

Effective field theories applied to nuclear systems

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NRC - CNRC

Outline

results with:

J.D. Holt



1. Introduction

2. Effective field theory and the renormalization group for nuclear interactions

R.J. Furnstahl, E. Anderson

E. Jurgenson, R.J. Perry

S. Ramanan



S.K. Bogner, P. Piecuch, M. Wloch

3. Three-nucleon interactions: a frontier in nuclear structure

G.E. Brown, T.T.S. Kuo

A. Nogga

Forschungszentrum Jülich
in der Helmholtz-Gemeinschaft



4. Summary and developments

D. Dean, G. Hagen

T. Papenbrock



OAK RIDGE NATIONAL LABORATORY

B. Friman, K. Hebeler, L. Tolos



C.J. Pethick



A.P. Zuker



P. Maris, J.P. Vary



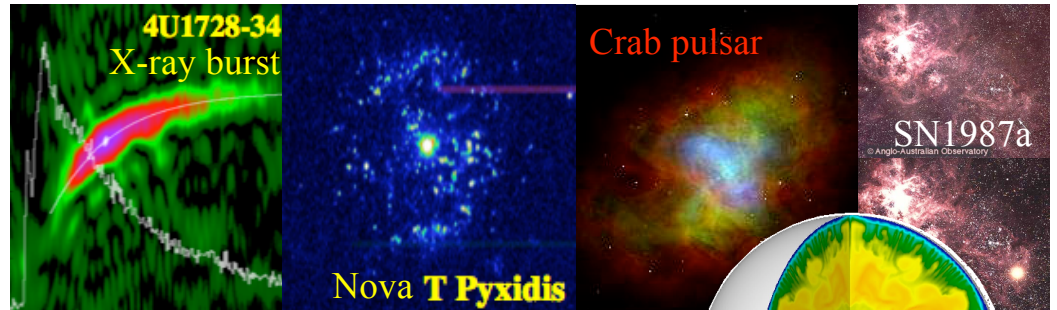
1. Strong interaction physics in the lab and cosmos

QCD + electroweak interactions

Matter at the extremes:

$$\rho \sim 10^{11} \dots 10^{15} \text{ g/cm}^3$$

$$Z/N \sim 0.05 \dots 0.6, T \sim \dots 30 \text{ MeV}$$



Interaction challenges:

QCD \Rightarrow EFT \Rightarrow Low-momentum interactions

Many-body challenges:

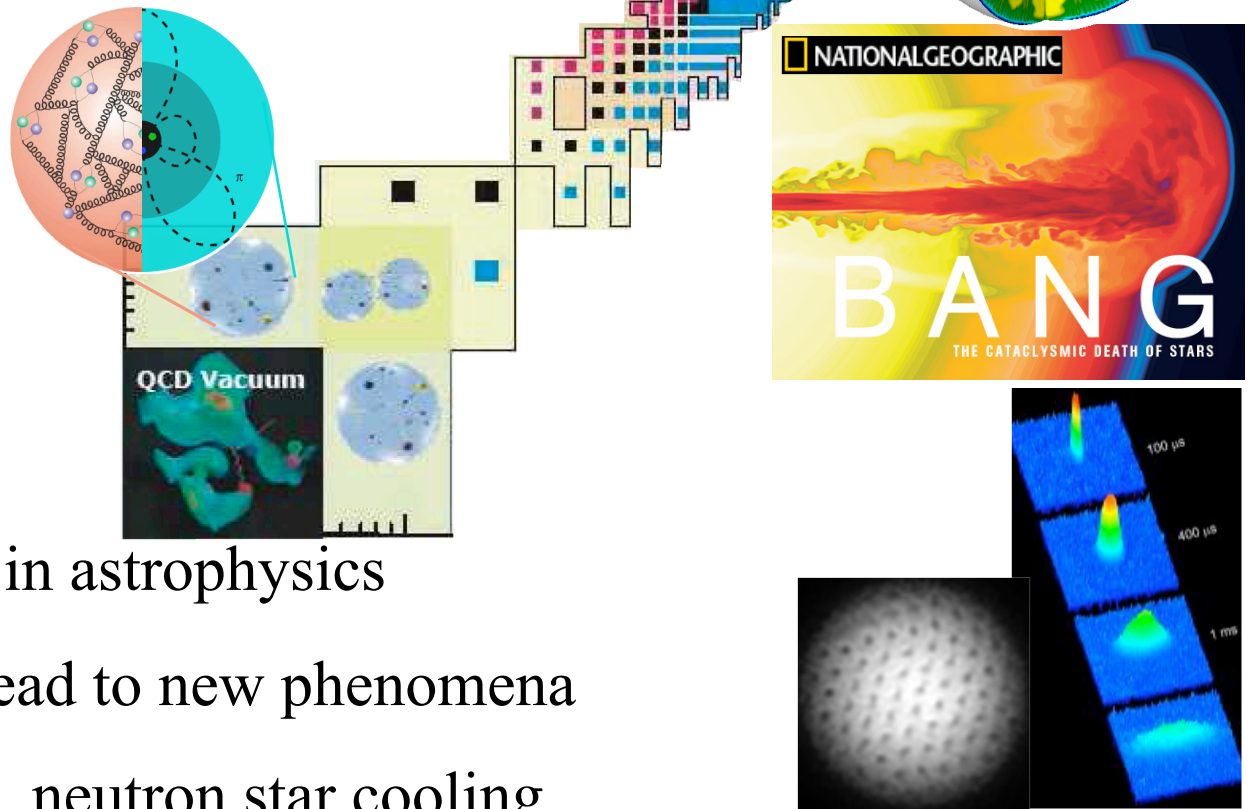
evolution with isospin
and asymmetric systems

pairing and superfluidity

impact on the universe,
nuclear equation of state in astrophysics

large scattering lengths lead to new phenomena

ν physics for supernovae, neutron star cooling



Interaction challenges and lattice QCD

pion-NN coupling g_A from full QCD

Edwards et al. (2006)

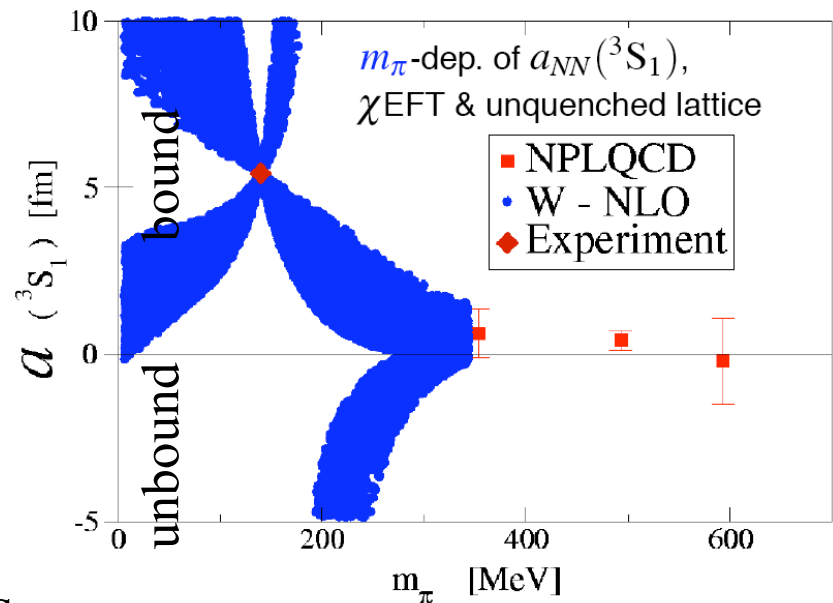
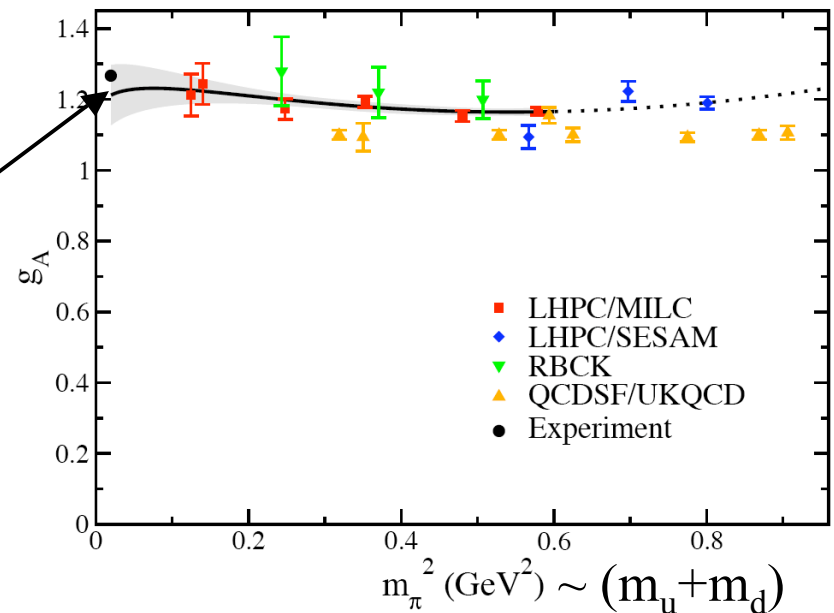
chiral extrapolation to physical pion mass agrees with experiment

How does the deuteron binding depend on quark masses? Beane et al. (2006)

First coherent effort to connect nuclear interactions to underlying QCD

Constrain more low-energy couplings

Constrain experimentally difficult observables: 3-neutron properties
 \Rightarrow isospin $T=3/2$ part of 3N interactions



Many-body methods

$A \lesssim 6$: exact few-body methods

PHYSICAL REVIEW C, VOLUME 64, 044001

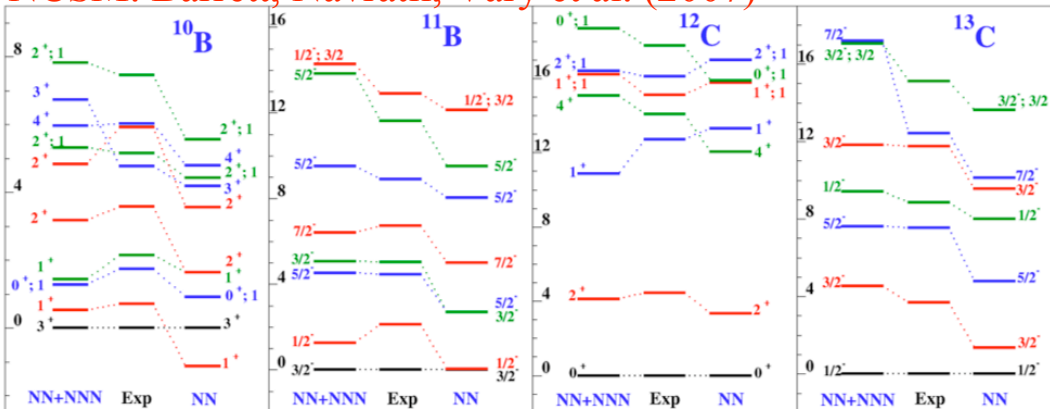
Benchmark test calculation of a four-nucleon bound state

In the past, several efficient methods have been developed to solve the Schrödinger equation for four-nucleon bound states accurately. These are the Faddeev-Yakubovsky, the coupled-rearrangement-channel Gaussian-basis variational, the stochastic variational, the hyperspherical variational, the Green's function Monte Carlo, the no-core shell model, and the effective interaction hyperspherical harmonic methods. In this article we compare the energy eigenvalue results and some wave function properties using the realistic AV8' NN interaction. The results of all schemes agree very well showing the high accuracy of our present ability to calculate the four-nucleon bound state.

$A \lesssim 16$: imaginary-time evolution GFMC, large basis diagonalizations NCSM

$A \lesssim 100$: Shell model with truncations to states of valence N on top of closed shells, Coupled Cluster theory

NCSM: Barrett, Navratil, Vary et al. (2007)



Faddeev/Faddeev-Yakubovsky:
Nogga et al. (2000)

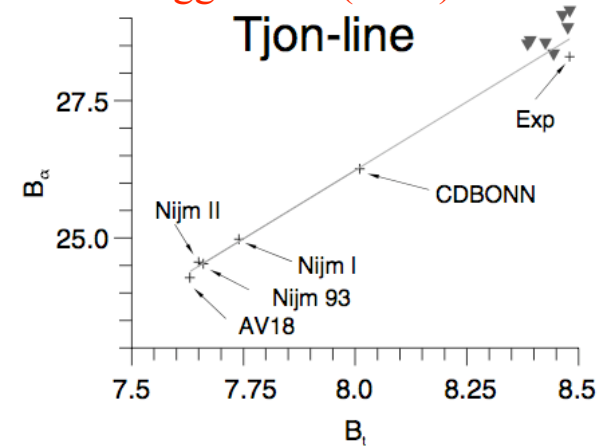
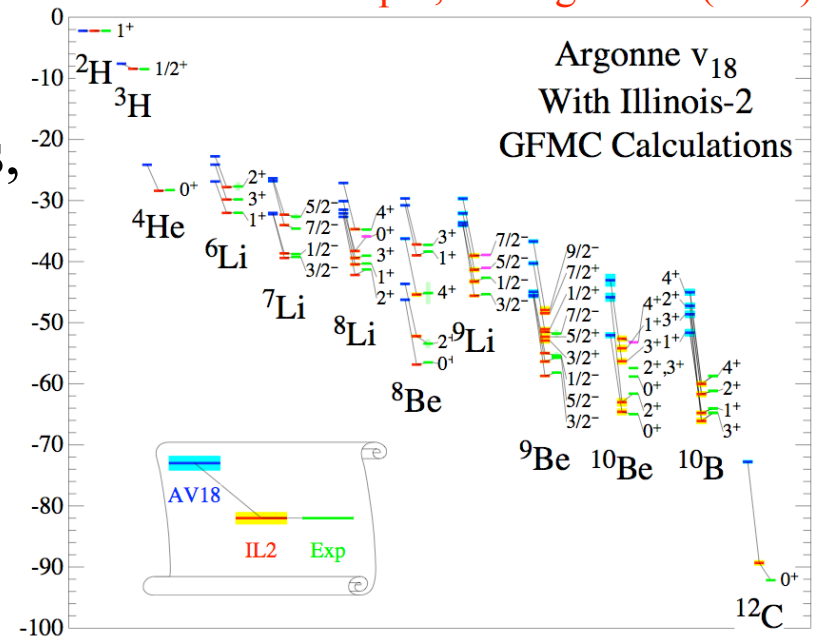


FIG. 1. Correlation of ${}^4\text{He}$ against ${}^3\text{H}$ binding energies in MeV for the different potentials. The triangles mark the predictions with 3NF from Table II.

GFMC: Pieper, Wiringa et al. (2007)



all highlight importance of 3N

Towards a universal nuclear density functional



from Kohn's 1998 Nobel Prize lecture

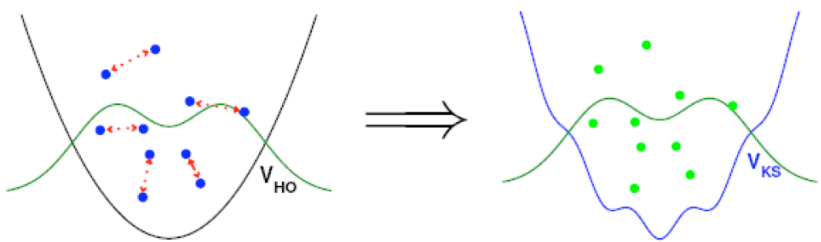
I begin with a provocative statement. *In general the many-electron wavefunction $\Psi(r_1, \dots, r_N)$ for a system of N electrons is not a legitimate scientific concept, when $N \geq N_0$, where $N_0 \approx 10^8$.*
I will use two criteria for defining "legitimacy": a) That Ψ can be calculated with sufficient accuracy and b) can be recorded with sufficient accuracy.

