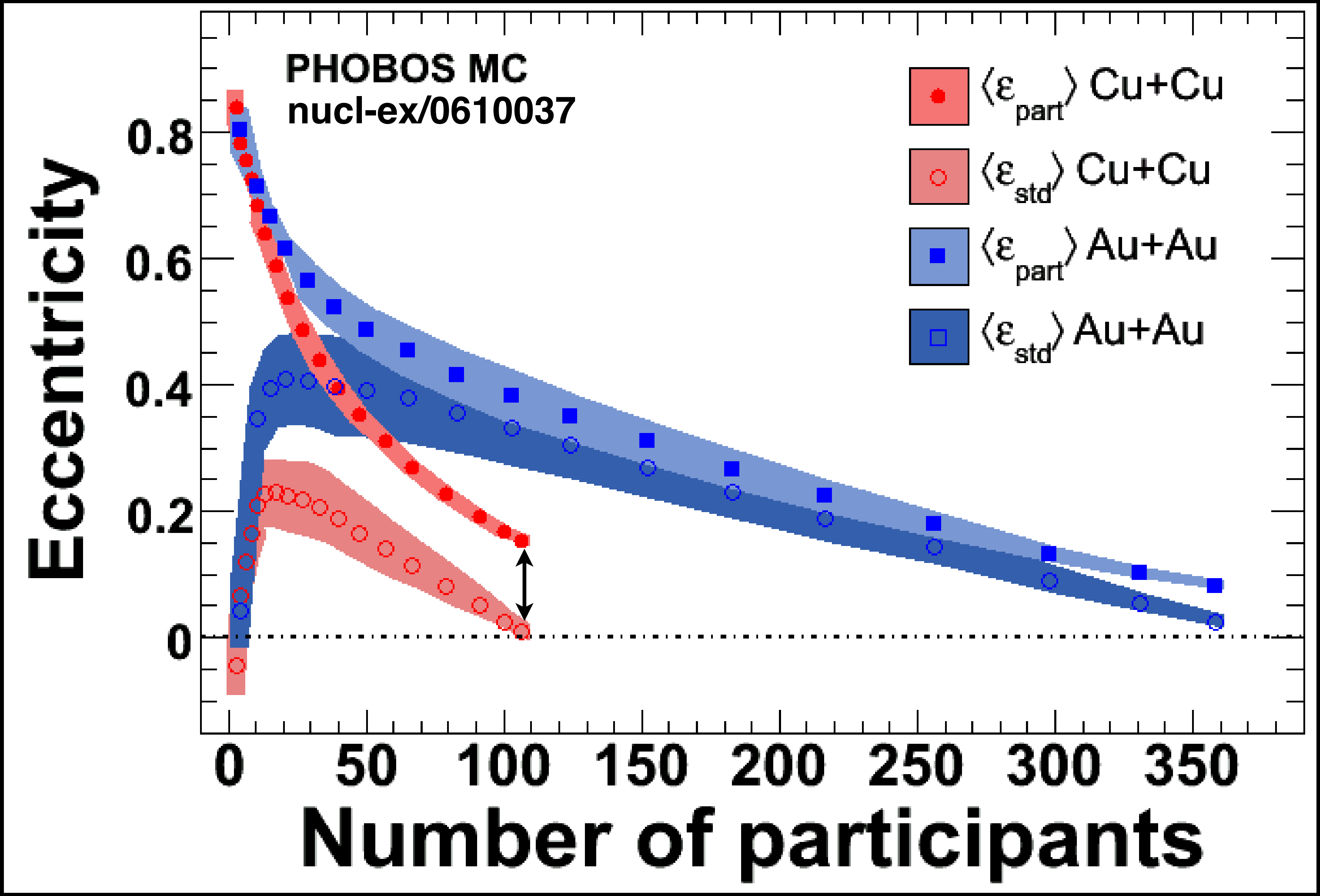


Au+Au

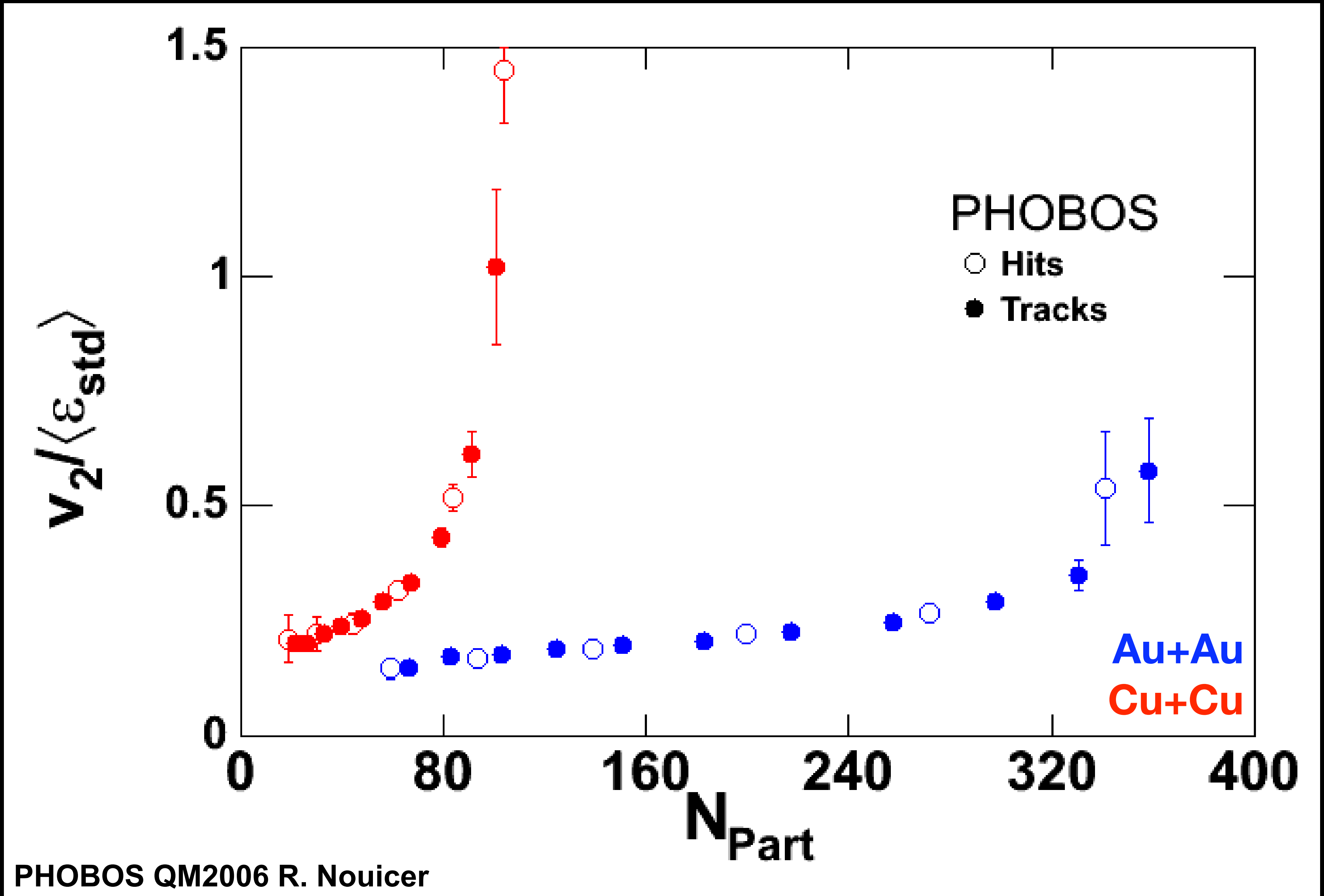
Participant Eccentricity



