Nuclear & Particle Physics of Compact Stars

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The Role of the Equation of State in Binary Mergers

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Jim

Sasa

Our Thoughts on this Subject

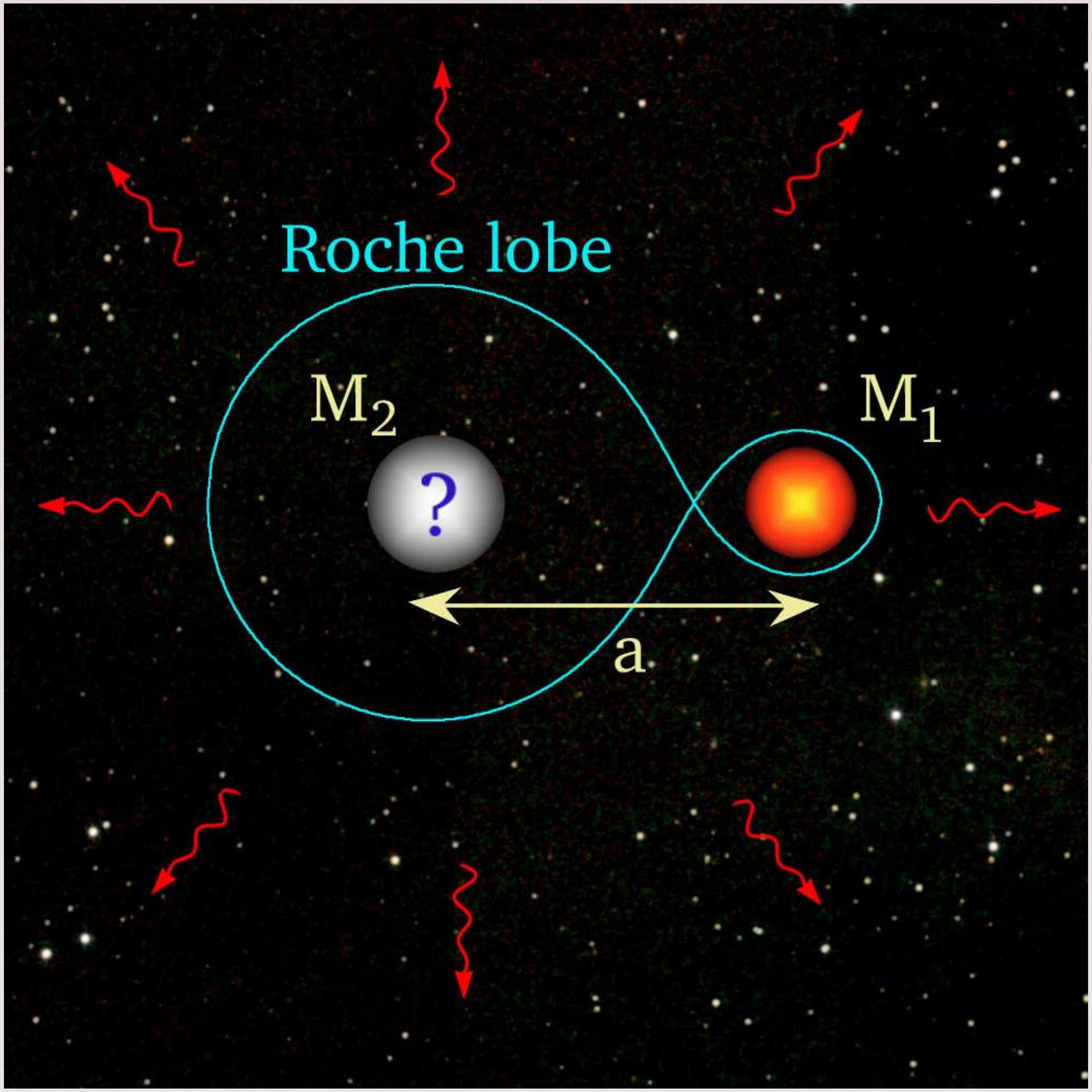
1. M. Prakash,
Jl. Phys. G.: Nucl. Part. Phys. **30**, S451 (2003)