Density functional theory (DFT) for all A , based on densities not wave functions, $\sim 500\text{keV}$ deviation

Hohenberg-Kohn theorem:

exists energy functional $E[\rho]$, minimized by density of a noninteracting system that yields exact ground-state energy and density

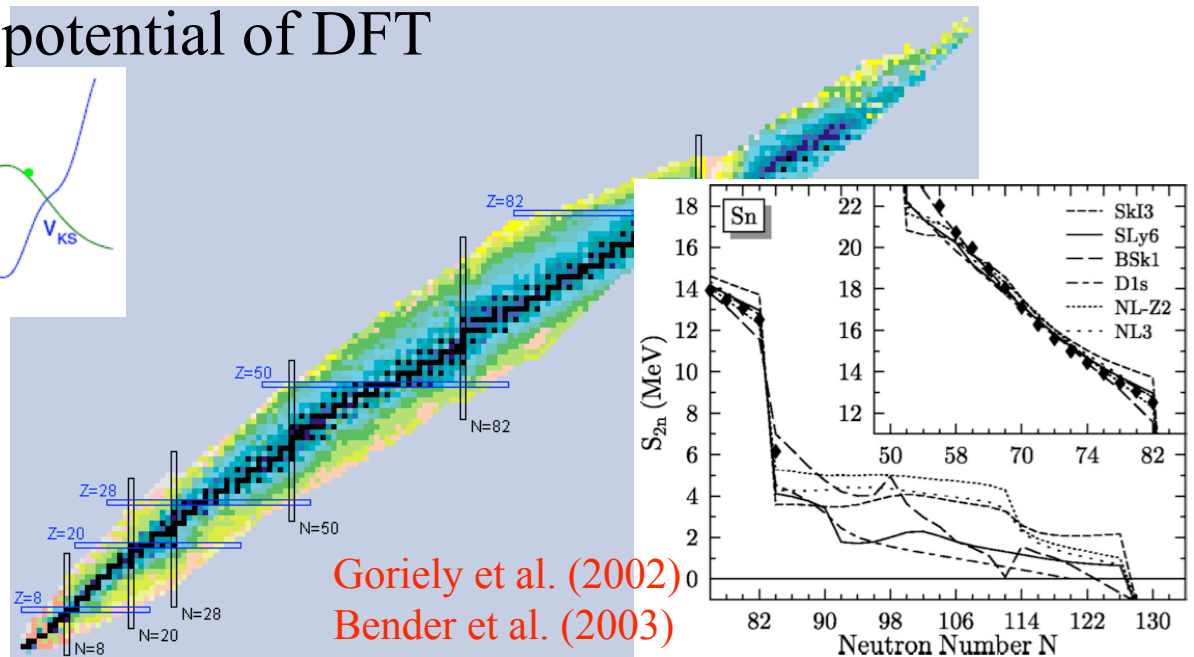
start from interacting fermions in external V_{HO} , turn off interactions and change external V such that density remains that of interacting system, new V is the Kohn-Scham potential of DFT



Challenges: many.....

reliable extrapolations

connect/constrain to NN and many-N interactions



Large scattering lengths and universal properties

neutron-neutron scattering length $a_{nn} = -18.5 \pm 0.3$ fm

large compared to interaction range $R \sim 1/m_\pi \approx 1.4$ fm

can imagine tuning quark masses to infinite a_{nn}

Beane, Savage (2002)

generate same properties by tuning scattering length of any dilute system to **universal regime**

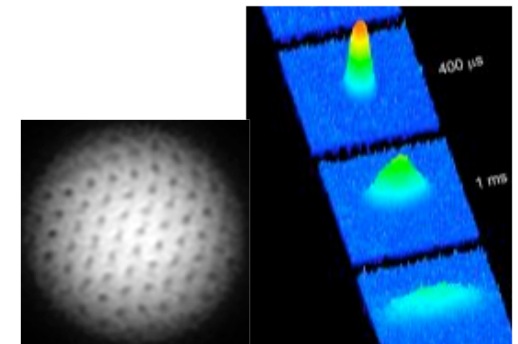
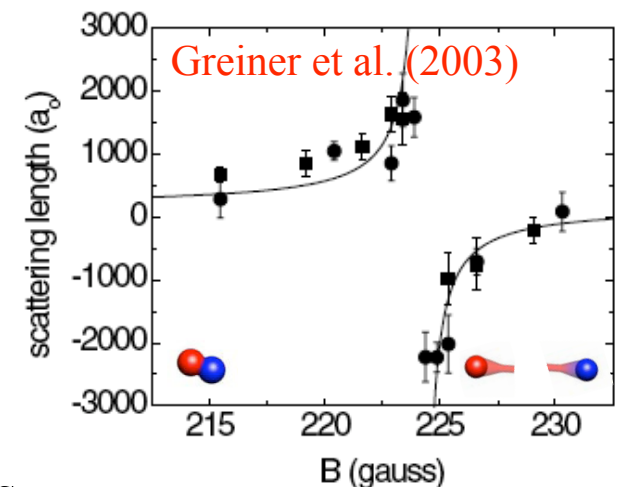
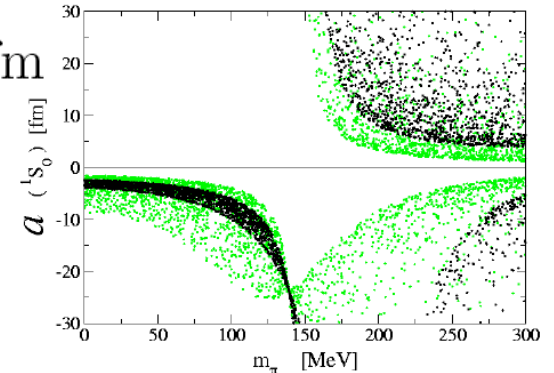
$$0 \leftarrow \frac{1}{a_s} \ll k_F \ll \frac{1}{r_e}, \frac{1}{R}, \dots \rightarrow \infty$$

strongly-interacting dilute

only Fermi momentum sets scale,
physics independent of interaction details

universal properties of fermionic ${}^6\text{Li}$ or ${}^{40}\text{K}$ atoms
and extremely low-density neutrons

controlled strong interactions in cold atoms
+ spin-polarization, m/M or Bose-Fermi mixtures,
rotation (vortices), optical lattices,.....



Nuclear physics dominated by large scattering lengths

Low-density neutron matter:

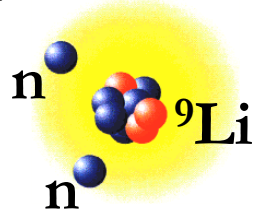
universal energy $\frac{E}{N} = \xi \left(\frac{E}{N} \right)_{\text{free}} = \xi \frac{3k_F^2}{10m}$

$\xi=0.42$ for cold atoms, ~ 0.5 for neutron matter

universal superfluid gap $\Delta \sim \epsilon_F$

critical $T_c \sim 0.2-0.3 T_F$ [Thomas et al. \(2006\)](#)

Borromean systems



Many large scattering lengths in nuclear astrophysics:

n - ^4He $a_{P_{3/2}} = -62.95 \text{ fm}^3$, ^4He - ^4He

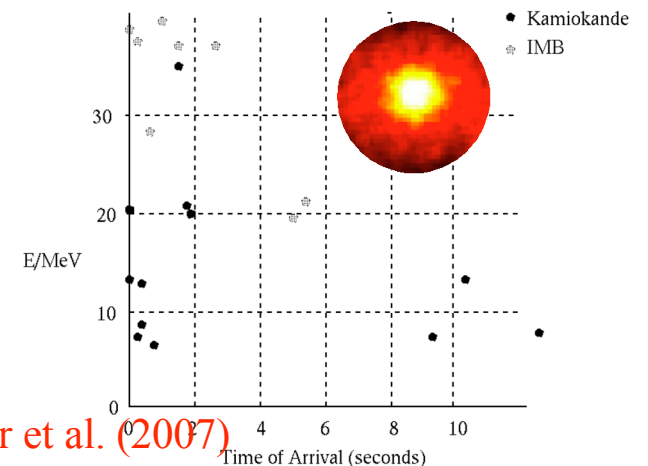
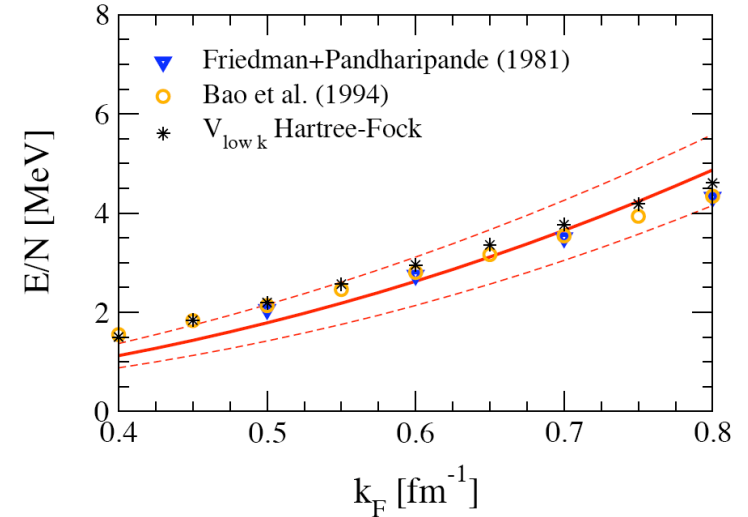
Physics of supernova neutrinosphere dominated by large scattering lengths

temperature $T \sim 4 \text{ MeV}$ from ~ 20 SN1987a events

density $n \sim 10^{11}-10^{12} \text{ g/cm}^3$

described by virial expansion [Horowitz, AS \(2006\)](#), [O'Connor et al. \(2007\)](#)

EFT for large a_s and r_e [AS, Pethick \(2005\)](#)

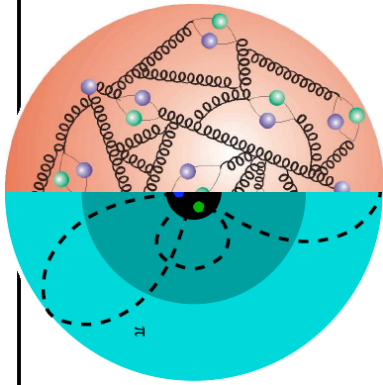


2. Effective field theory and the renormalization group

Resolution scale dependence of nuclear interactions

with high-energy probes:
deconfined quarks+gluons

cf. scale/scheme dependence
of parton distribution functions



Lattice QCD

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory

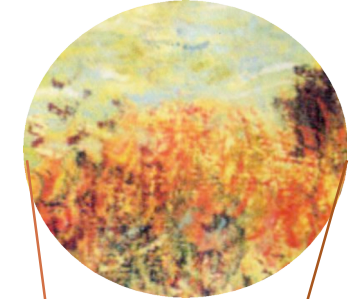
nucleons, interacting with pion exchanges and contact interactions,
note typical Fermi momentum in nuclei $k_F \sim m_\pi$

$Q \ll m_\pi = 140 \text{ MeV}$ - pion not resolved:

pionless effective field theory

nucleons and contact interactions

applicable to loosely-bound, dilute systems, reactions at astro energies



Idea of the renormalization group (RG)

integrate out high-momentum modes that are not resolved
and incorporate their effects in low-energy couplings

schematically

$$\begin{aligned} Z &= \int dx \int dy e^{-S(x,y)} = \int dx \int dy e^{-a(x^2+y^2)-b(x^2+y^2)^2} \\ &= \int dx e^{-S_{\text{eff}}(x)} = \int dx e^{-a'x^2-b'x^4-c'x^6+\dots} \end{aligned}$$

separate into slow and fast modes $\phi(\omega, k) = \begin{cases} \phi_{<}(\omega, k) & \text{for } \omega, k < \Lambda \\ \phi_{>}(\omega, k) & \text{else} \end{cases}$
and integrate out fast modes

$$\begin{aligned} Z &= \int \prod d\phi_{<}(\omega, k) e^{-S_{\text{free}}[\phi_{<}]} \int \prod d\phi_{>}(\omega, k) e^{-S_{\text{free}}[\phi_{>}] - S_{\text{int}}[\phi_{<}, \phi_{>}]} \\ &= \int \prod d\phi_{<}(\omega, k) e^{-S_{\text{eff}}[\phi_{<}]} \end{aligned}$$

effective action $S_{\text{eff}}[\phi_{<}]$ with couplings $g_i(\Lambda)$

when we integrate out momentum modes by $\delta\Lambda$ the couplings evolve

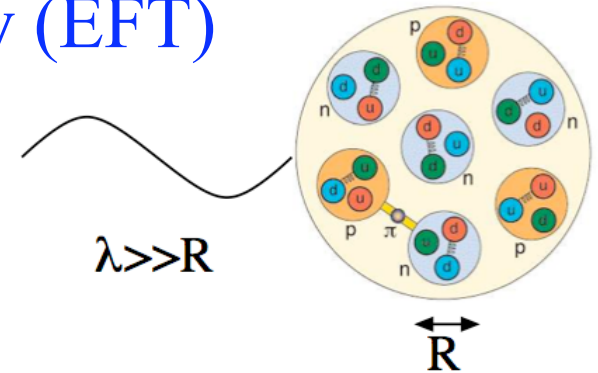
$$g_i(\Lambda - \delta\Lambda) = g_i(\Lambda) - \frac{\delta\Lambda}{\Lambda} \beta_i(\{g_j(\Lambda)\}, \Lambda)$$

leads to renormalization group equations $\Lambda \frac{d}{d\Lambda} g_i(\Lambda) = \beta_i(\{g_j(\Lambda)\}, \Lambda)$

Idea of effective field theory (EFT)

Separation of scales $\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$

limited resolution at low energies,
can expand in powers Q/Λ_b



details of short-distance physics not resolved, capture in few low-energy constants in effective action, fit couplings to data

include long-range physics explicitly (pions for chiral EFT)

systematic: can work to desired accuracy and obtain error estimates

NN scattering in pionless EFT,
general expansion

$$\mathcal{L}_{\text{EFT}} = \psi^\dagger \left[i \frac{\partial}{\partial t} + \frac{\vec{\nabla}^2}{2m} \right] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} [(\psi \psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + \text{h.c.}]$$

NN S-wave

$$+ \frac{C'_2}{8} (\psi \overleftrightarrow{\nabla} \psi)^\dagger \cdot (\psi \overleftrightarrow{\nabla} \psi) + \mathcal{O}\left(\frac{\nabla^4}{\Lambda_b^4}\right)$$

NN P-wave

$$- \frac{D_0}{6} (\psi^\dagger \psi)^3 + \mathcal{O}\left(\frac{\nabla^2}{\Lambda_b^2}\right)$$

3N S-wave

Kaplan, Savage, Wise,
van Kolck, Bedaque,
Hammer, Phillips,
Griesshammer, ...

with EFT potential $\langle \mathbf{k} | V_{\text{EFT}} | \mathbf{k}' \rangle = C_0 + \frac{1}{2} C_2 (\mathbf{k}^2 + \mathbf{k}'^2) + C'_2 \mathbf{k} \cdot \mathbf{k}' + \mathcal{O}\left(\frac{Q^4}{\Lambda_b^4}\right)$