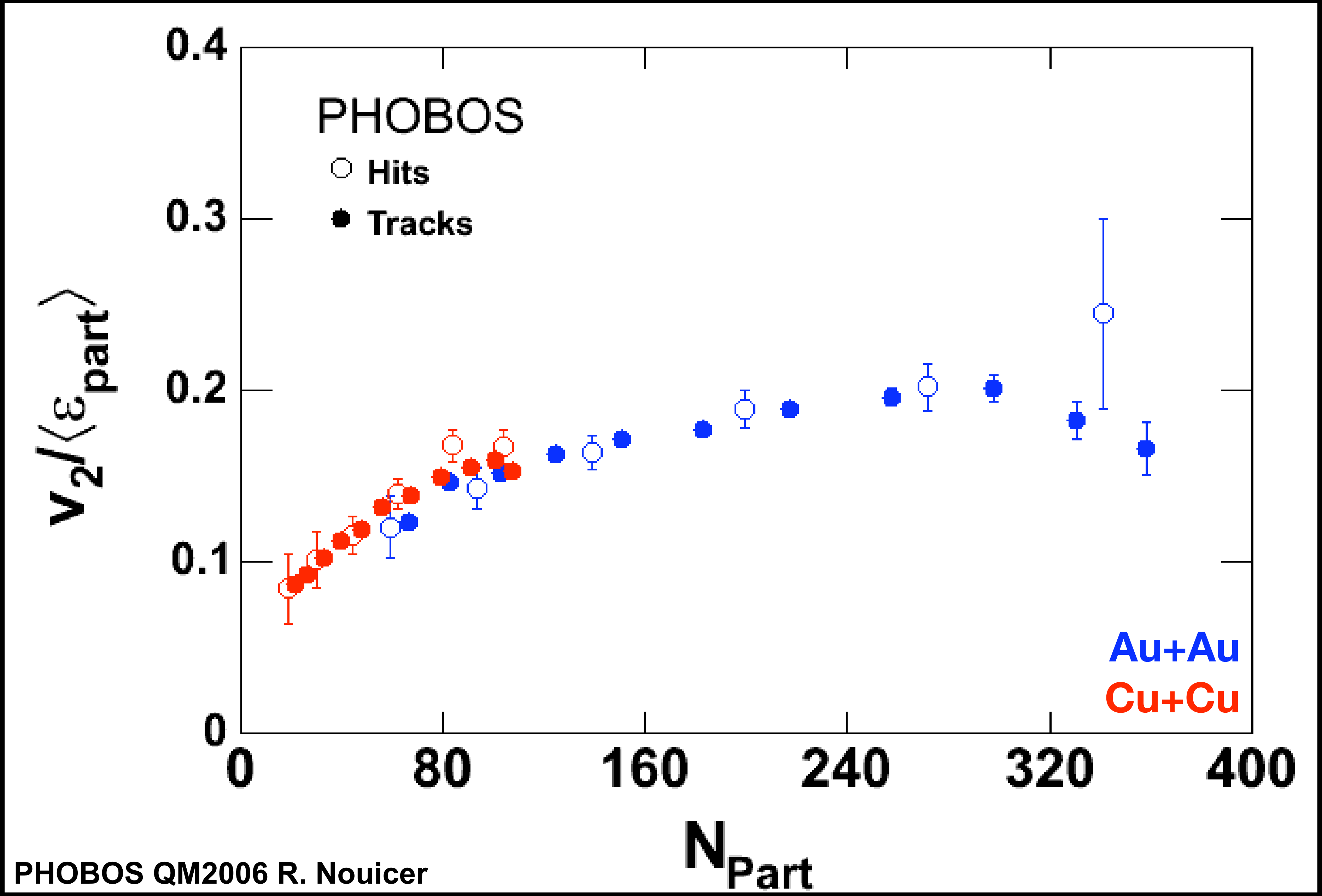
Participant vs. Standard



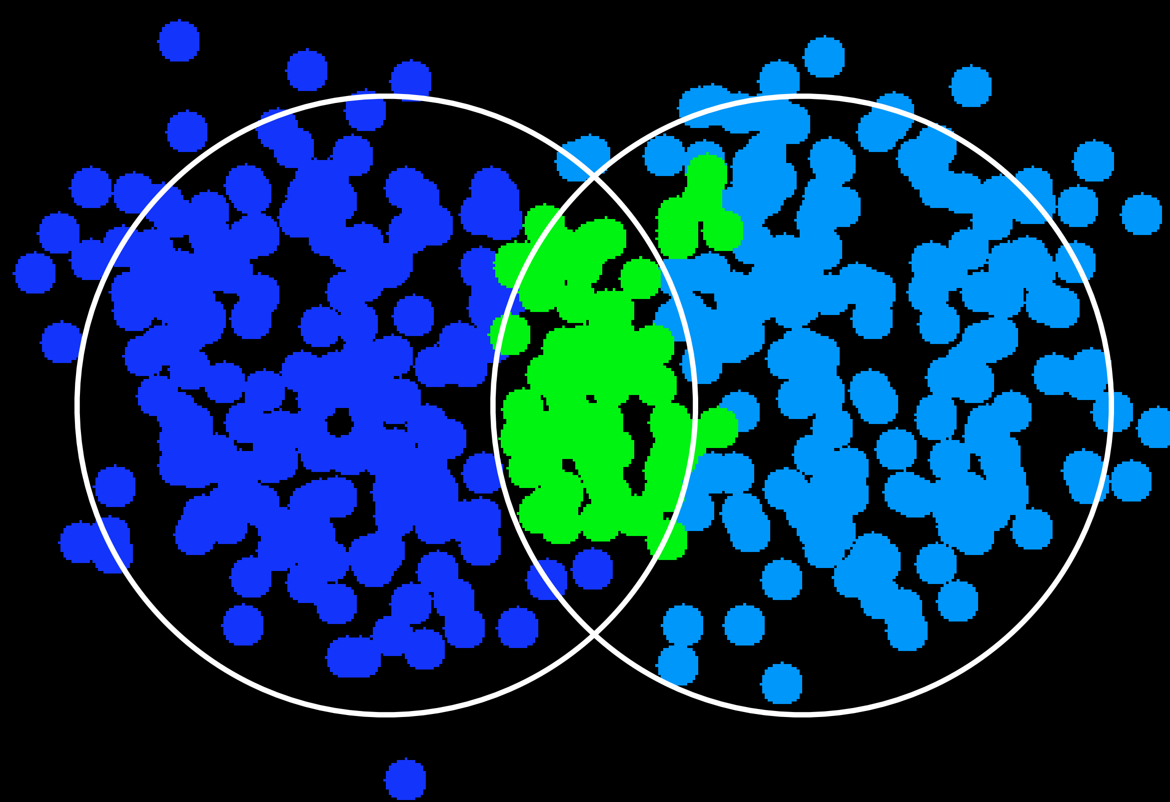
Something wrong...



...leads to scaling

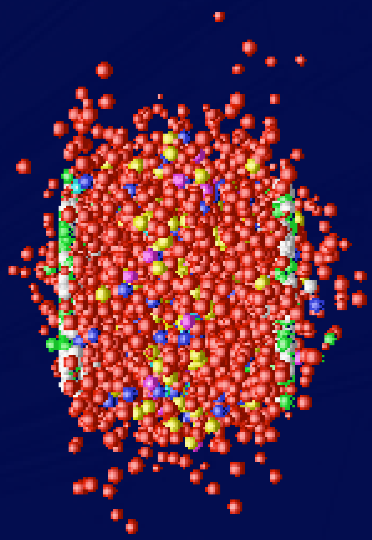


“Freeze-in”