2. S. Ratkovic, M. Prakash & J. M. Lattimer,
Jl. Phys. G.: Nucl. Part. Phys. **30**, S1279 (2004)

3. S. Ratkovic, M. Prakash & J. M. Lattimer,
astro-ph/0512133; 0512136
ApJ (2006), To be published.

The Binary Merger Experience

The Ultimate Heavy-Ion Collision



- $M_1 \leq M_2$
- radial separation: $a(t)$
- M_1 - NS or SQM
- M_2 - BH, NS, …
- GW emission ⇒

$$L_{GW} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{I\!\!\!-}_{jk} \dddot{I\!\!\!-}_{jk} \rangle = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^6}$$

orbit shrinks

- Mass transfer

Einstein's General Relativity

$$G^{\alpha\beta}\left[g, \partial g, \partial^2 g\right] = 8\pi\, T^{\alpha\beta}\left[g\right]$$

- $G^{\alpha\beta}$: 2^{nd}-order nonlinear differential operator acting on $g_{\alpha\beta}$
- $T^{\alpha\beta}$: Stress-energy tensor of matter fields

Parametrized Post-Newtonian (PPN) Formulation

In weak field limit,

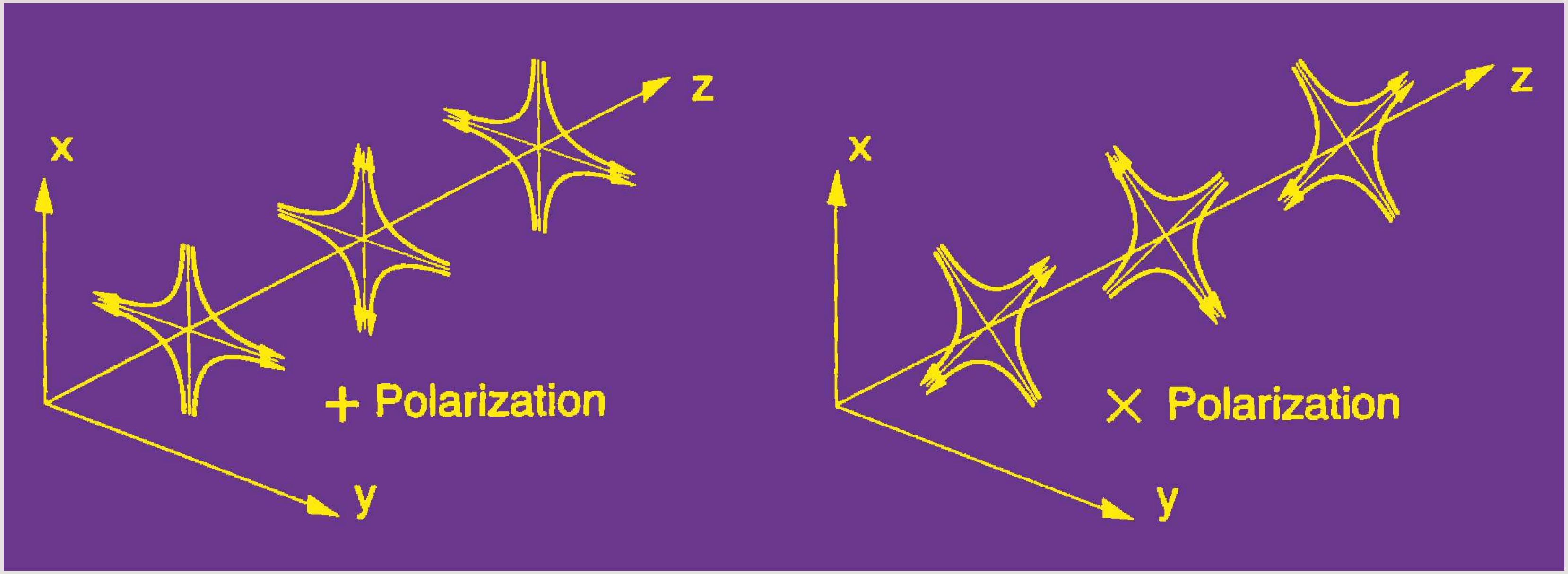
$$g^{PPN}_{\mu\nu} = \eta_{\mu\nu} + h^{1PN}_{\mu\nu}(M) + h^{2PN}_{\mu\nu}(M) + h^{3PN}_{\mu\nu}(M) + \cdots$$