NN scattering in pionless EFT

sum iterated leading-order NN interactions, yields scattering amplitude

$$T_0(k) \begin{array}{c} \text{P}/2 + k \\ \swarrow \quad \searrow \\ \text{---} \text{---} \\ \nwarrow \quad \nearrow \\ \text{P}/2 - k \end{array} = \begin{array}{c} \text{P}/2 + k' \\ \swarrow \quad \searrow \\ \text{---} \text{---} \\ \nwarrow \quad \nearrow \\ \text{P}/2 - k' \end{array} = \begin{array}{c} \text{---} \text{---} \\ \swarrow \quad \searrow \\ \text{---} \text{---} \\ \nwarrow \quad \nearrow \end{array} + \begin{array}{c} \text{---} \text{---} \\ \swarrow \quad \searrow \\ \text{---} \text{---} \\ \nwarrow \quad \nearrow \end{array} + \begin{array}{c} \text{---} \text{---} \\ \swarrow \quad \searrow \\ \text{---} \text{---} \\ \nwarrow \quad \nearrow \end{array} + \dots$$

$$C_0(\Lambda) \quad C_0(\Lambda) I(k, \Lambda) C_0(\Lambda)$$

sum over intermediate states is divergent, regularize, here with cutoff Λ

$$I(k, \Lambda) = \int^{\Lambda} \frac{d^3 q}{(2\pi)^3} \frac{1}{\frac{k^2}{m} - \frac{q^2}{m} + i\epsilon} = \frac{m}{4\pi} \left[\frac{2}{\pi} \Lambda - ik + \mathcal{O}\left(\frac{k^2}{\Lambda^2}\right) \right] + \text{renormalization:}$$

match scattering amplitude to low-energy effective range expansion/data

$$T_0(k) = \frac{C_0(\Lambda)}{1 - I(k, \Lambda) C_0(\Lambda)} \longleftrightarrow T_0(k) = -\frac{4\pi}{m} \frac{1}{k \cot \delta_0(k) - ik} = \frac{4\pi}{m} \frac{1}{\frac{1}{a_s} - \frac{1}{2} r_e k^2 + ik}$$

yields

$$C_0(\Lambda) = \frac{4\pi}{m} \frac{1}{\frac{1}{a_s} - \frac{2}{\pi} \Lambda}$$

with cutoff-indep.
low-energy NN
observables

(i) for weak scattering, can chose $\Lambda a_s \ll 1$ over wide cutoff range \Rightarrow weak-coupling $C_0(\Lambda) \simeq \frac{4\pi a_s}{m}$

(ii) for $\frac{1}{a_s} = 0$ fine-tuned $C_0(\Lambda) = -\frac{2\pi^2}{m\Lambda}$

(iii) find $r_e \sim \frac{1}{\Lambda}$ implies trunc error $\mathcal{O}\left[\left(\frac{Q^2}{\Lambda_b^2}\right), \left(\frac{Q^2}{\Lambda^2}\right)\right]$

Lepage plots

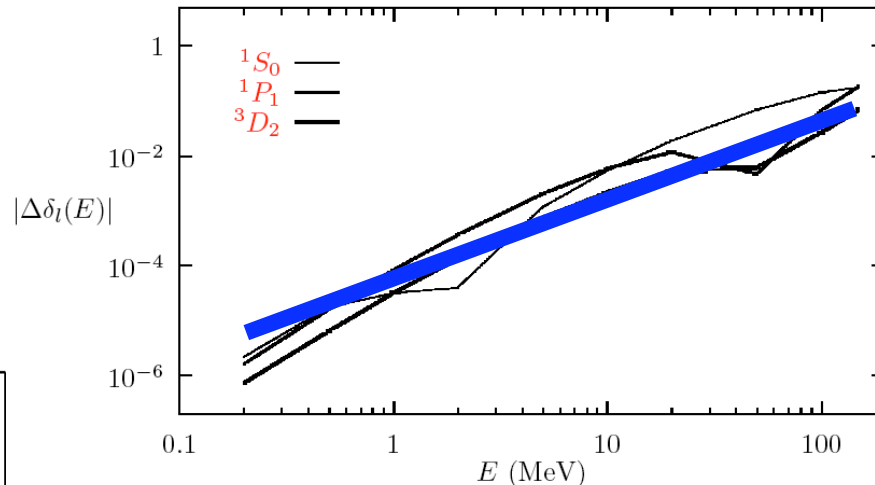
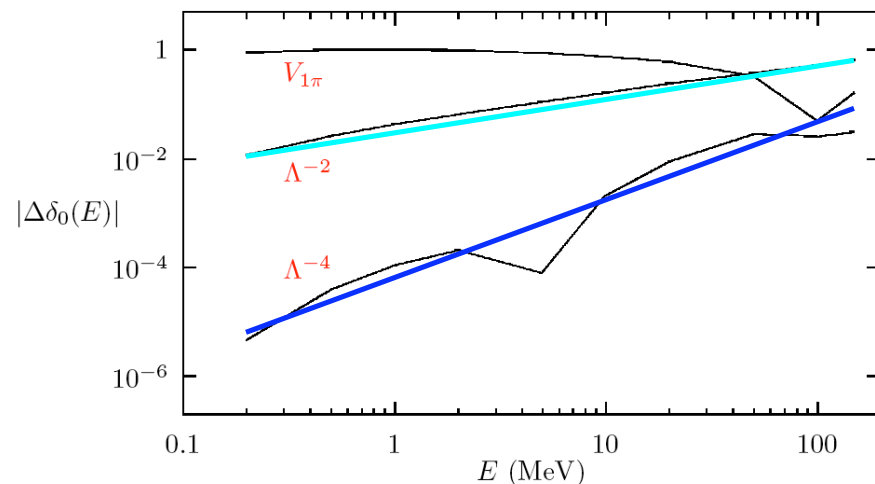
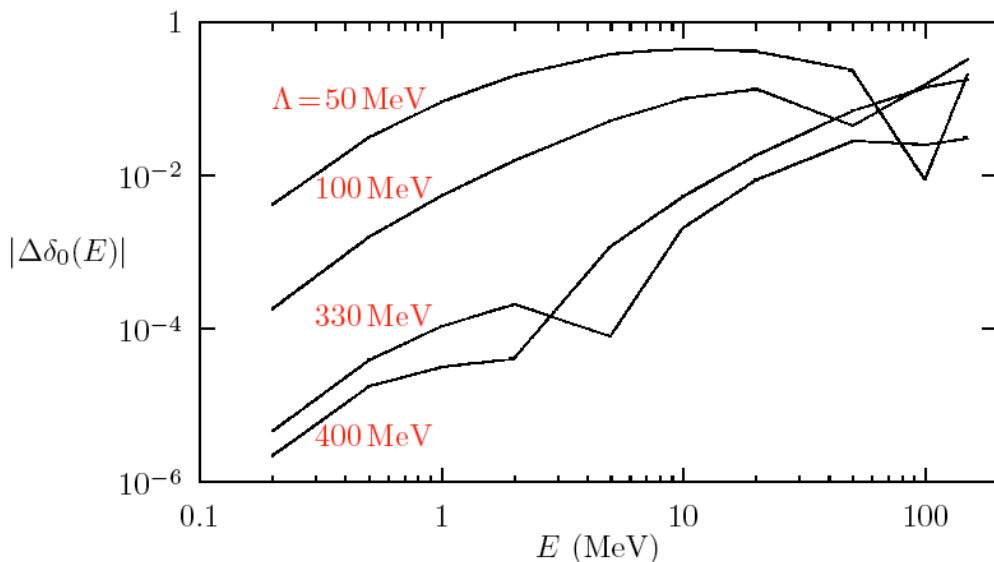
Lepage (1997)

Log-log plots of relative errors vs. E
 steeper slope with subsequent orders,
 same in all channels: S,P,D,... waves

$$V(r) = -\alpha_\pi v_\Lambda(r) + c \frac{\delta_{1/\Lambda}^3(\mathbf{r})}{\Lambda^2} - d \frac{\nabla^2 \delta_{1/\Lambda}^3(\mathbf{r})}{\Lambda^4}$$

$$P: V_{\pi,P} + \frac{\nabla \delta_{1/\Lambda}^3 \nabla}{\Lambda^4} + \mathcal{O}(1/\Lambda^6)$$

$$D: V_{\pi,D} + \mathcal{O}(1/\Lambda^6)$$



Cutoff dependence:

truncation errors decrease with increasing cutoff

no advantages for cutoffs \gg

breakdown scale (nonlinearities)

Pionless EFT applied to the few-body sector

Leading-order NN contact interactions $^1S_0 + ^3S_1$ $C_0(\Lambda)$ lead to divergence in triton channel, cutoff dependence generates Phillips line [Bedaque et al. \(1999\)](#)

and band around Tjon line in $A=3,4$ system [Platter et al. \(2005\)](#)

promote 3N contact interaction $D_0(\Lambda)$ to leading order, coupling fixed by triton energy exhibits RG limit cycle

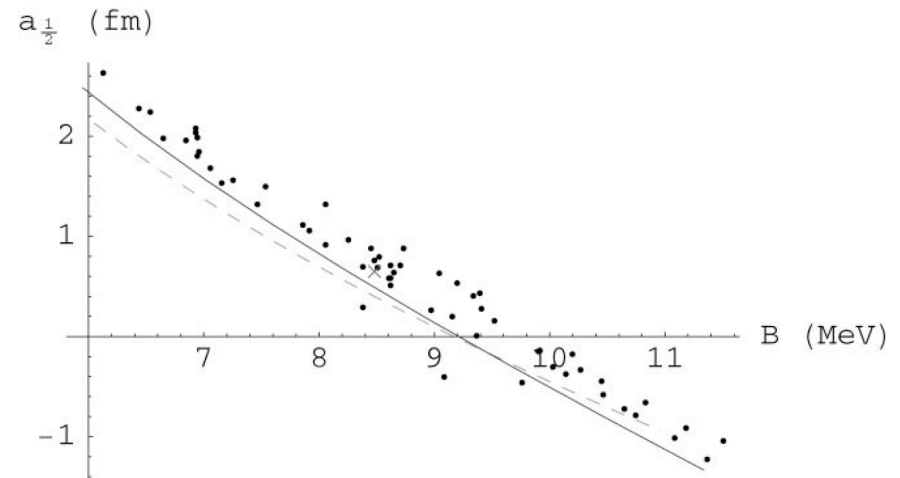


Figure 9 Correlation between the doublet S -wave nucleon-deuteron scattering length and the triton binding energy (Phillips line): predictions of different models (points), EFT at LO (light dashed line) and NLO (dark solid line), and experimental value (cross).

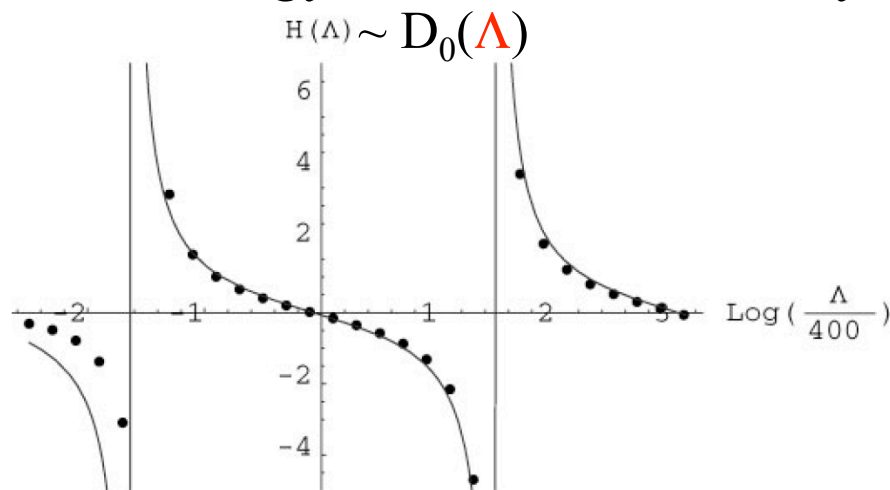
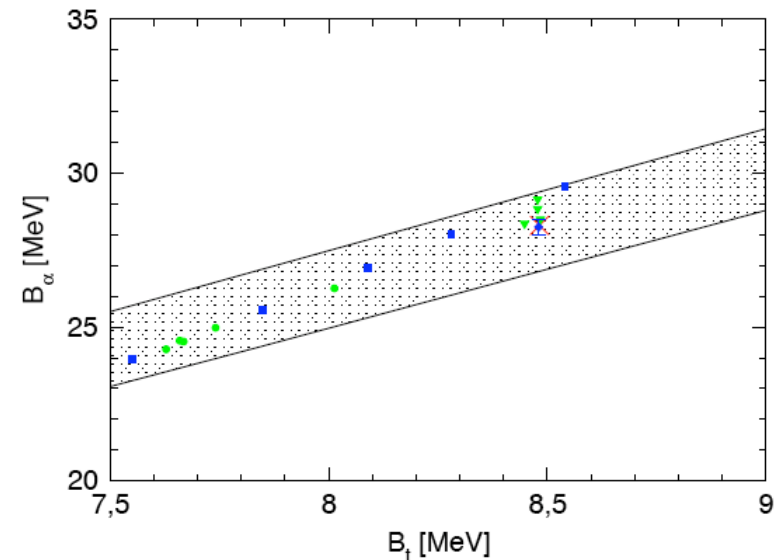


Figure 7 Three-body force coefficient $H(\Lambda)$ computed analytically (line) and numerically (points) as a function of $\log(\Lambda/400 \text{ MeV})$.



Large scattering lengths lead to Efimov effect

LO 2- and 3-body contact interactions:

Efimov spectrum for 3 distinguishable particles or 3 bosons with identical pair-wise scattering lengths

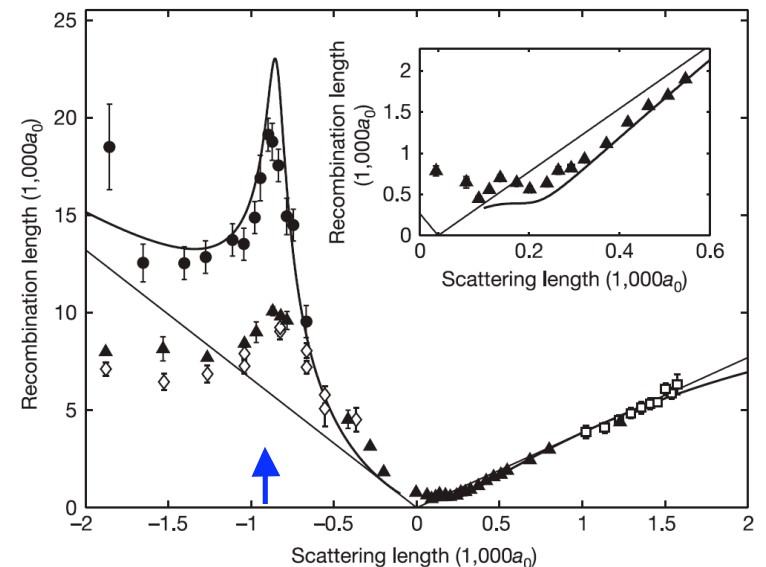
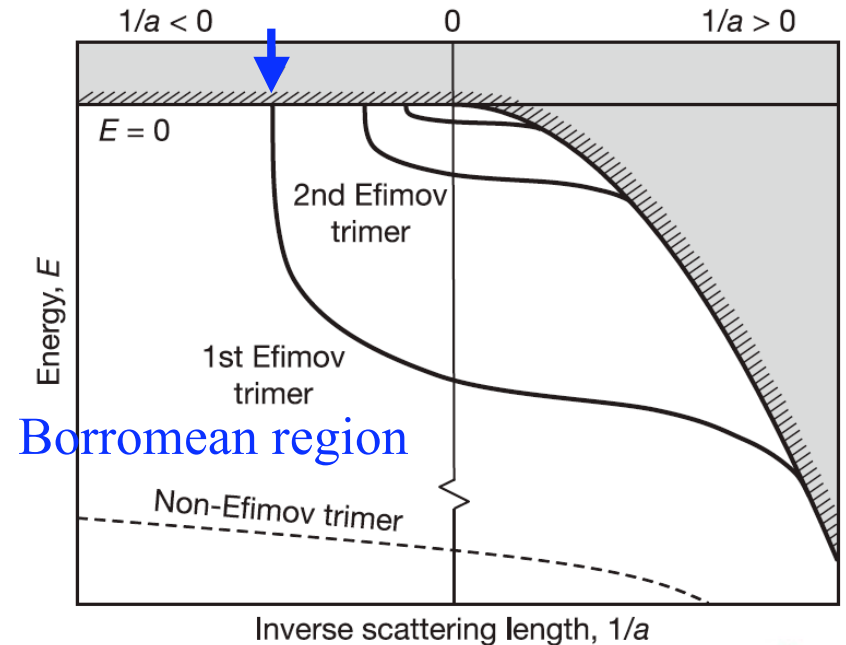
predicts Borromean states:
quantum three-body bound states,
all two-body subsystems unbound

