The Nuclear Equation of State

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Outline

- EOS of Hot Dense Matter
- Variational Theory for Finite T EOS
- The Hamiltonian
- Results
- Conclusions

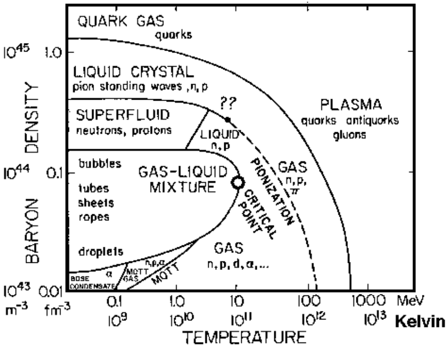
- **Nuclear Matter**
  Infinite Matter with Neutrons + Protons only

  - *Symmetric Nuclear Matter*
    Equal Number of Neutrons and Protons
  - *Pure Neutron Matter*
    Neutrons only
  - *Asymmetric Nuclear Matter*
    Unequal number of neutrons and protons

Why is the EOS interesting/useful ?

- Supernova evolution
- Cooling of Neutron Stars
- Heavy Ion Collisions
- Novel Many Body Systems

EOS of Hot Dense Matter



Challenges for a microscopic theory:

1. Include correlations beyond mean field
2. Good description of the excited states (for $T \neq 0$)
3. Calculate the energy, $E(\rho, T)$ **accurately**
4. Calculate the entropy, $S(\rho, T)$ **accurately**

Correlated Basis States

Landau Fermi Liquid Theory

Eigenstates of Interacting System $\overset{\text{one - one}}{\longleftrightarrow}$ Fermi Gas States (**FGS**)

Correlated Basis States (CBS)

$$|\Psi_I) = |\mathbf{CBS}) = \frac{\mathcal{G}\,|\mathbf{FGS}]}{[\mathbf{FGS}\,|\mathcal{G}^\dagger\mathcal{G}|\,\mathbf{FGS}]}$$

Correlation Operator

$$\mathcal{G} = \mathcal{S} \prod_{i<j} \mathcal{F}(\mathbf{r}_{ij})$$

- Generalization of Jastrow method of pair correlation functions
- Correlations are included in the functions $\mathcal{F}(\mathbf{r}_{ij})$

$$\boxed{(\Psi_I|H|\Psi_I) \quad \longrightarrow \text{VCS (FHNC/SOC)}}$$

$$\boxed{(\Psi_I|H|\Psi_J) \quad \longrightarrow \text{No such method}}$$

Gibbs-Bogoliubov variational principle for $T \neq 0$

$$F \leq F_V \equiv \mathrm{Tr}(\rho_V H) + T\mathrm{Tr}\rho_V \mathrm{ln}\rho_V \ .$$

F_V is an approximation for F

Analogy : Rayleigh-Ritz variational principle for $T = 0$

$$E_0 \leq (\Phi_0 | \, H \, | \Phi_0)$$

e.g. the Akmal-Pandharipande-Ravenhall EOS Phys. Rev. C 58, 1804(1998)

What we want to do

Variational 'Hamiltonian'

$$H_v|\Psi_I) = E_v^I|\Psi_I) = \sum_k \epsilon_T(k) n_I(k)|\Psi_I)$$

Variational density matrix

$$\rho_v(T) \propto \mathrm{e}^{-H_v/T} = \mathrm{e}^{-E_v^I/T}|\Psi_I)(\Psi_I|$$

$$S_v(T) = -\sum_k (\bar{n}(k)\ln\bar{n}(k) + (1-\bar{n}(k))\ln(1-\bar{n}(k)))$$

$$\mathrm{Tr}\,(\rho_v H) = \frac{1}{\sum_I \mathrm{e}^{-E_v^I/T}} \sum_I \mathrm{e}^{-E_v^I/T}(\Psi_I|H|\Psi_I)$$

The Orthogonality Problem

The Correlated basis states are not **Orthonormal** to each other

$|\Psi_I)(\mathrm{CBS}) \longrightarrow |\Theta_I\rangle \equiv$ Orthonormal Correlated Basis States (OCBS)

$$H_v|\Theta_I\rangle = E_v^I|\Theta_I\rangle = \sum_k \epsilon_T(k) n_I(k)|\Theta_I\rangle$$

$$\mathrm{Tr}\,(\rho_v H) = \frac{1}{\sum_I \mathrm{e}^{-E_v^I/T}} \sum_i \mathrm{e}^{-E_v^I/T}[(\Psi_i|H|\Psi_i) + \Delta\mathbf{E_I}]$$

$\Delta\mathbf{E_I}$ (**Orthogonality Corrections**) $\longrightarrow (\Psi_I|H|\Psi_J)$, *non diagonal* matrix elements

Our Solution

- Microcanonical ensemble instead of a canonical ensemble

$$\rho_V^{MC} = \frac{1}{\mathcal{N}_{\mathcal{M}}} \sum_I |\Theta_I\rangle\langle\Theta_I|$$

$\mathcal{M}(T) \equiv$ the microcanonical ensemble at temperature T
$\mathcal{N}_{\mathcal{M}} =$ # of elements in $\mathcal{M}$

- **Mixed** orthonormalization procedure (Lówdin + Gram-Schmidt).