- $\eta_{\mu\nu}$: flat-space Minkowski metric
- M : incorporates dependence on matter fields
- $1PN, 2PN, \cdots \Rightarrow [\mathcal{O}(v^2/c^2)]^\epsilon$ with $\epsilon = 1, 2, \cdots$

For vacuum gravitational fields (in transverse traceless gauge),

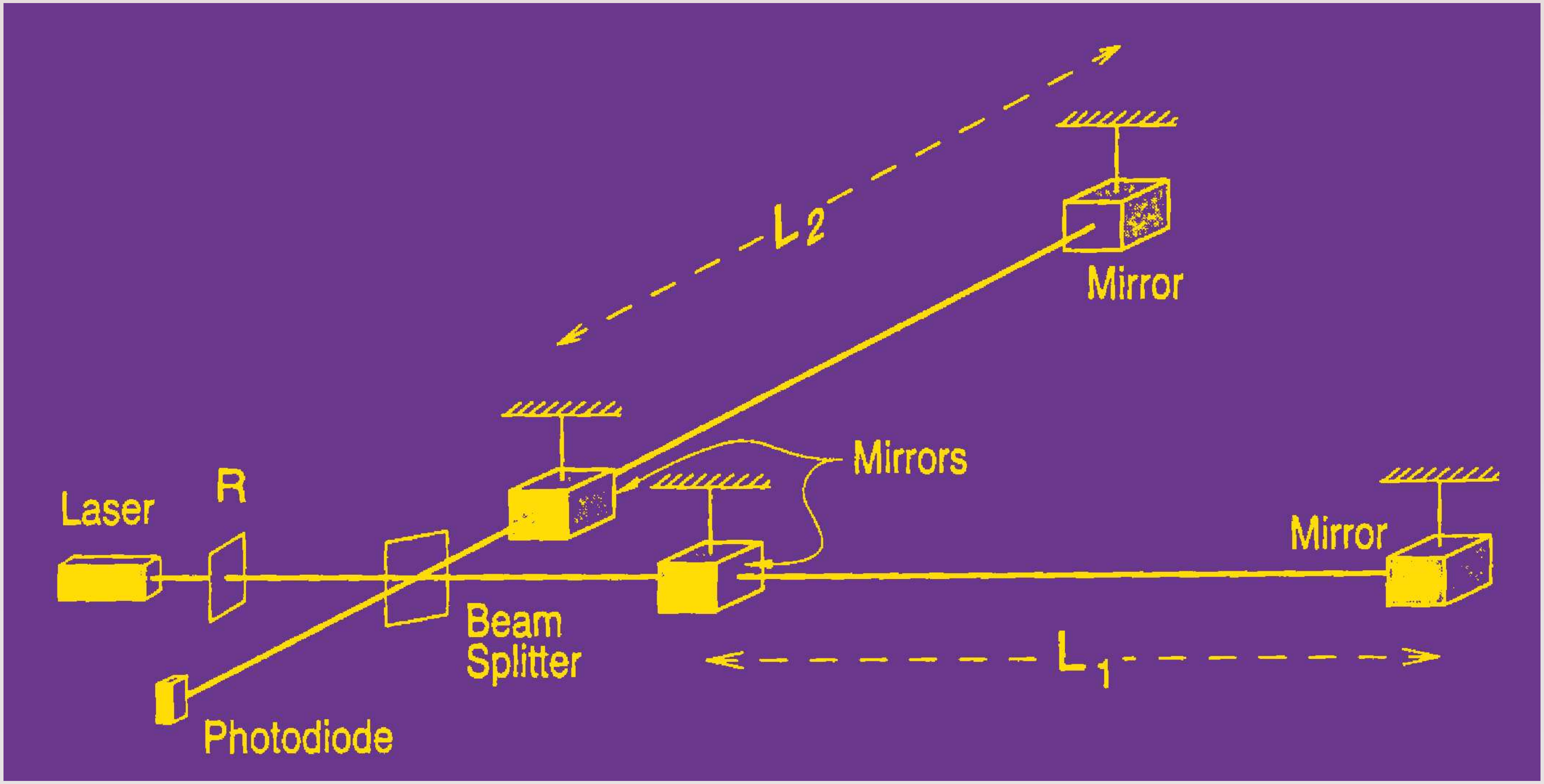
$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{\times/+} = 0$$

GW Lines of Force



GW's have two transverse polarizations, h_+ & $h_\times$.