Universal physics on resonance:
infinite bound excited states with
scaling $E_n/E_{n+1} = 515$

observed first Efimov resonance in cold atoms by 3-body losses for Cs bosons

[Kraemer et al. \(2006\)](#)

predicted Borromean state for three ${}^6\text{Li}$ cold atoms [Luu, AS \(2007\)](#)

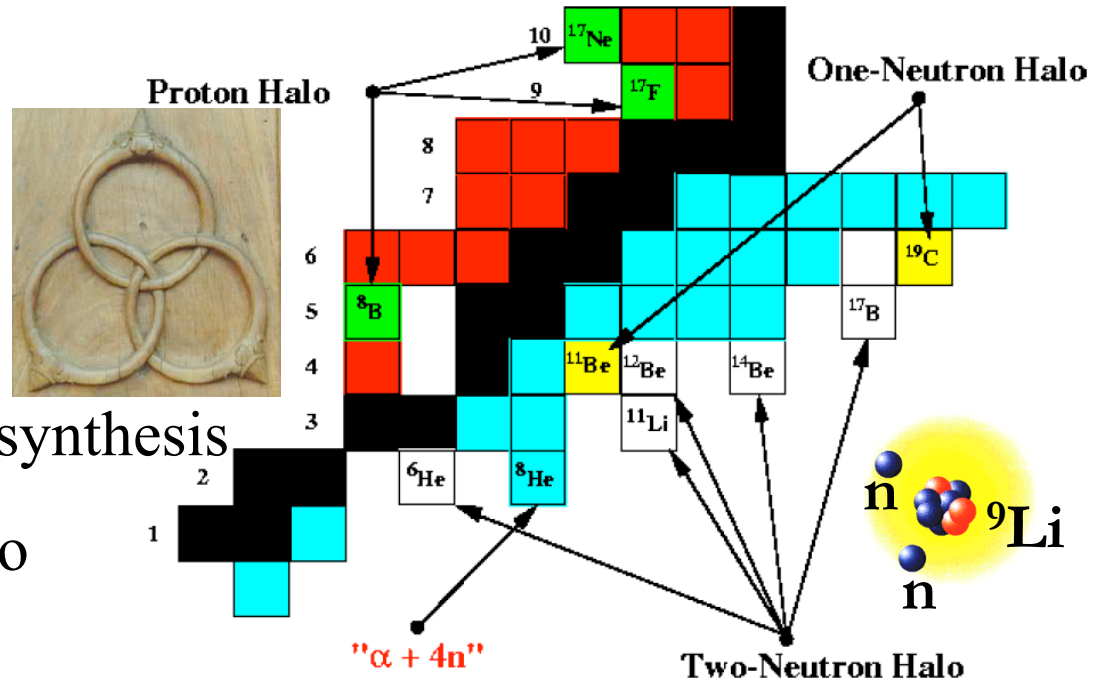


Borromean states in nuclei

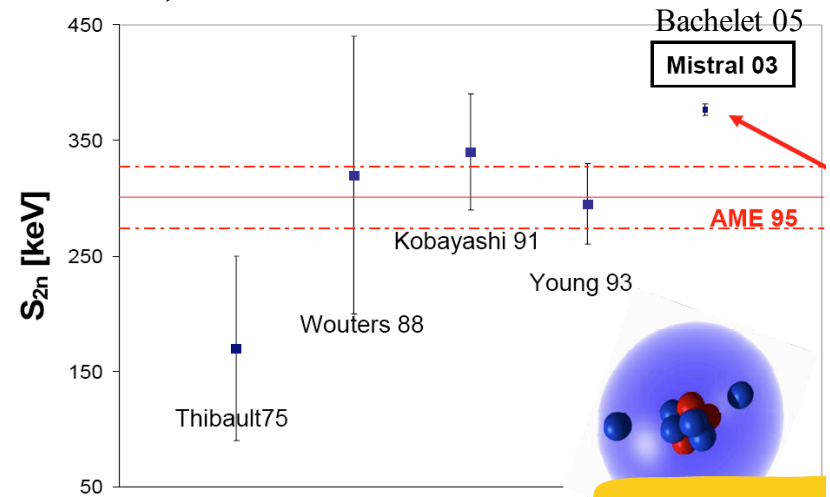
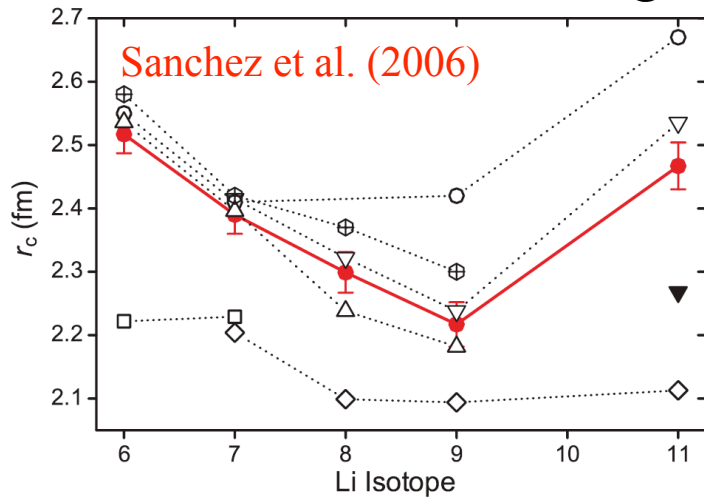
nn halos near neutron drip-line
are the only known examples
of Borromean states in nature

Partly responsible for $A=5$
mass gap, important for nucleosynthesis

Impact of medium-mass nn halo
discoveries on r-process?

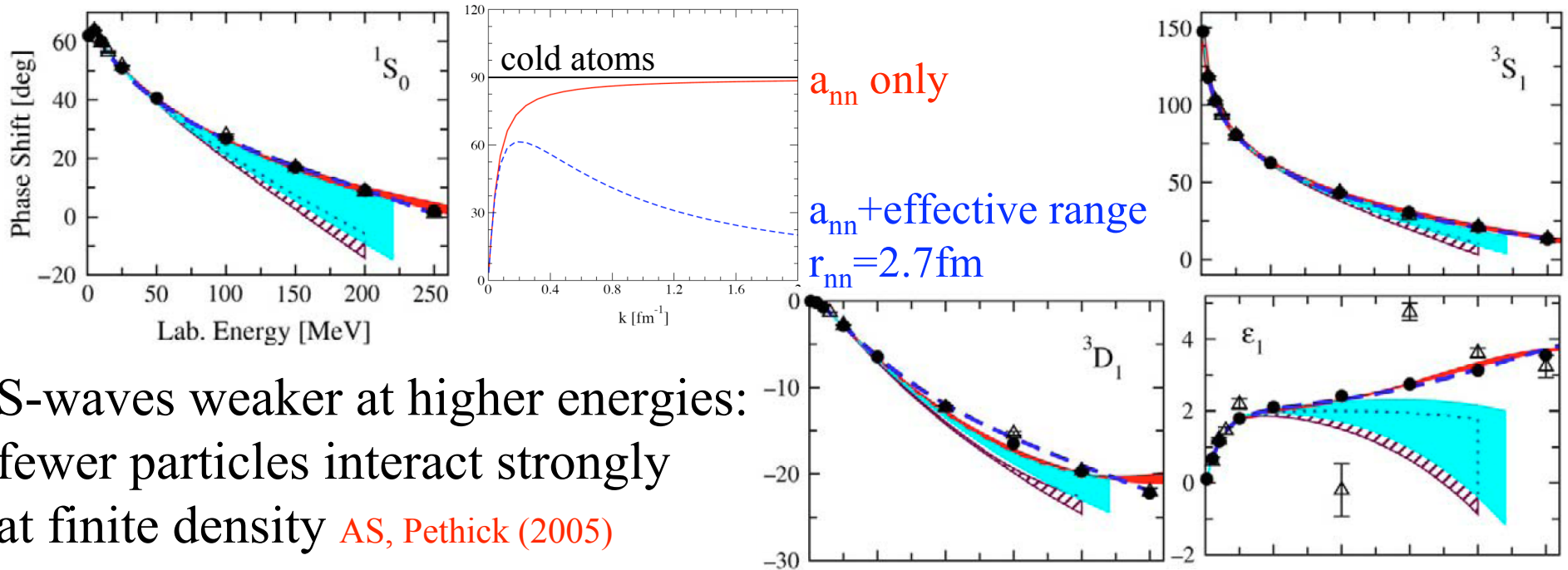


Recent ^{11}Li results: charge radius (isotope shift), mass still unsettled



NN partial wave analysis and phase shifts

Nijmegen PWA93 (filled circles) <http://www.nn-online.org>



S-waves weaker at higher energies:
fewer particles interact strongly
at finite density *AS, Pethick (2005)*

Tensor force from 3S_1 - 3D_1 mixing ϵ_1 in spin=1 channel

Epelbaum (2006)
Machleidt (1998)

Spin-orbit (LS) force
from triplet ${}^3P_{J=0,1,2}$
phase shifts
C=central, T=tensor only

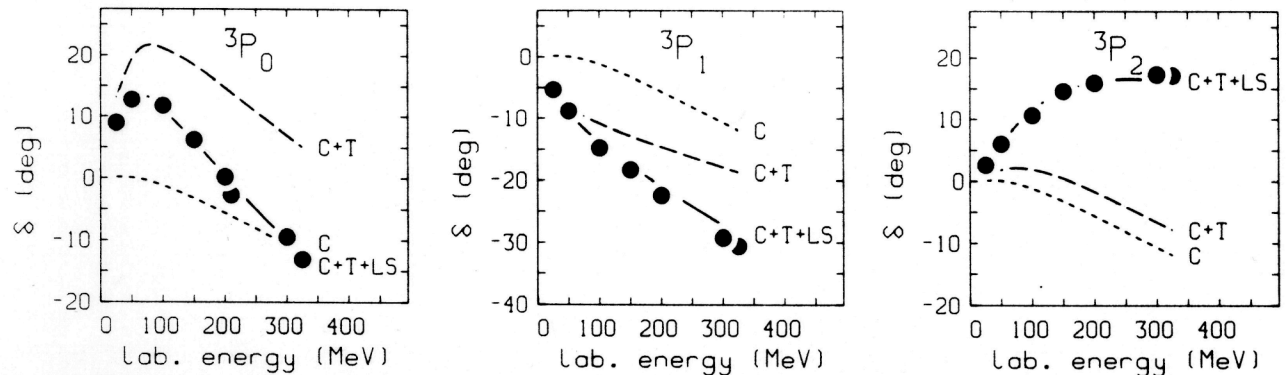
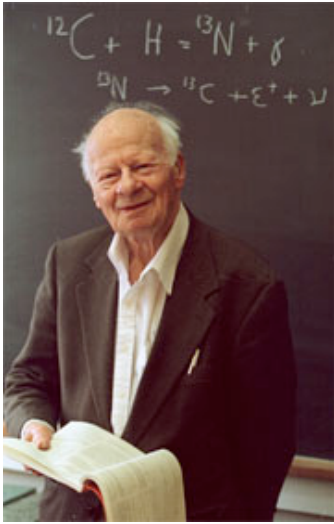


Fig. 3.3. NN phase shifts in triplet P waves. Shown are predictions using a central



Nuclear interactions

Bethe (1953): "more man-hours ... given to this [nuclear force] problem than to any other scientific question in the history of mankind"

Effective theory for NN, many-N interactions, depend on resolution scale

$$H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + V_{\text{4N}}(\Lambda) + \dots$$

All nuclear interactions are effective interactions

never have "bare" interactions for composite particles, "bare" only useful to specify reference Hamiltonian

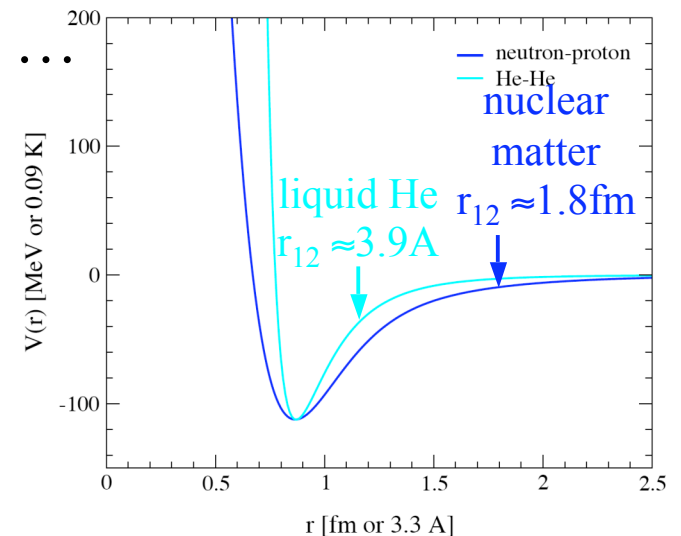
Phenomenological hierarchy: $\text{NN} > \text{3N} > \text{4N} > \dots$

NN: constrained by ~ 4500 NN data, $\chi^2 \sim 1$

3N: fit to few-body data, $\chi^2 \sim \text{poor}$

4N, ...: estimates small

Last 30+ yrs, most accurate calculations with hard **NN** interactions



Chiral effective field theory for nuclear forces

systematic expansion in low-momenta Q over breakdown scale Λ

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt, ...

	NN	3N	4N
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$	C_0 	—	—
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$	C_2 	—	—
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			—
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

Explains pheno hierarchy:

NN > 3N > 4N > ...

NN-3N, πN , $\pi\pi$, electro-weak, ...

consistency

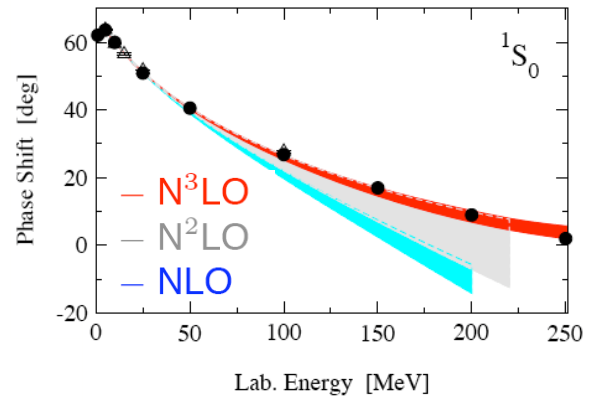
3N: only 2 new couplings to N³LO!

used to constrain ν -deuteron breakup reactions for SNO

theoretical error estimates

.....
limited cutoffs used/explored

Δ less vs. full
 $m_\Delta - m_N \sim 2m_\pi$



$\chi^2_{\text{+}} \sim 1$

+ ...