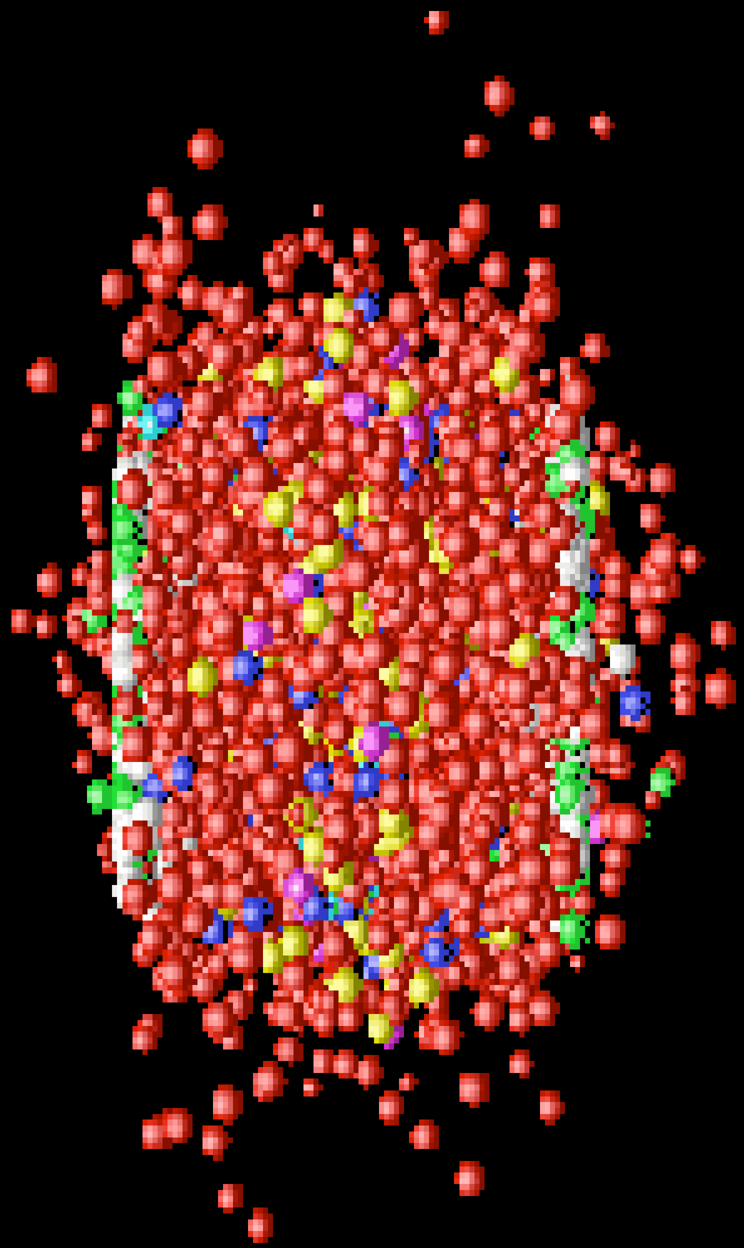
Configuration established <u>early</u> and <u>preserved</u>: substantial viscosity would generate new entropy under different geometric conditions

Near-Perfect Fluid?



How does this all relate to QCD?

What is the fluid made of?