Laser Interferometer GW Detector



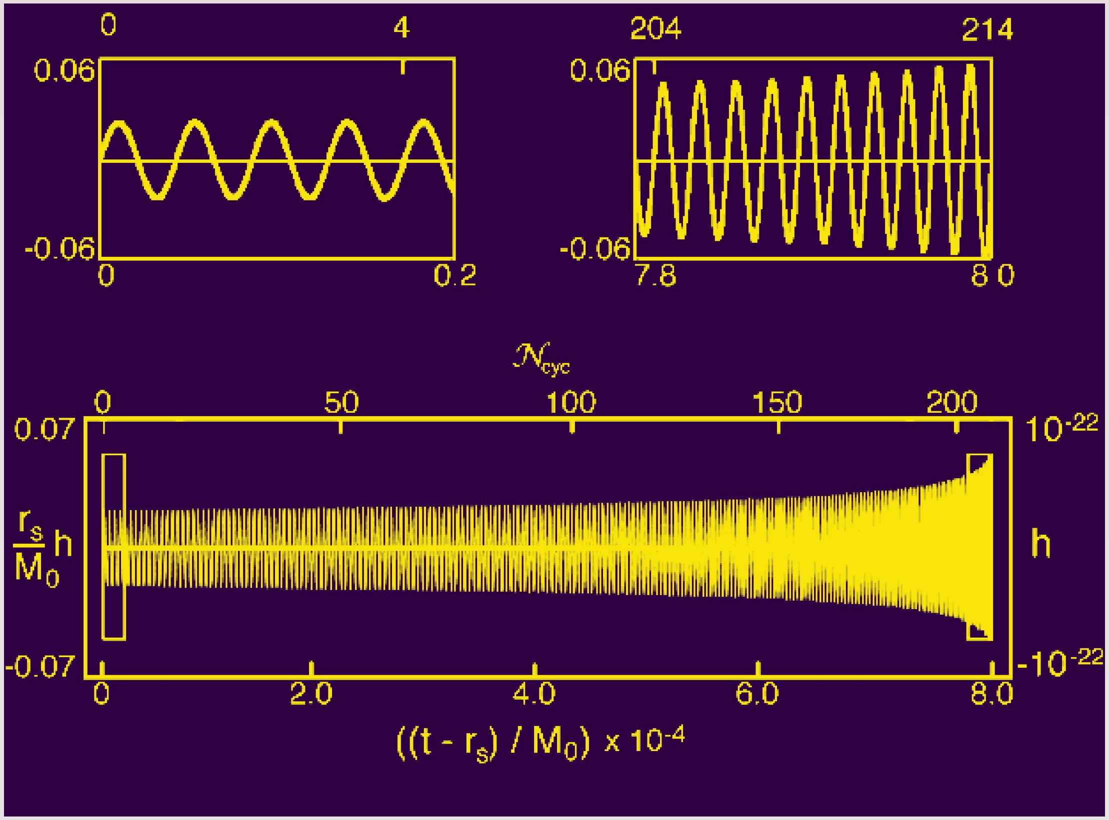
For a readable account,
see K. Thorne, arXiv:gr-qc/9506084

Gravitational Wave Detection

- GW Strain : $h(t) = F_\times h_\times(t) + F_+ h_+(t)$
  - $F_{\times,+}$: Constants of order unity
  - $h_{\times,+} \sim \frac{\delta L}{L_0} \sim \frac{1}{c^2} \frac{4G(E^{ns}_{kin}/c^2)}{r}$: Gravitational waveforms
    - L_0: Unperturbed length of detector arm
    - δL : Relative change in length
    - E^{ns}_{kin} : Nonspherical part of the internal kinetic energy
  - ELF : 10^{-15} - 10^{-18} Hz   VLF : 10^{-7} - 10^{-9} Hz*
  - LFB : 10^{-4} Hz - 1 Hz,   HFB : 1 Hz - 10^{4} Hz
- Astrophysical Sources Radiating GW′s in the HFB