$\Delta \mathbf{E_I}$ (**Orthogonality Corrections**) $\longrightarrow 0$

Details: A. Mukherjee and V.R. Pandharipande , Phys. Rev. C **75**, 035802 (2007)

The Hamiltonian

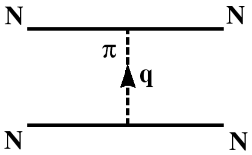
$$H = \sum_i -\frac{\hbar^2 \nabla^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_{i<j} \delta v_{ij} \; .$$

- v_{ij} = NN potential (Argonne $v18$)
- V_{ijk} = NNN potential (Urbana IX)
- δv_{ij} = Relativistic Boost Corrections

Argonne $v18$

Av18 $=$ Long Range Part + Intermediate Range Part + Short Range Part

• Long Range Part $\sim$ One Pion Exchange Potential



- Intermediate Range Part : Motivated by Meson Exchange Theory
- Short Range Part : Phenomenological

Urbana IX and Relativistic Boost Corrections

Urbana IX

- UIX = Two Pion Exchange + Phenomelogical Repulsion
- Fits Binding Energy of Light nuclei + Binding of nuclear matter at saturation

Urbana IX and Relativistic Boost Corrections

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Relativistic Boost Corrections

- Leading order corrections due to Special Relativity

- The (temperature and density dependent) eigenvalue spectrum of H_V is an additional 'parameter' to be varied.

$$E_v^I = \sum_k \epsilon_T(k)\bar{n}(k)$$

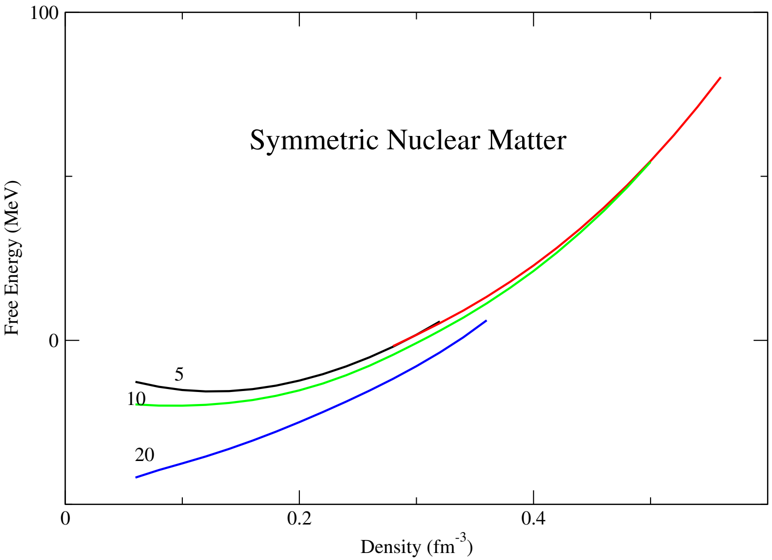
Effective mass approximation

$$\epsilon_T(k) = \frac{\mathbf{k}^2}{2m^*(\rho, T)} + U(\rho, T)$$

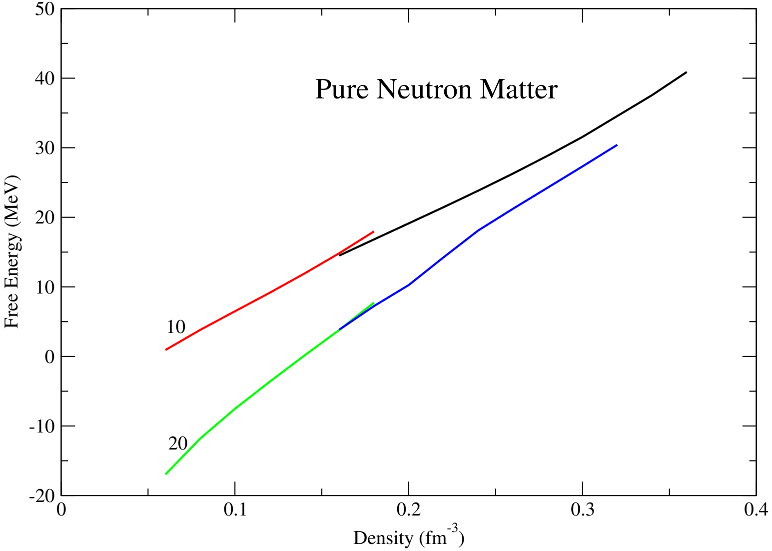
- Minimize the energy of the two body cluster at finite temperature
- Long range part of the wavefunctions constrained by the model (uncorrelated) wavefunction