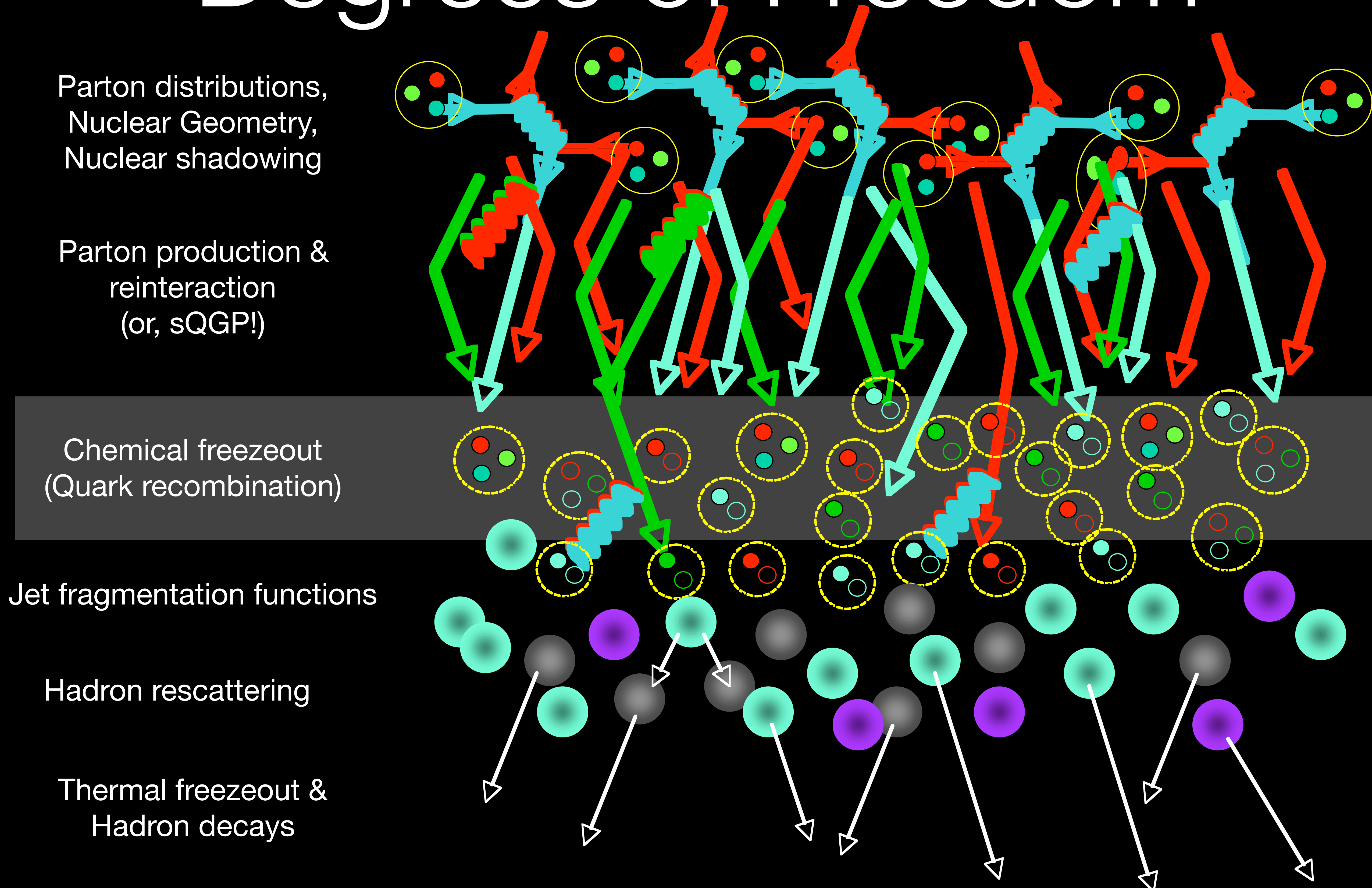
<u>Rapidly</u> thermalized matter

$$\tau_0 \ll 1 fm/c$$

But of what? and how so fast?

Quarks & gluons?
Is it a real "quark-gluon plasma" (QGP)?