Supernovae	at 10 Mpc	$h \geq 10^{-25}$
Supernovae	Milky Way	$h \sim 10^{-18}$
$1.4 M_\odot$ NS Binaries	at 10 Mpc	$h \sim 10^{-20}$
$10 M_\odot$ BH Binaries	at 150 Mpc	$h \sim 10^{-20}$

Inspiral Waveform



Chirp signal:

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

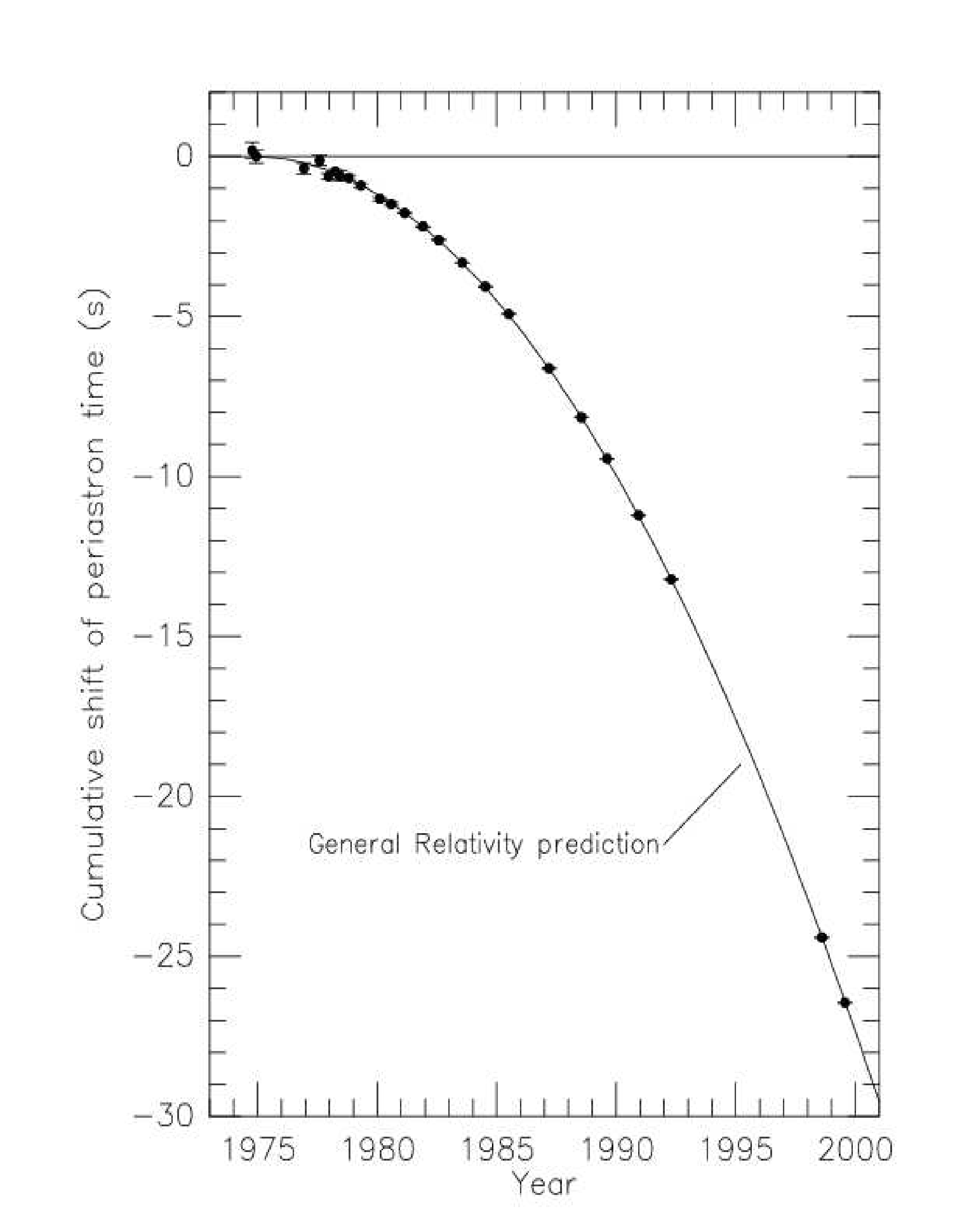
$$f = K_0 \mathcal{M}^{-5/8} (t_c - t)^{-3/8}$$