+ ...

Difficulties of conventional nuclear interactions

due to high momenta, large cutoffs, very small resolution scale

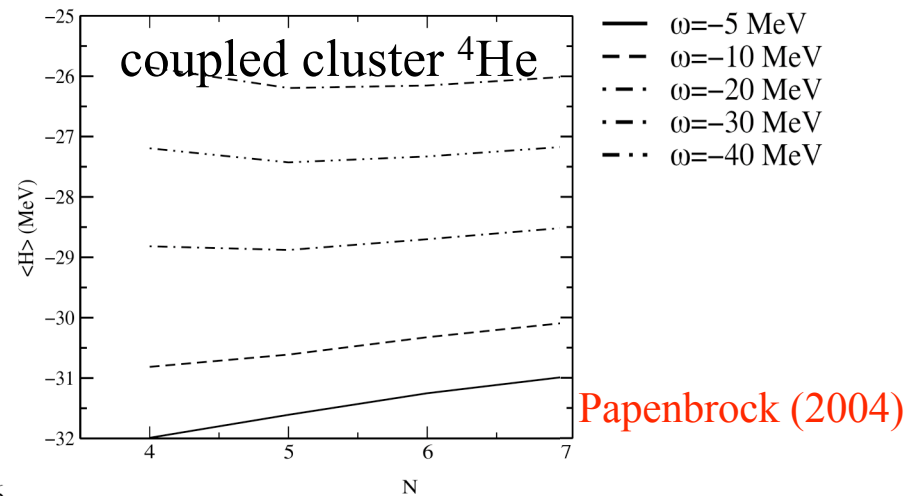
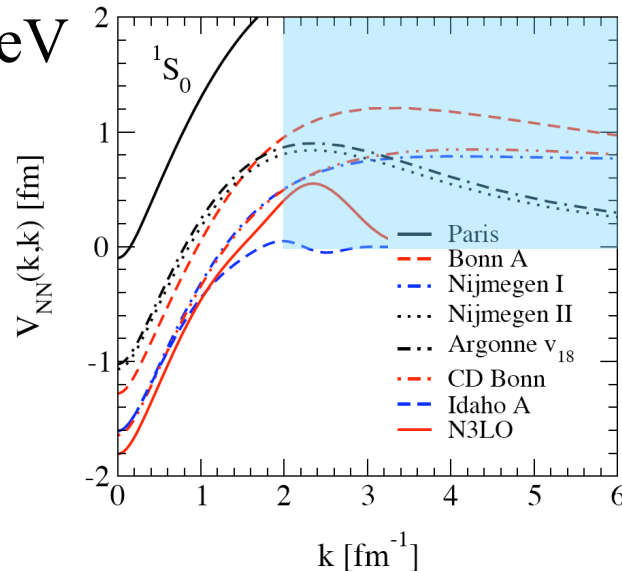
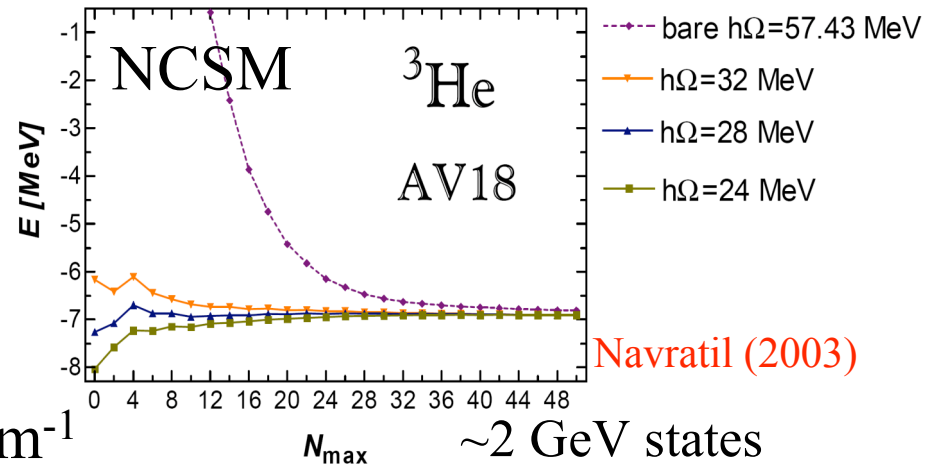
leads to non-perturbative flipped-potential bound states of $-\lambda V$ for small λ , hinder any perturbative expansion, radius of convergence=0

requires resummation due to slow convergence with basis size

resummation introduces uncontrolled starting-energy ω in G matrix

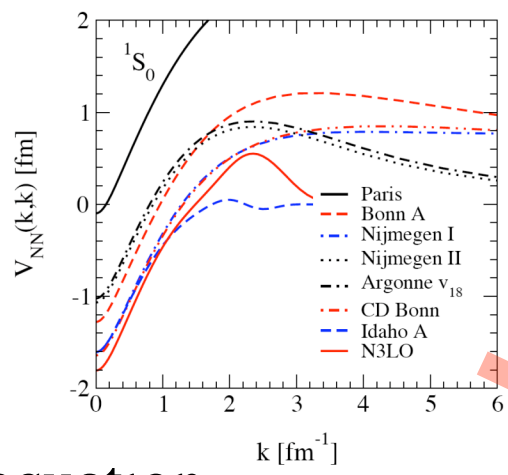
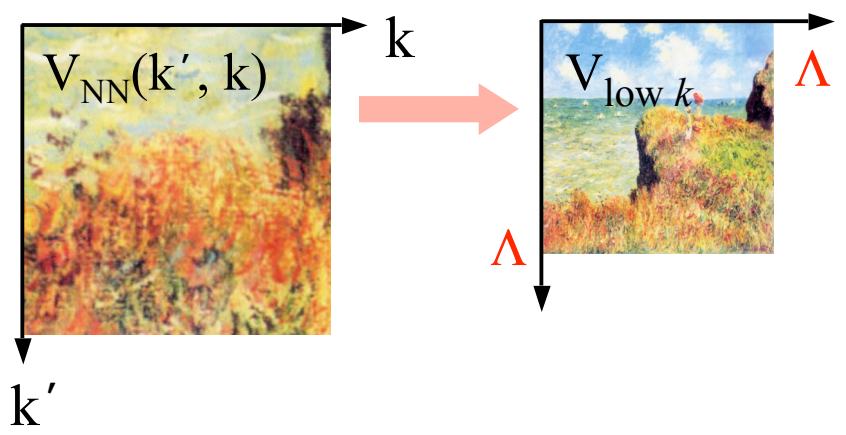
but only fit to NN scattering for $k < 2 \text{ fm}^{-1}$

or $E_{\text{lab}} < 350 \text{ MeV}$

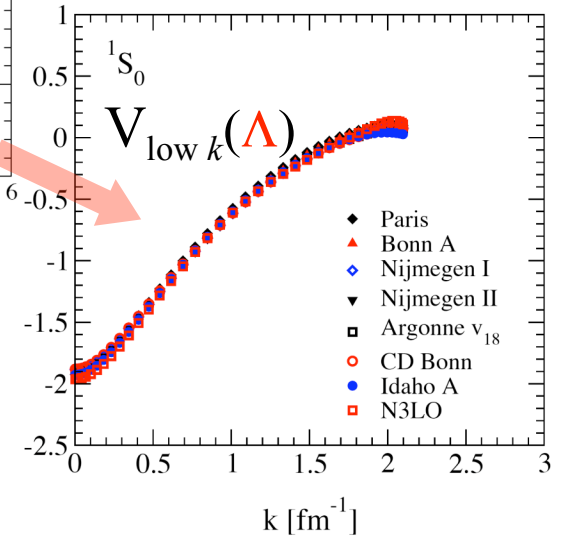


Low-momentum interactions from the Renormalization Group

for low-energy physics, can evolve to lower resolution scale, integrate out high-momentum modes using exact RG equation



reproduces low-energy NN observables



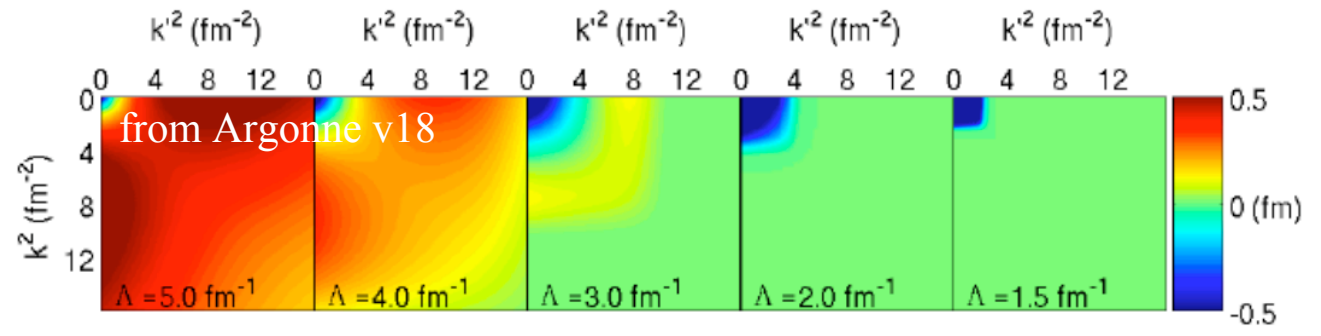
implemented by discretized RG equation

$$\frac{d}{d\Lambda} V_{\text{low } k}(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}(k', \Lambda) T(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

universal low-momentum $V_{\text{low } k}(\Lambda)$ for $\Lambda \lesssim 2.1 \text{ fm}^{-1}$

Bogner, Kuo, AS (2003)

evolution to $V_{\text{low } k}(\Lambda)$ weakens off-diagonal coupling



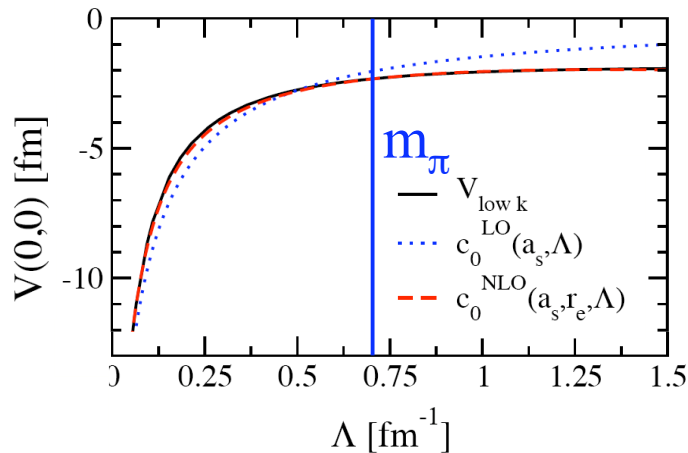
Evolution to $V_{\text{low } k}(\Lambda)$

collapse due to same long-distance pion exchange and fit to same NN data

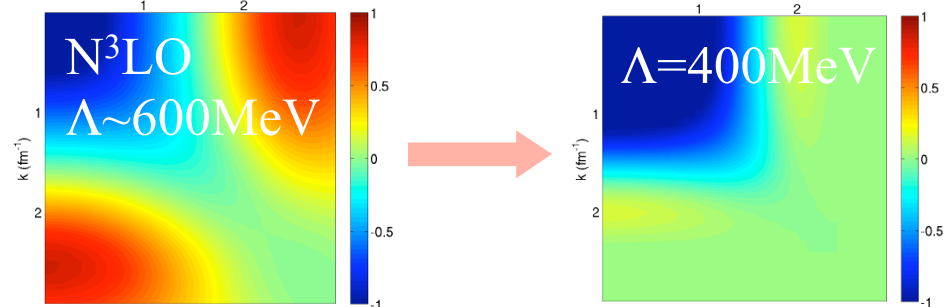
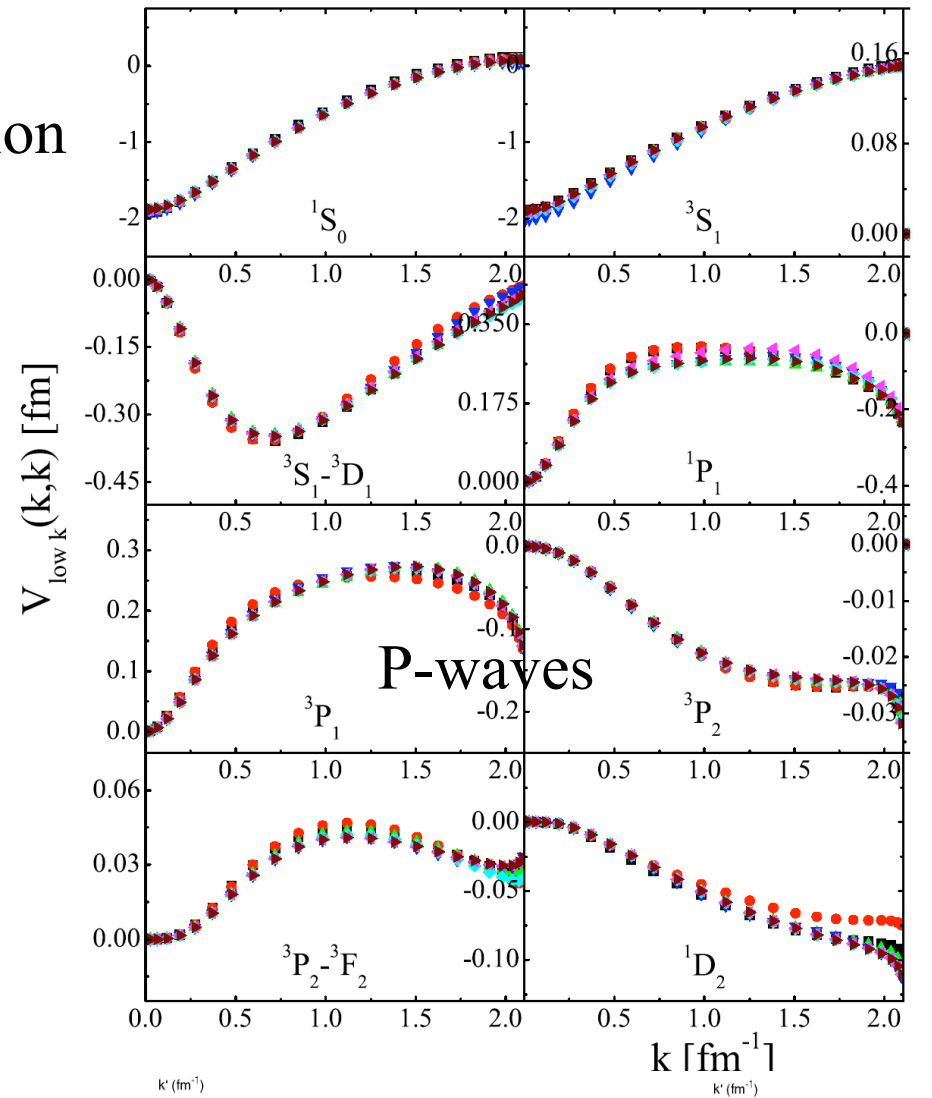
small differences correlate with fits to data

defines new class of NN interactions

evolution of $V_{\text{low } k}(0,0;\Lambda)$ follows contact interaction $c_0(\Lambda)$ at NLO in pionless EFT regime



evolution of chiral EFT interactions to low-momentum beneficial



Advantages of lower cutoffs for nuclear structure

tractable in an oscillator basis,
direct convergence in few-/many-body
calculations

improved convergence for
smooth cutoffs

Bogner, Furnstahl, Ramanan, AS (2007)