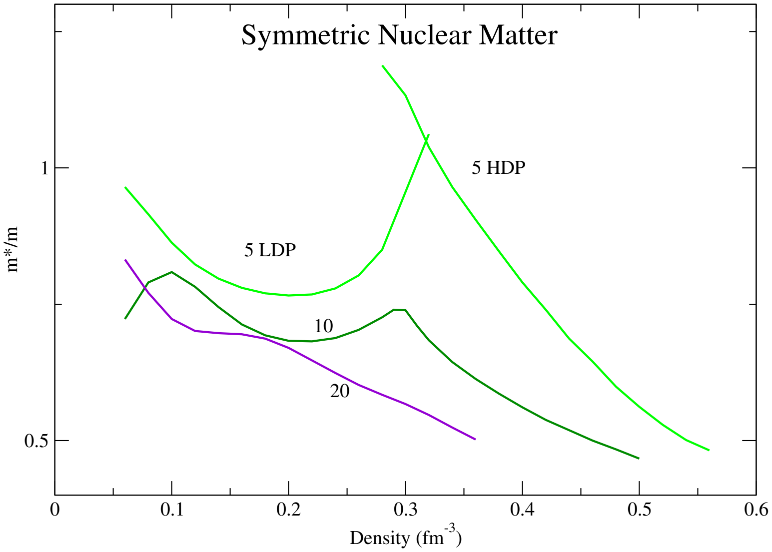
$$\text{Euler-Lagrange equations} \rightarrow \text{Correlation functions}(\mathcal{F})$$

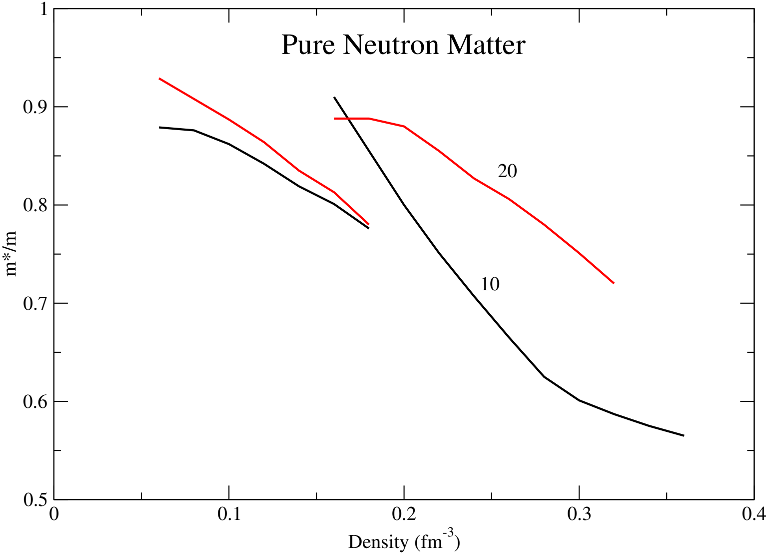
- $\mathcal{F}$ has variational parameters : 2 correlation lengths (d_c, d_t), spin-isospin quenching parameter (α) and the effective mass (m^*)


100
0
0
0.2
0.4
Free Energy (MeV)
Density (fm^{-3})
Symmetric Nuclear Matter
5
10
20

Results


Pure Neutron Matter
Free Energy (MeV)
Density (fm^{-3})
10
20
50
40
30
20
10
0
-10
-20
0
0.1
0.2
0.3
0.4


Symmetric Nuclear Matter
m*/m
Density (fm^{-3})
5 HDP
5 LDP
10
20
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6