with the "chirp mass":

$$\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}$$

and the constant:

$$K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G} \right)^{5/8}$$



- Binary pulsar PSR 1913+16
- Period: 7 h 45 min
- $M_{NS} = 1.4408 \pm 0.0003 \ \mathrm{M}_\odot$
- $M_c = 1.3873 \pm 0.0003 \ \mathrm{M}_\odot$
- Distance: 7.13 kpc
- Merger in 300 Myr

Merger Rates of Binary Systems

Author(s)	Information	Type	Merger Rate
Phinney (1991)	pulsar lifetimes, distributions	cons. bguess	5×10^{-8} 7×10^{-6}
Van den Heuval & Lorimar (1996)	pulsar detectability, distribution	cons. bguess	3×10^{-7} 8×10^{-6}
Bailes (1996)	galactic pulsar birth rates	lbound ubound	10^{-7} 10^{-5}
Potegies Zwart & Yungelson (1998)	"scenario machine" w/ supernova kicks		$0.2 - 3$ $\times 10^{-5}$
Bethe & Brown (1998)	common envelope hypercritical accretion	ubound	10^{-5}

Rates in $\mathrm{yr}^{-1}\ \mathrm{Mpc}^{-3}$ $\quad$ $1\ \mathrm{pc} = 3 \times 10^{18}$ cm.

Discovery of Double-Pulsar System

Pulsar	PSR J0737-3039A	PSR J0737-3039B
Pulse Period P (ms)	22.69937855615(6)	2773.4607474(4)
Period derivative $\dot{P}$	$1.74(5) \times 10^{-18}$	$0.88(13) \times 10^{-15}$
Orbital period P_b (day)	0.102251563(1)	–
Eccentricity e	0.087779(5)	–
Characteristic age (My)	210	50
Magnetic field B_s	6.3×10^9	1.6×10^{12}
Spin-down luminosity $\dot{E}$ (erg/s)	5.8×10^{33}	1.6×10^{30}
Distance (kpc)	~ 0.6	–
Stellar mass	1.337(5)	1.250(5)

Merger expected in 85 Myr, a factor 3.5 shorter than PSR 1913+16

A.G Lyne et al., Science, **303**, 1153 (2004)

Kalogera et al. (2004): Revisions w/ PSR J037-3039 imply 1 event per 1.5 yr for initial LIGO (for advanced LIGO, 20-1000 events per yr).

PSR J0737 3039 and LIGO

- Merger rate $R \propto N/\tau$
- Binary pulsar lifetime: $\tau = \tau_{BIRTH} + \tau_{COAL.}$

$$\frac{\tau_{1913}}{\tau_{0737}} = \frac{365 \text{ Myr}}{185 \text{ Myr}} \approx 2$$

- scaling factor $N \propto L_{400}^{-1}$

$$\frac{N_{0737}}{N_{1913}} = \frac{L_{1913}}{L_{0737}} = \frac{200 \text{ mJy kpc}^2}{30 \text{ mJy kpc}^2} \approx 6$$

$2 \times 6 = 12 \Rightarrow$ an order of magnitude increase of merger rates!