or SRG interactions

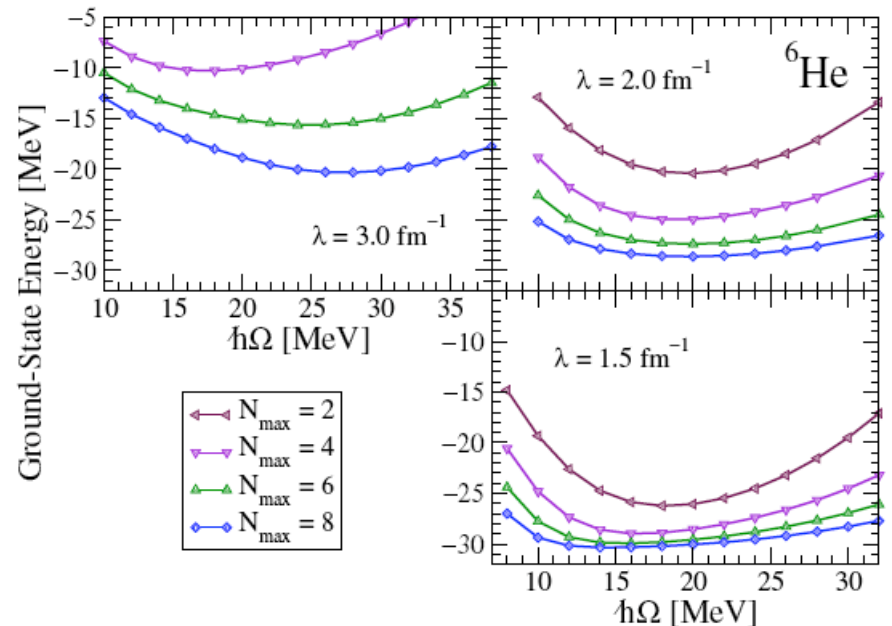
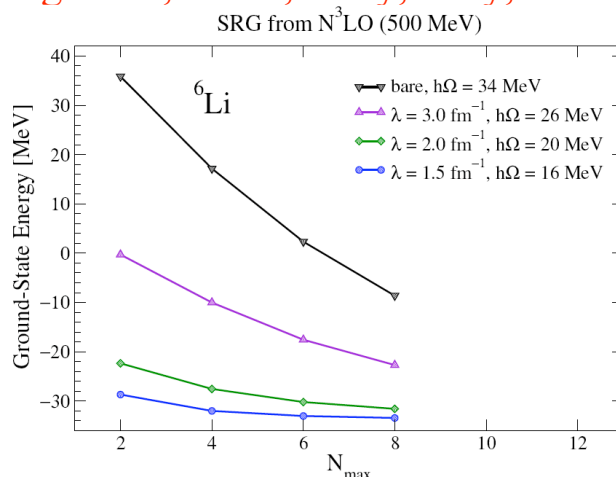
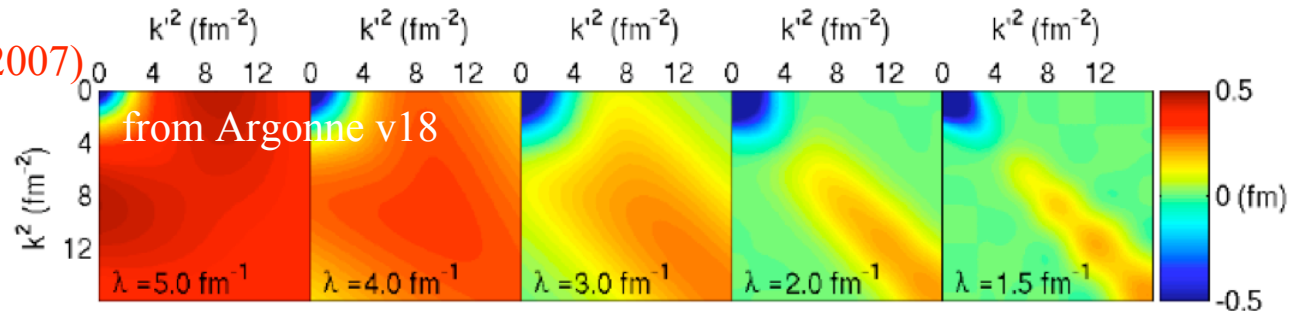
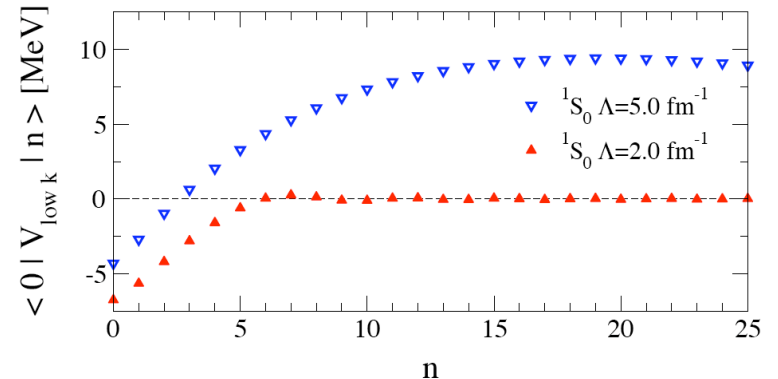
Bogner, Furnstahl, Perry (2007)

talks by Eric Anderson and

Eric Jurgenson

very promising convergence in NCSM

Bogner, Furnstahl, Jurgenson, Maris, Perry, Vary, AS



Weinberg eigenvalues

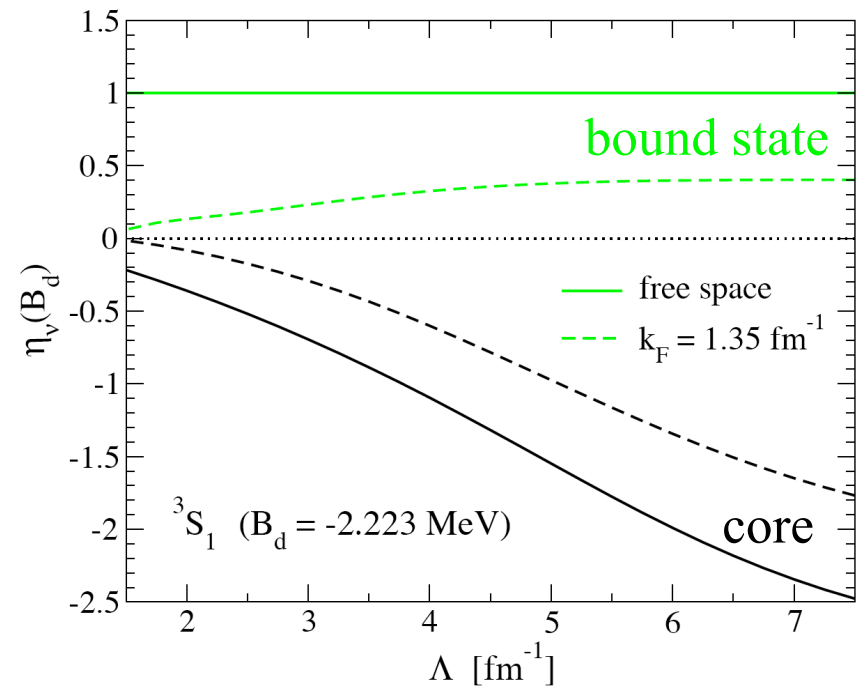
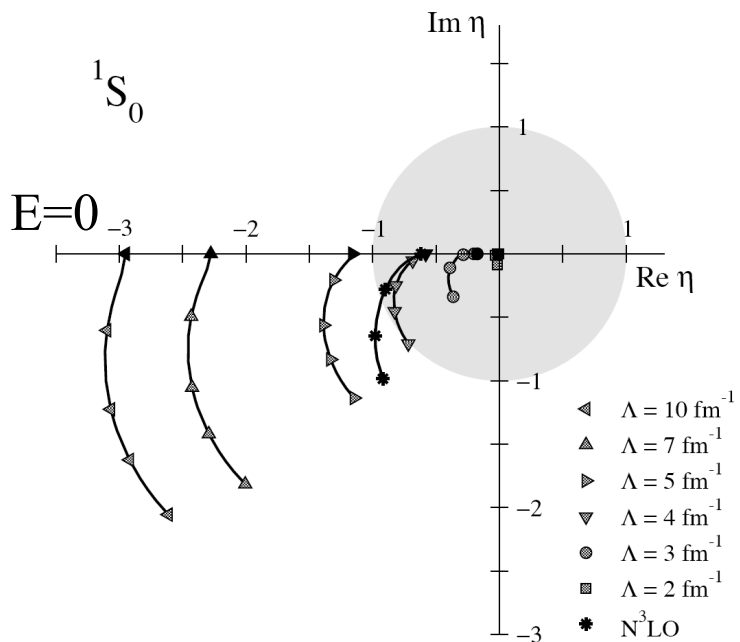
study spectrum of $G_0(z)V |\Psi_\nu(z)\rangle = \eta_\nu(z) |\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z) |\Psi_\nu(z)\rangle = (1 + \eta_\nu(z) + \eta_\nu(z)^2 + \dots) V |\Psi_\nu(z)\rangle$

can write as Schrödinger eqn $(H_0 + \frac{1}{\eta_\nu(z)} V) |\Psi_\nu(z)\rangle = z |\Psi_\nu(z)\rangle$

Repulsive core eigenvalues small for lower cutoffs,

Born series always nonperturbative for interactions with cores



Bogner, AS, Furnstahl, Nogga (2005)

2-body nonperturbative at low energies due to near-bound states,

in nuclear matter Pauli blocks low energies \Rightarrow **deuteron eigenvalue small**

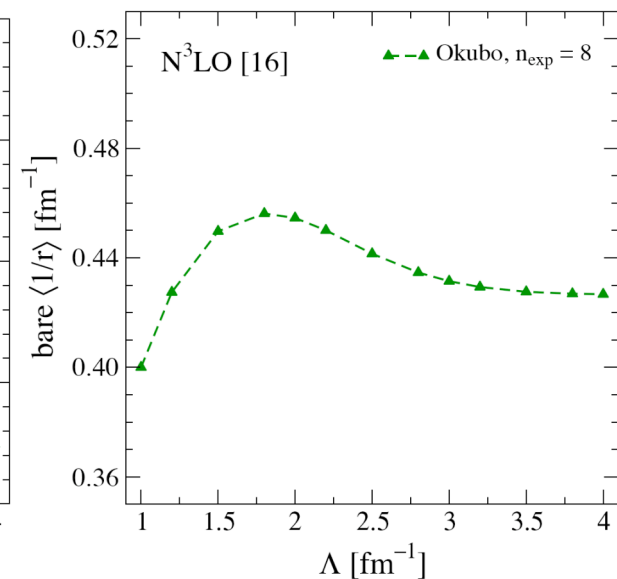
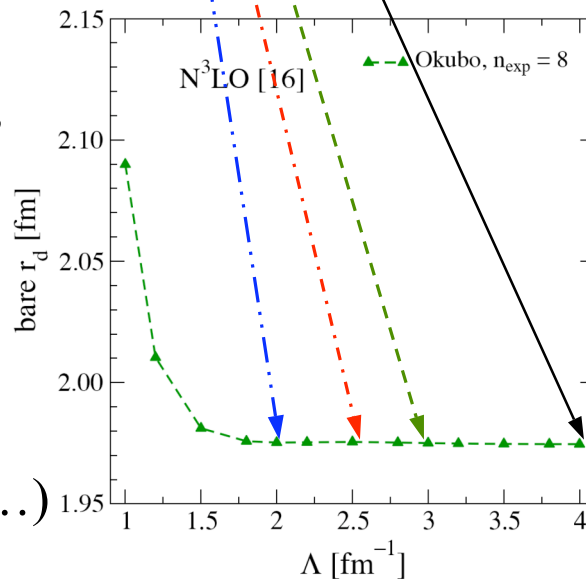
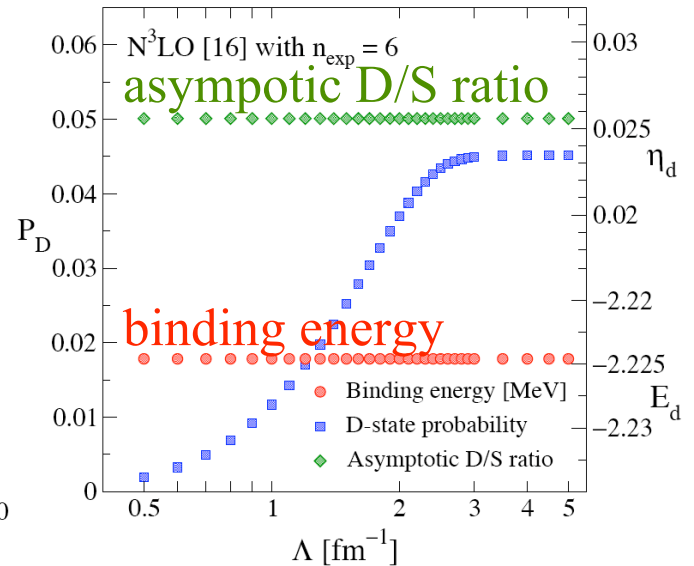
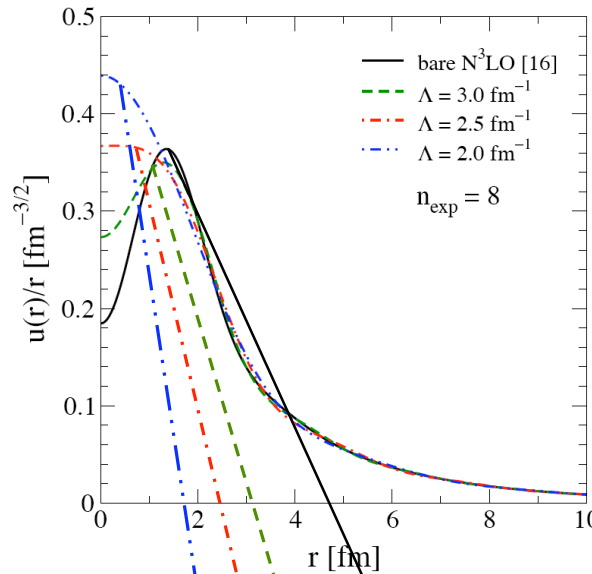
Correlations ???

RG preserves long-range parts of interactions, deuteron **observables**, with dramatically different wave functions/correlations

short-range correlations depend on resolution scale, cf. parton df

weak renormalization of long-range operators $r, Q, 1/r, \dots$

short-range operators always have correction terms (exchange currents, ...)