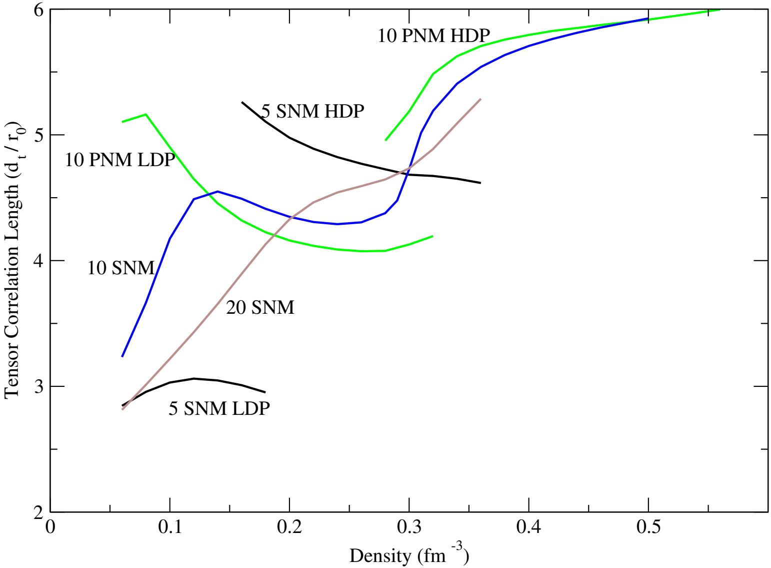

Pure Neutron Matter
20
10
m*/m
Density (fm^{-3})
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4

Pion Condensation

- π_s^0 condensation = Softening of the spin-isospin sound mode (same quantum number as a pion)

- Pions are absorbed in the NN interaction

- Pionic modes $\sim$ Tensor interactions
  Pion condensation $\implies$ enhancement of the tensor correlations
  d_t increases dramatically in the HDP


10 PNM HDP
5 SNM HDP
10 PNM LDP
10 SNM
20 SNM
5 SNM LDP
Tensor Correlation Length (d$_t$ / r$_0$)
Density (fm^{-3})
6
5
4
3
2
0
0.1
0.2
0.3
0.4
0.5

- The method of correlated basis functions which has in the past been successfully applied to study the ground state of dense strongly interacting systems has been extended to study the thermodynamics and response at finite temperature.

- An EOS for nuclear matter at low and moderate temperatures is being developed.

- In future the influence of thermal pions at very high temperature and pairing at low densities will be examined.

Correlated Basis States

Correlation operator

$$\mathcal{G} = \mathcal{S} \prod_{i<j} \mathcal{F}_{ij}$$

$\mathcal{F}_{ij}$ contains the same operators as the NN interaction.
Generalization of the Jastrow correlation functions.
Correlated Basis States (CBS)

$$|\Psi_I) = \frac{\mathcal{G}|\Phi_I\rangle}{\sqrt{\langle\Phi_I|\mathcal{G}^\dagger\mathcal{G}|\Phi_I\rangle}}$$

Orthogonalization

The CBS are not orthogonal to each other

$$|\Phi_I) \quad \text{(CBS)} \qquad \xrightarrow{\text{orthonormalization}} \qquad |\Theta_I\rangle \quad \text{(OCBS)}$$

OCBS = Orthonormalized Correlated Basis States

$$\langle\Theta_I|H|\Theta_I\rangle = (\Phi_I|\, H \,|\Phi_I) + \textbf{Orthogonality Corrections}$$

Variational Hamiltonian

$$
\begin{aligned}
H_V|\Theta_I\{n_I(\mathbf{k},\sigma_z)\}\rangle &= \left[\sum_{\mathbf{k},\sigma_z} n_I(\mathbf{k},\sigma_z)\epsilon_V(\mathbf{k},\sigma_z)\right]|\Theta_I\{n_I(\mathbf{k},\sigma_z)\}\rangle \ , \\
&= E_I^V|\Theta_I\{n_I(\mathbf{k},\sigma_z)\}\rangle
\end{aligned}
$$

For present Calculations:

$$
\epsilon_V = \frac{\hbar^2 k^2}{2m^*(\rho, T)} + \text{constant}
$$

We define

$$\rho_V = \frac{1}{\mathcal{N}_{\mathcal{M}}} \sum_{I \in \mathcal{M}} |\Theta_I\rangle\langle\Theta_I|$$

$\mathcal{M}$ = Microcaninical ensemble of the Variational Hamiltonian H_V

$$\begin{aligned} F_V &= \frac{1}{\mathcal{N}_{\mathcal{M}}} \sum_{I \in \mathcal{M}} \frac{(\Psi_I |H| \Psi_I)}{} \\ &+ \text{Orthogonality Corrections} - TS_V(T) \end{aligned}$$

Vanishing Orthogonality corrections

We showed that Orthogonality Corrections $\rightarrow$, in the thermodynamic limit.

$$F_V = \frac{1}{\mathcal{N}_{\mathcal{M}}} \sum_{I \in \mathcal{M}} (\Psi_I | H | \Psi_I) - TS_V(T)$$

First Term : Variational Chain Summation = Fermi Hypernetted Chain summation + Single Operator Chain summation
Second Term : Trivial