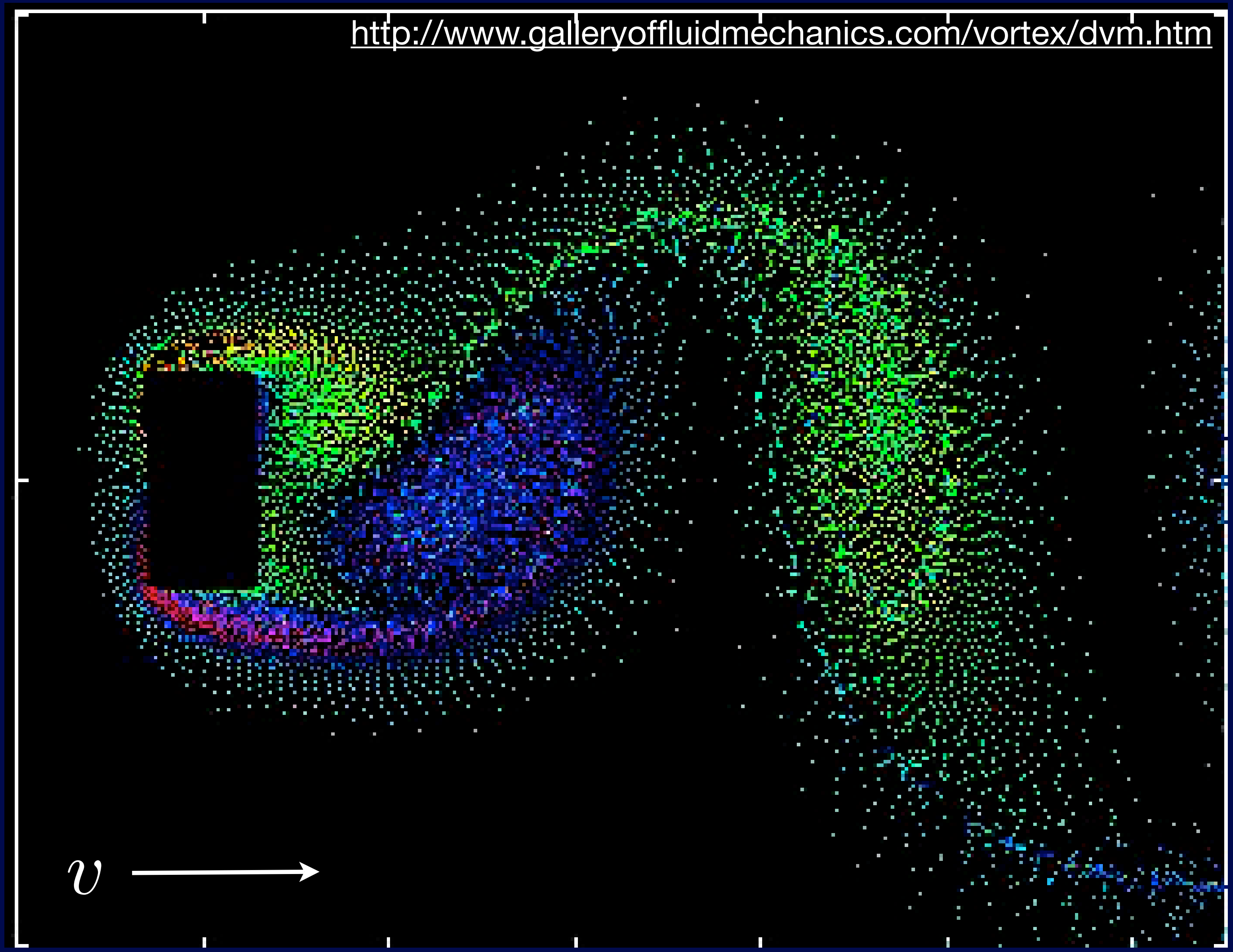
Degrees of Freedom



Viscosity

- **A genuine microscopic length scale relevant to the evolution violates assumption of continuum hypothesis**
  - This is a good rule of thumb for why “**viscosity**” is important
- **Viscosity is a new, hot issue in heavy ion physics**
  - Measures the “non-equilibrium” physics
  - Jet quenching, parton scattering, etc.
  - Breaks scale invariance!

Viscous Phenomena

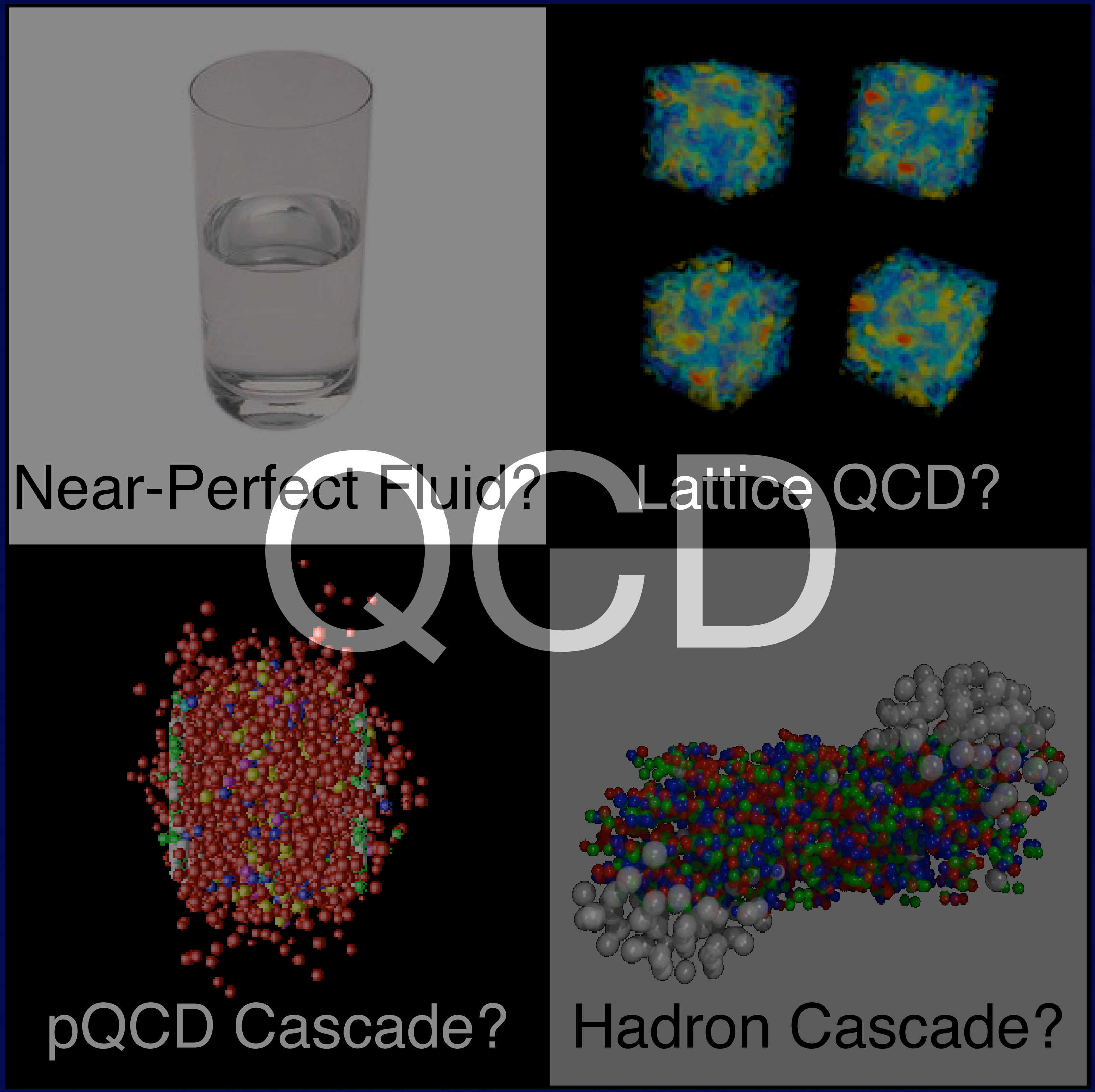


Viscosity introduces new dimensions to hydrodynamic phenomena

Dynamical Regimes of QCD

Large opacity

Small opacity

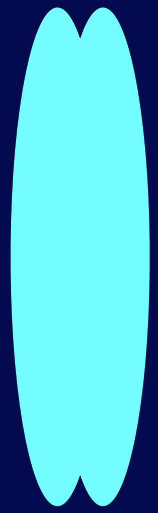


Short times

Long times

Three Dimensions?