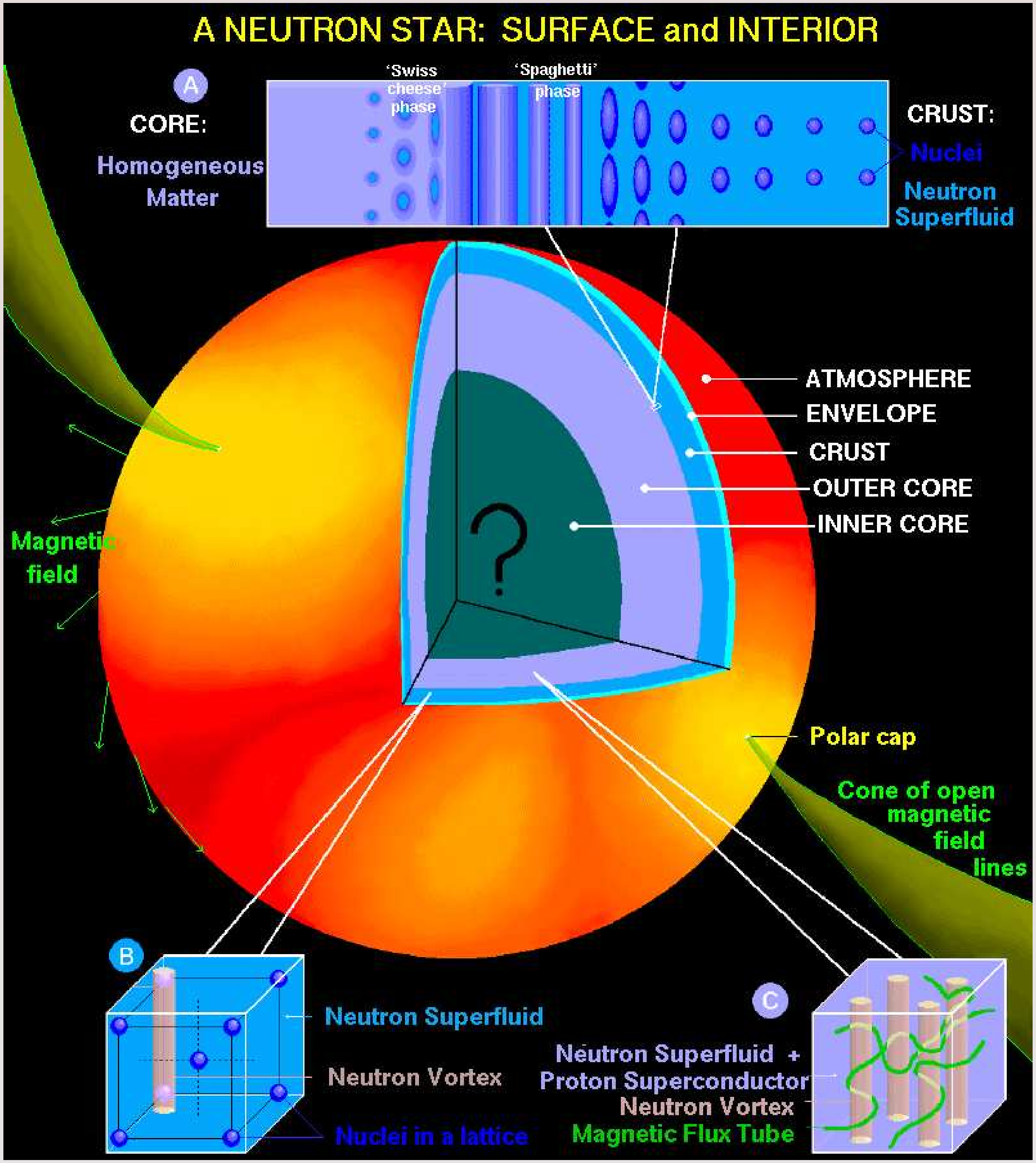
GW Detectors & Expected Gains

- Ground-Based Laser Interferometers
  - LIGO, VIRGO, GEO, TAMA, ...
- The Laser Interferometer Space Antenna (LISA)

- GW's provide valuable new information "orthogonal" to electromagnetic observations
  - First direct test of GR
  - Precise ($\pm$ a few $\%$) determination of Hubble's constant H_0
  - Calibration of distance measurements
  - Masses of NS, BH (large scale structure formation)
  - ......