Few-nucleon systems

$V_{\text{low } k}(\Lambda)$ defines class of NN interactions with cutoff-independent NN observables

cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

Cutoff dependence explains Tjon line due to neglected 3N interactions, 3N required by renormalization

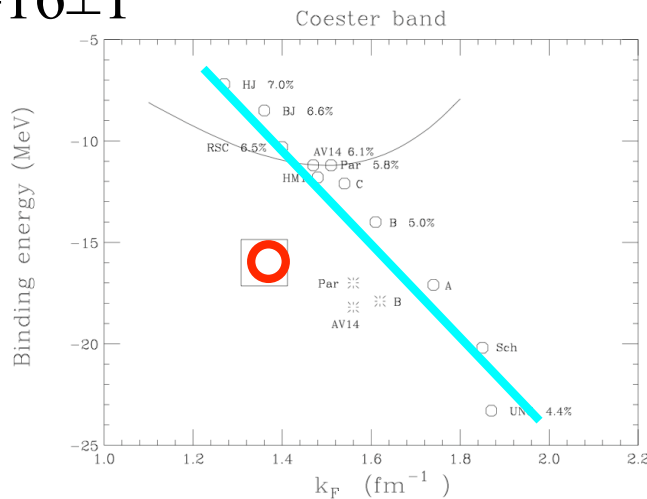
Experiment breaks from lines

Tjon lines in $A=16 \pm 1$

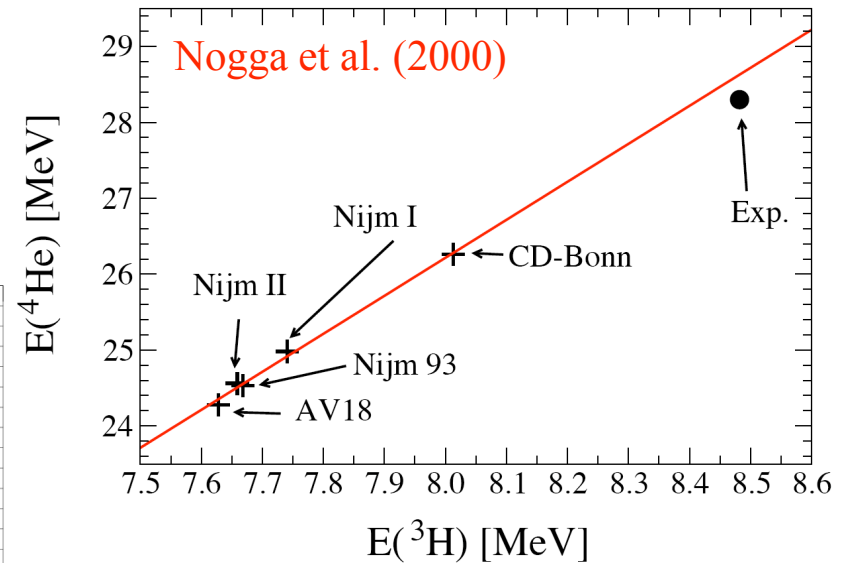
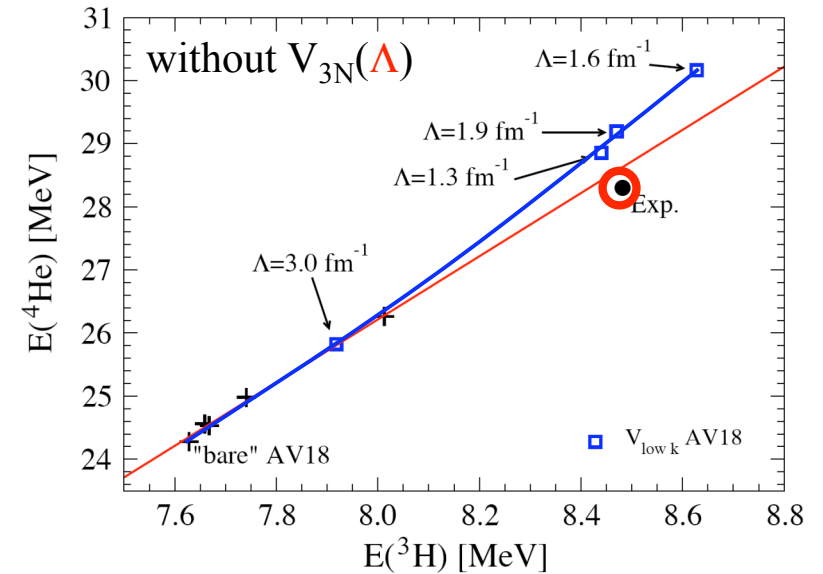
Hagen, AS, in prep.

Coester line in nuclear matter

Coester et al. (1970)
from Baldo et al. (2003)



Nogga, Bogner, AS (2004)



3. Three-nucleon interactions: a frontier in nuclear structure

Effect of 3N interactions are amplified in nuclei,
 constrain 3N with few- and **many-body** data \Rightarrow controlled predictions

3N interaction crucial:

to break off Tjon, Coester,... lines

spin-orbit dependence and A_y puzzle

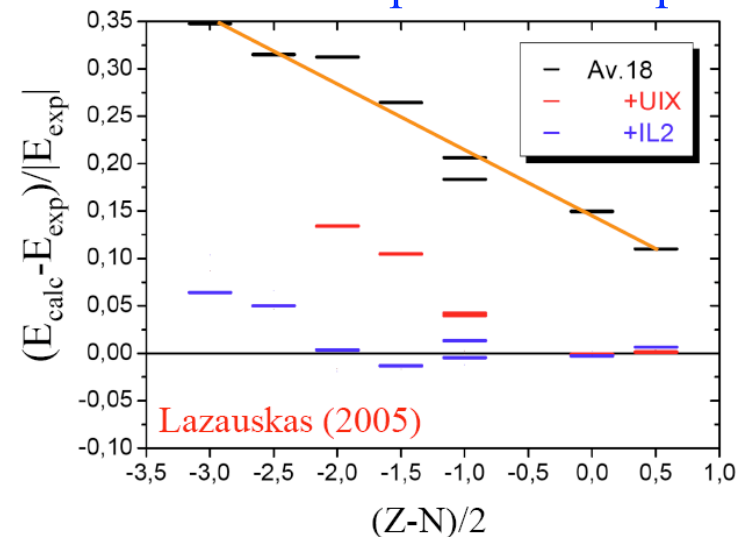
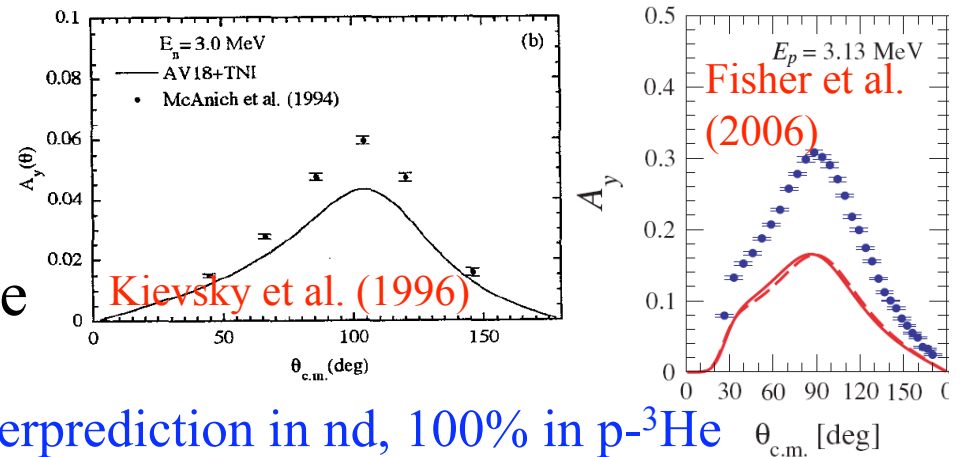
$$A_y = \frac{\frac{d\sigma}{d\Omega} \Big|_{\uparrow} - \frac{d\sigma}{d\Omega} \Big|_{\downarrow}}{\frac{d\sigma}{d\Omega} \Big|_{\uparrow} + \frac{d\sigma}{d\Omega} \Big|_{\downarrow}}$$

30% underprediction in nd, 100% in $p\text{-}^3\text{He}$
 $\sim 0\%$ in $p\text{-}^3\text{H} \Rightarrow$ isospin

isospin dependence, n/p-rich systems

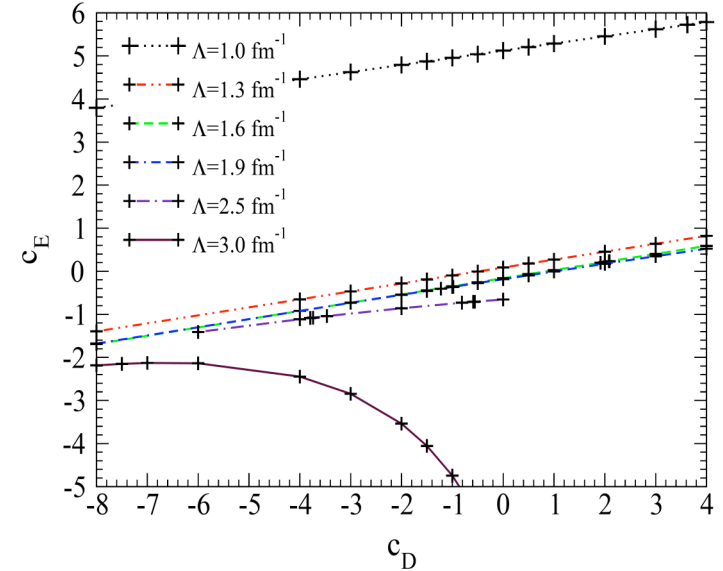
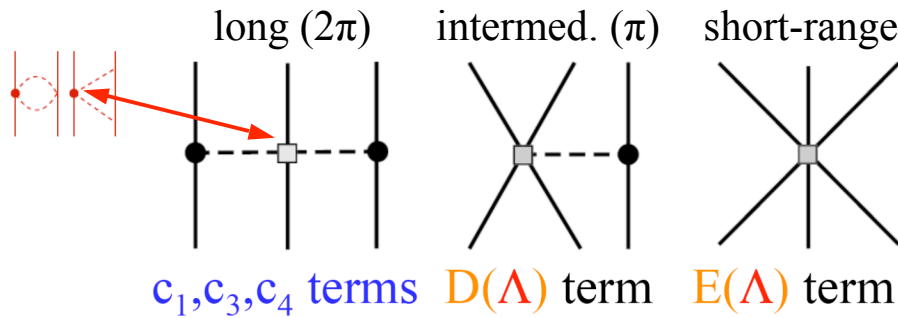
impact on nuclear matter, saturation

pivotal for extrapolations
 to extremes of astrophysics



Low-momentum 3N interactions $V_{3N}(\Lambda)$

corresponding $V_{3N}(\Lambda)$ from chiral EFT



c_i determined from πN or NN scattering

c_3, c_4 important for structure

$$c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.2}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

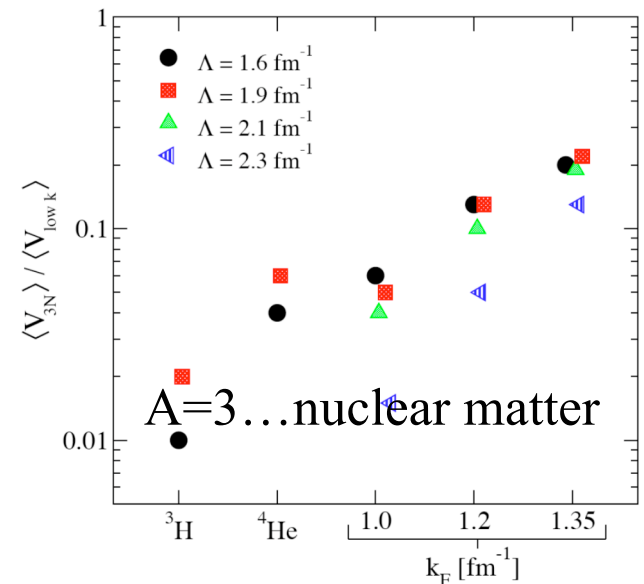
fit D,E couplings to $A=3,4$ binding energies
for range of cutoffs, linear dependences in fits

3N interactions perturbative for $\Lambda \lesssim 2 \text{ fm}^{-1}$

Nogga, Bogner, AS (2004)

nonperturbative at larger cutoffs, cf. chiral EFT $\Lambda \approx 3 \text{ fm}^{-1}$

3N expectation values natural $\sim (Q/\Lambda)^3 V_{NN}$



Radii and light nuclei

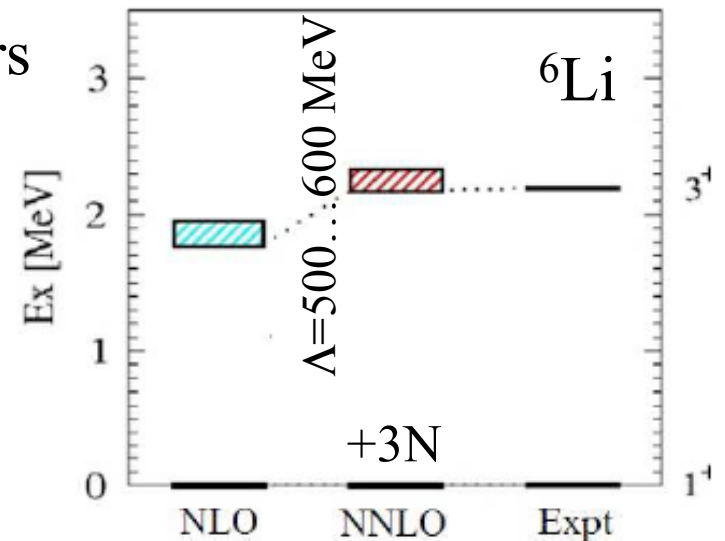
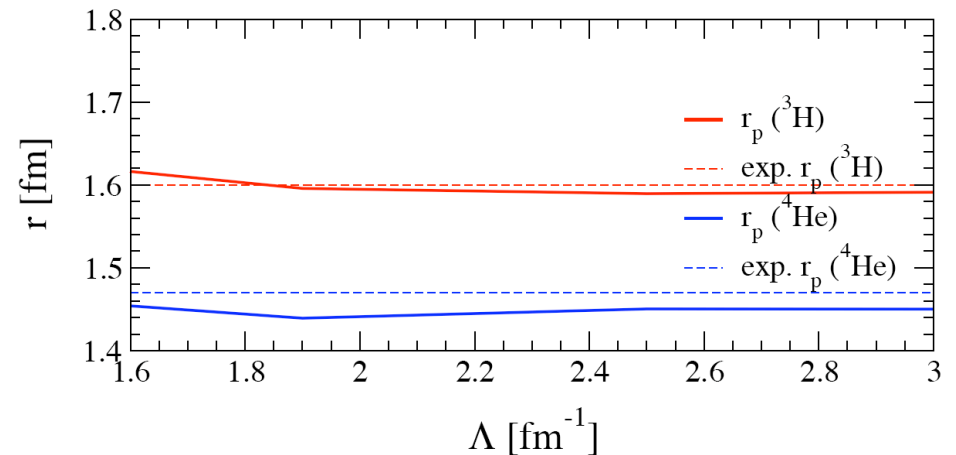
$V_{\text{low } k}(\Lambda)$ + leading chiral $V_{3N}(\Lambda)$ interaction

Cutoff dependence of observables probes neglected many-body int.

Radii with 3N interaction approx. cutoff-independent, agree with exp.