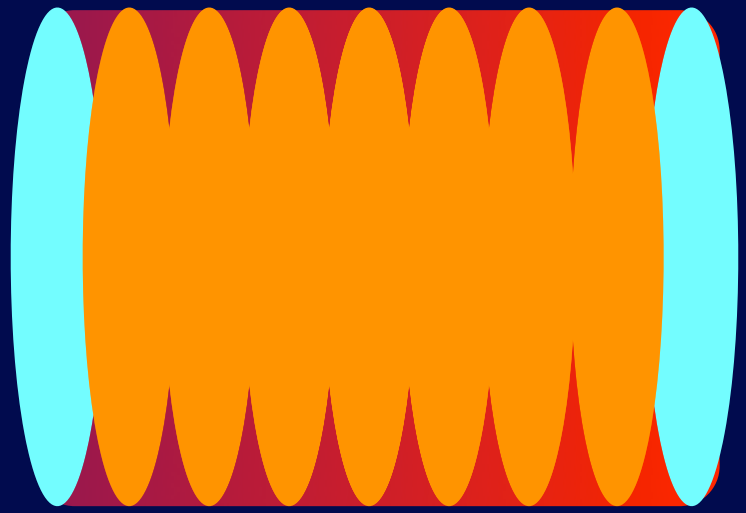
Landau



Total stopping, immediate thermalization & longitudinal **3D** re-expansion

$$\tau_0 \sim \frac{1}{\sqrt{s}} fm/c$$

Bjorken



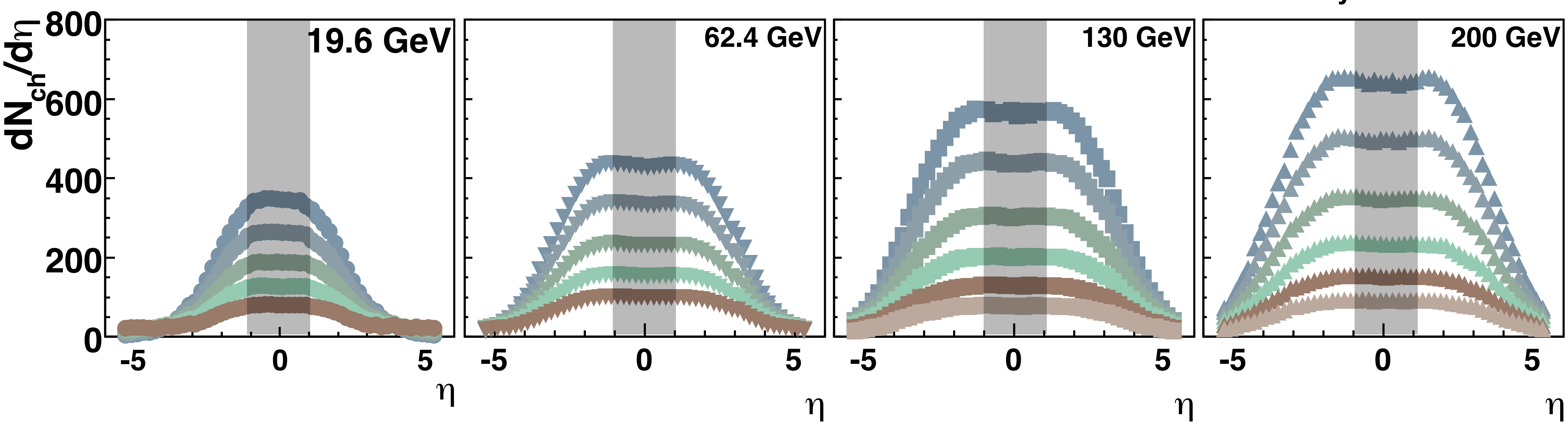
Partial stopping, "boost-invariant" **2D** dynamics

$$\tau_0 \sim 1 fm/c$$

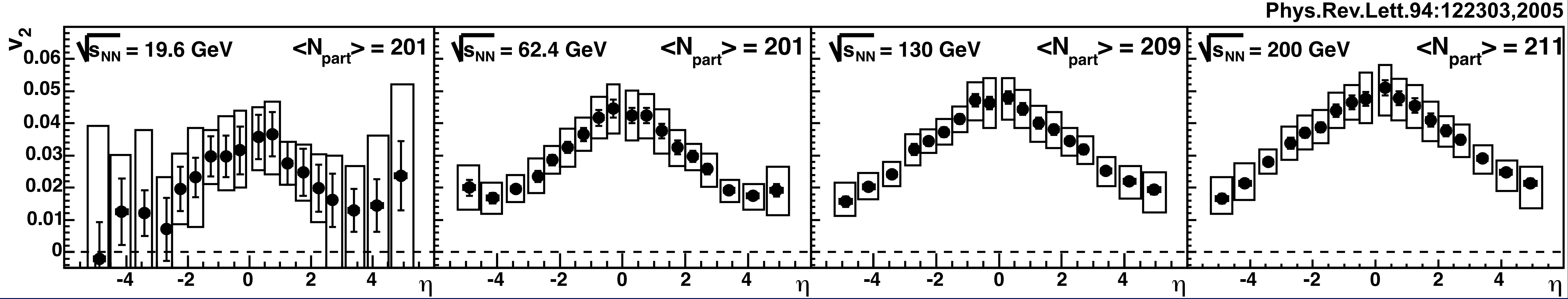
<u>Same</u> hydro, different initial conditions!

Three Dimensions

Phys.Rev.C74:021901,2006



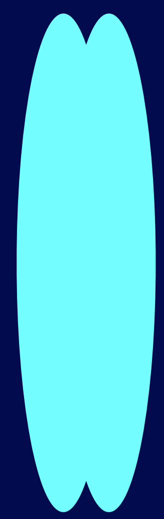
Phys.Rev.Lett.94:122303,2005



Elliptic flow shows strong pseudorapidity dependence, suggestive of real longitudinal hydrodynamic evolution

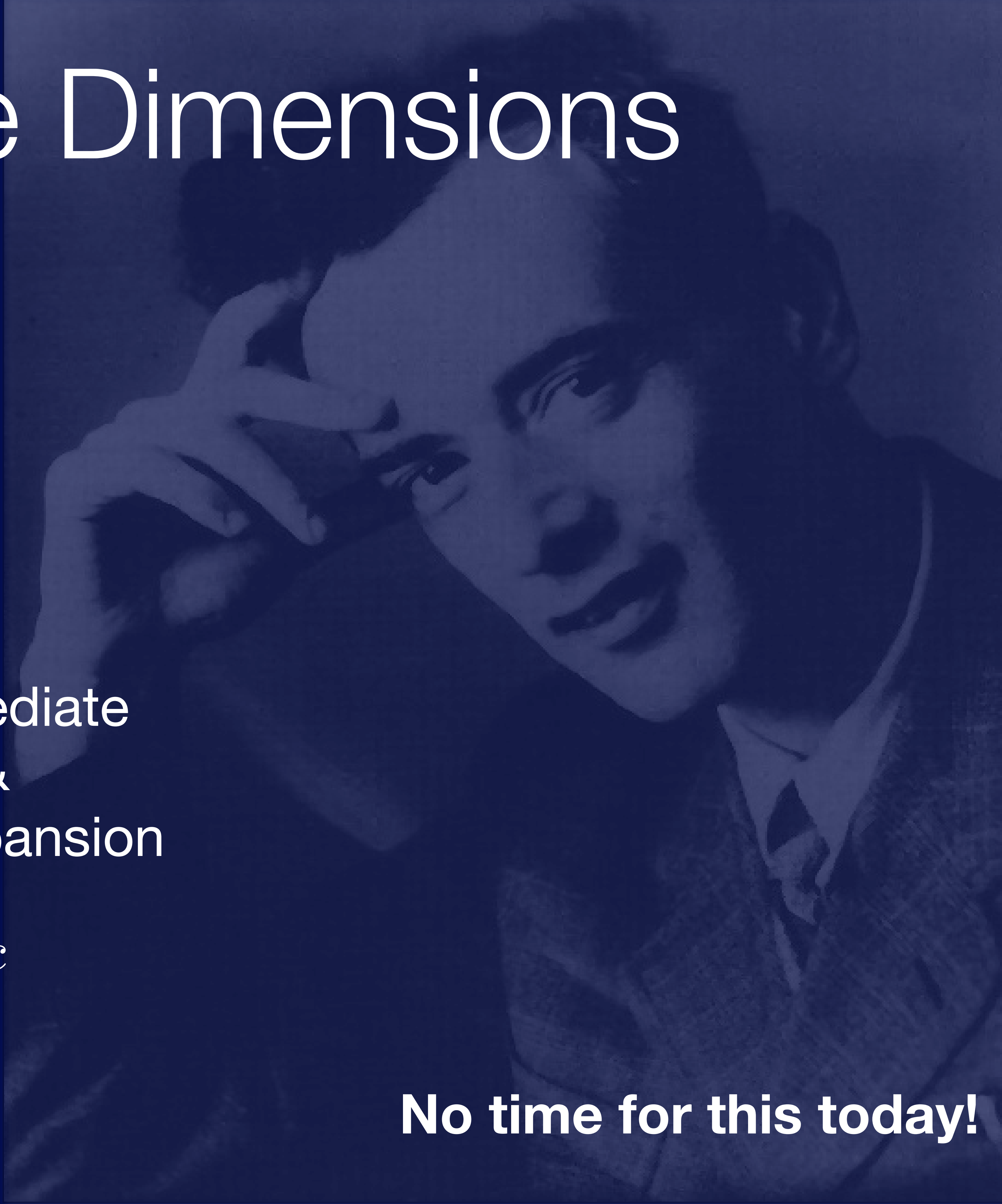
Three Dimensions

Landau



Total stopping, immediate thermalization & longitudinal **3D** re-expansion

$$\tau_0 \sim \frac{1}{\sqrt{s}} fm/c$$



No time for this today!