Can provide lower limits on theoretical errors

long-term:
important for nuclear matrix elements
needed in fundamental symmetry tests



NCSM results with chiral EFT interactions from A. Nogga

Towards 3N interactions in medium-mass nuclei

based on low-momentum $V_{\text{low } k}(\Lambda) + V_{3N}(\Lambda)$

Hagen et al., arXiv:0704.2854.

developed coupled-cluster theory with 3N interactions, first benchmark for ${}^4\text{He}$

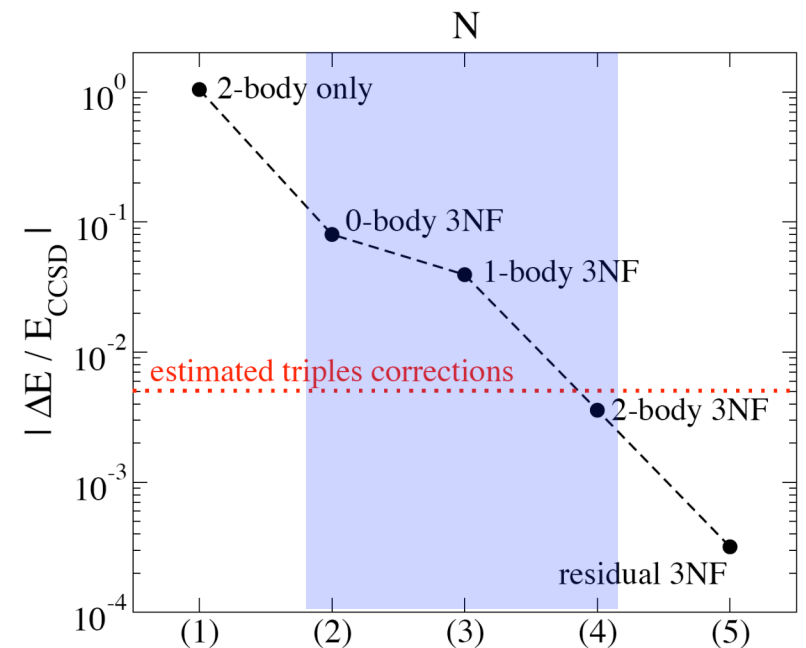
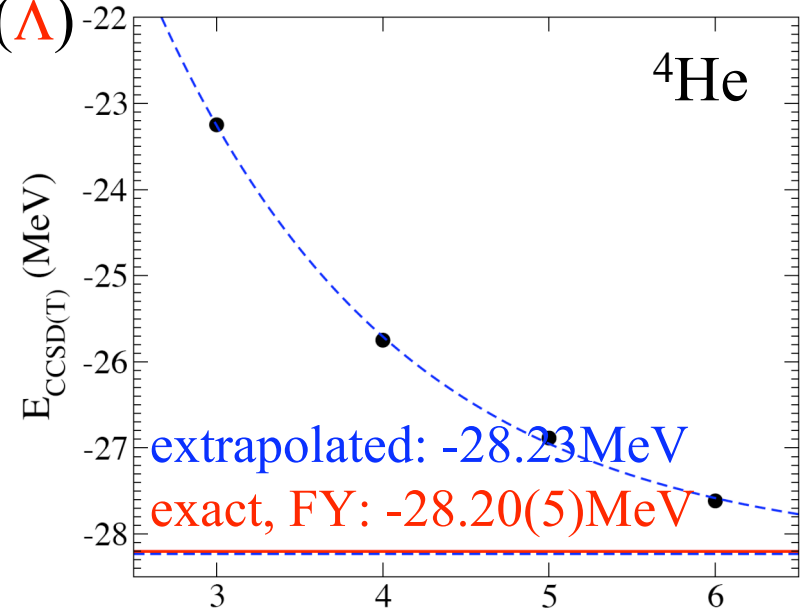
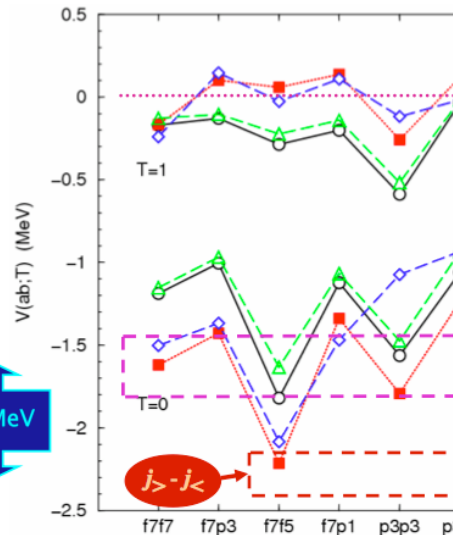
Results show that 0-, 1- and 2-body parts of 3N interaction dominate

residual 3N interaction can be neglected!

very promising

supports: “monopole” corrections for SM interactions due to 3N

0.6 MeV



Coupled-cluster theory: pushing the limits to $A=40$

based on $V_{\text{low } k}(\Lambda)$ only, + full $V_{3N}(\Lambda)$ in progress

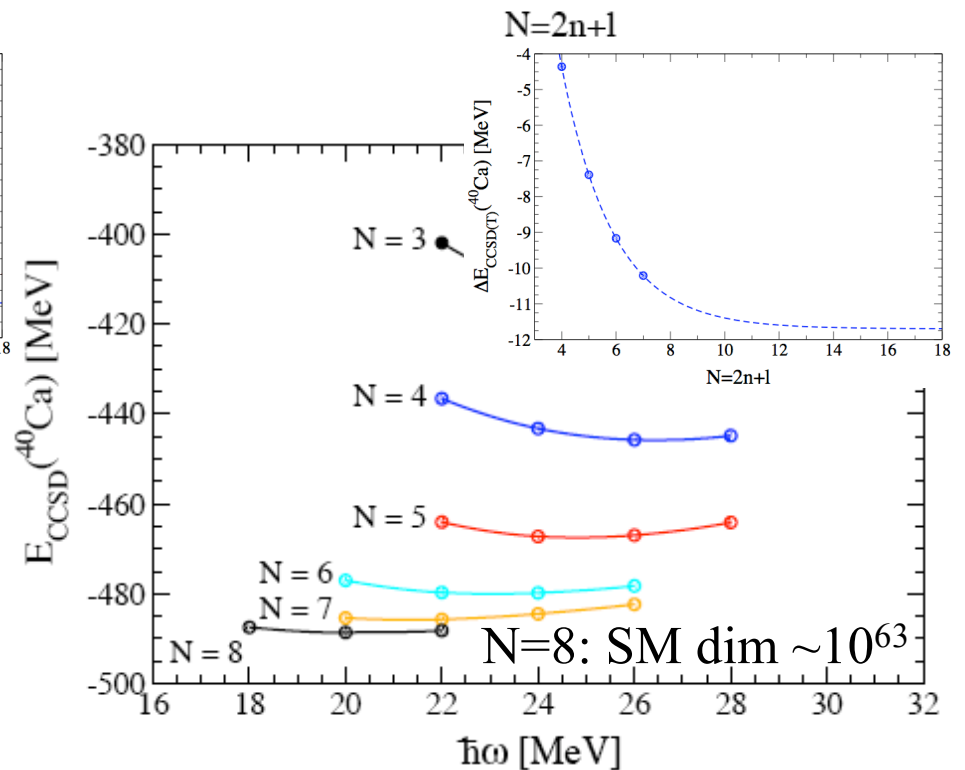
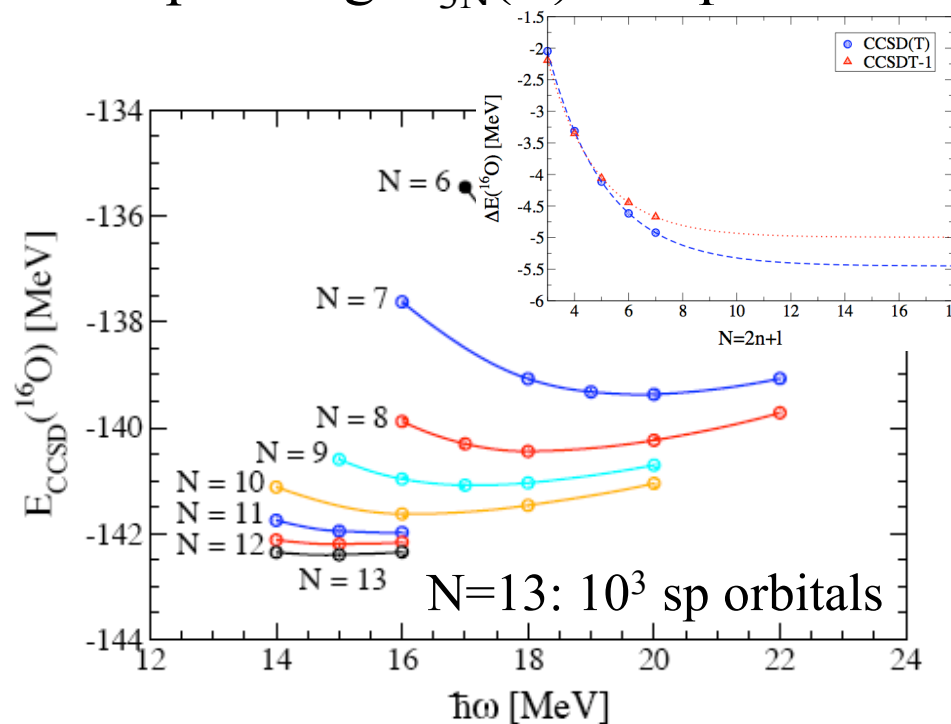
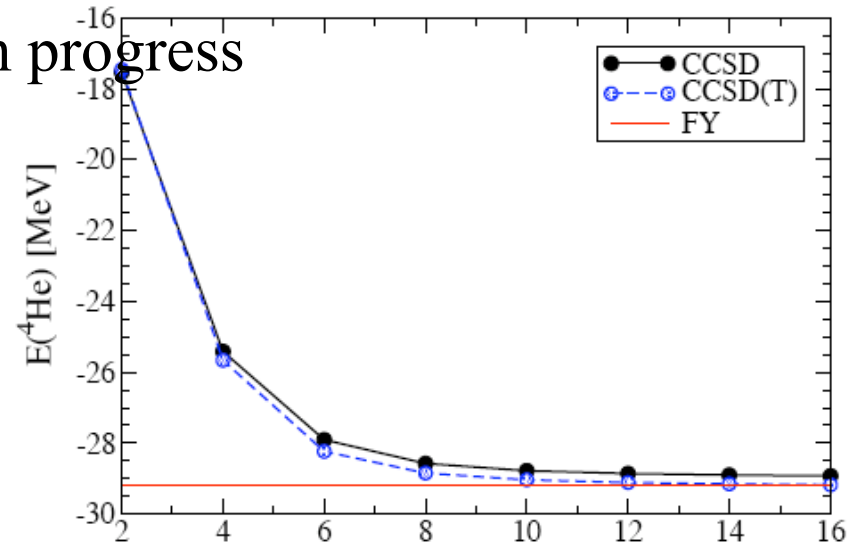
Hagen et al., to be posted soon.

CC theory meets and sets benchmarks:

within 10 keV of Faddeev-Yakubovsky

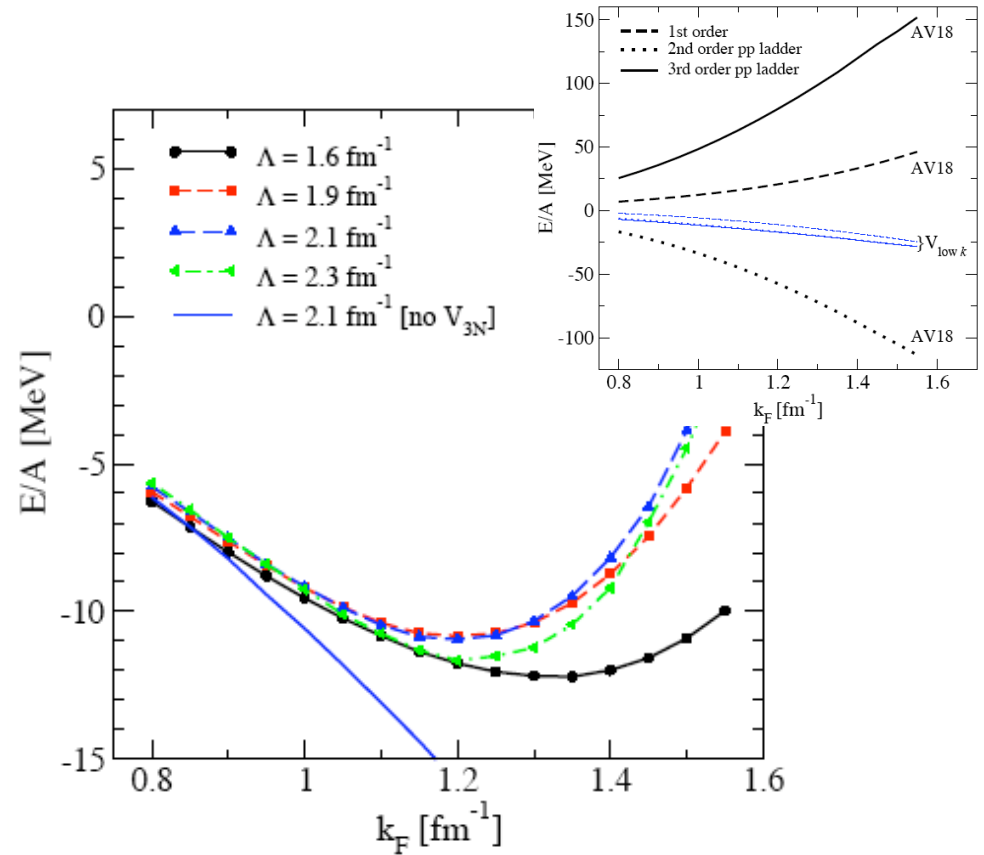
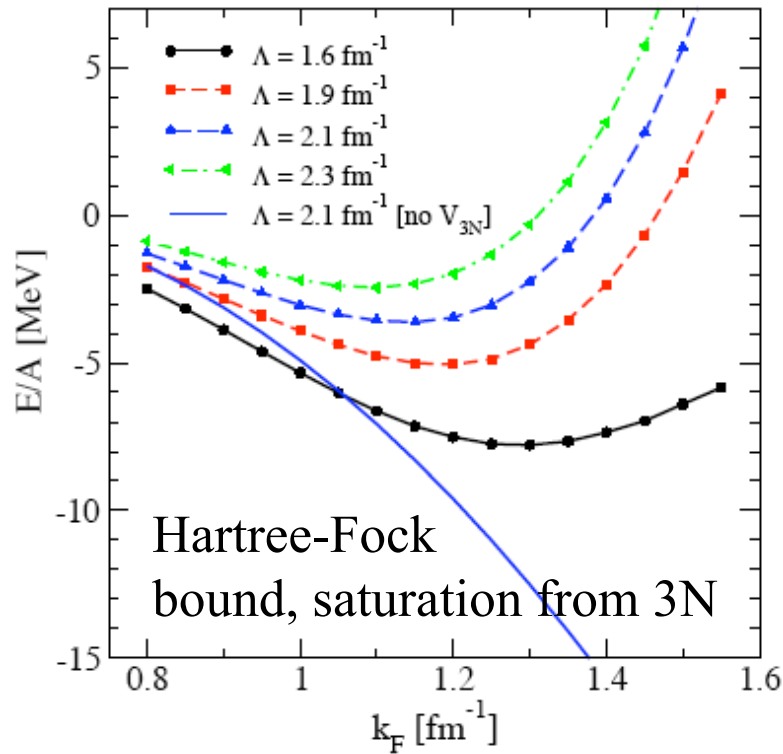
essentially converged for ^{16}O and ^{40}Ca

corresponding $V_{3N}(\Lambda)$ is repulsive



Possibility of perturbative nuclear matter with NN and 3N

motivated by Weinberg analysis



Hartree-Fock + \approx 2nd-order
cutoff dep. strongly reduced

Bogner, AS, Furnstahl, Nogga (2005)

3N drives saturation but expectation values natural, consistent with EFT
will provide key guidance to microscopic DFT for

4. Summary and many on-going developments

Exciting era with advances on many fronts!

NCSM, CC calculations in progress

3N contributions to shell model [Jason Holt](#)

new power counting for nuclear matter

microscopic guidance for DFT [Bogner, Furnstahl, Platter et al.](#)

constructing microscopic interactions for HF+GCM+... [Rotival, Duguet et al.](#)

superfluidity in neutron stars

[Hebeler et al.](#)

finite temperature equation of state
for supernovae, neutron-star mergers

[Tolos, Friman, AS \(2006\)](#)

.....