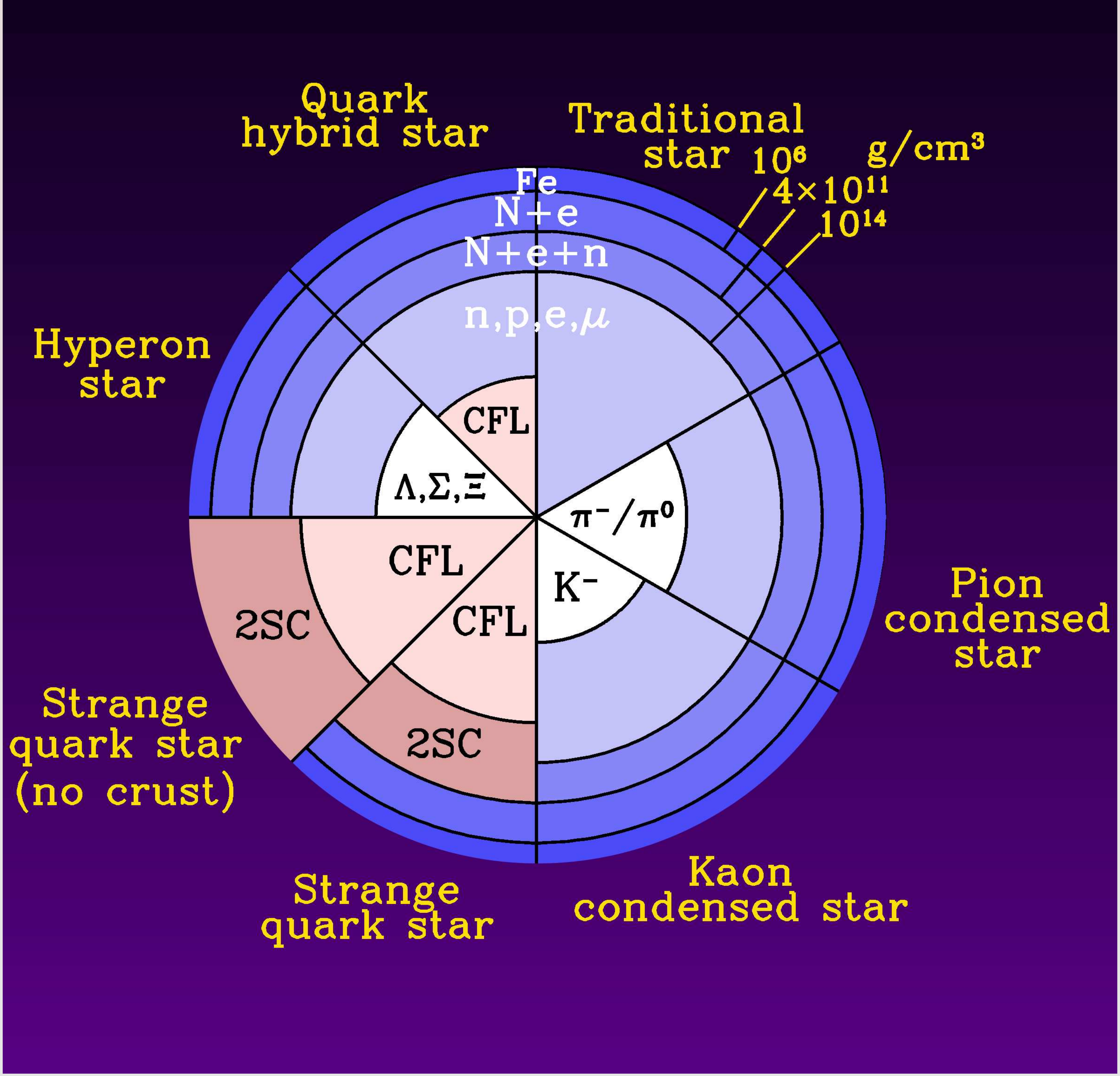
Objectives

- Explore EOS dependence of GW signals from mergers.
  - Specifically, look at differences between "normal" stars and "self-bound" (e.g., SQM) stars.
    - EOS parameter : $\alpha(M_1) \equiv d\ln(R_1)/d\ln(M_1)$
    - $\alpha_{NS} \leq 0$, while $\alpha_{SQM} \geq 0$ $(\approx 1/3)$
- Incorporate analysis to include GR (2PN, ...) orbital dynamics.
  - Extend the Roche lobe analysis from Newtonian to 2PN, ... GR makes stable mass transfer easier.
  - Utilize pseudo-GR potential to account for innermost circular orbit changes as a function of mass ratio. Study effects on results for existence of stable mass transfer.
- Explore astrophysical consequences of differences in $\alpha(M_1)$ in (1) merger time scales and (2) GW signals.

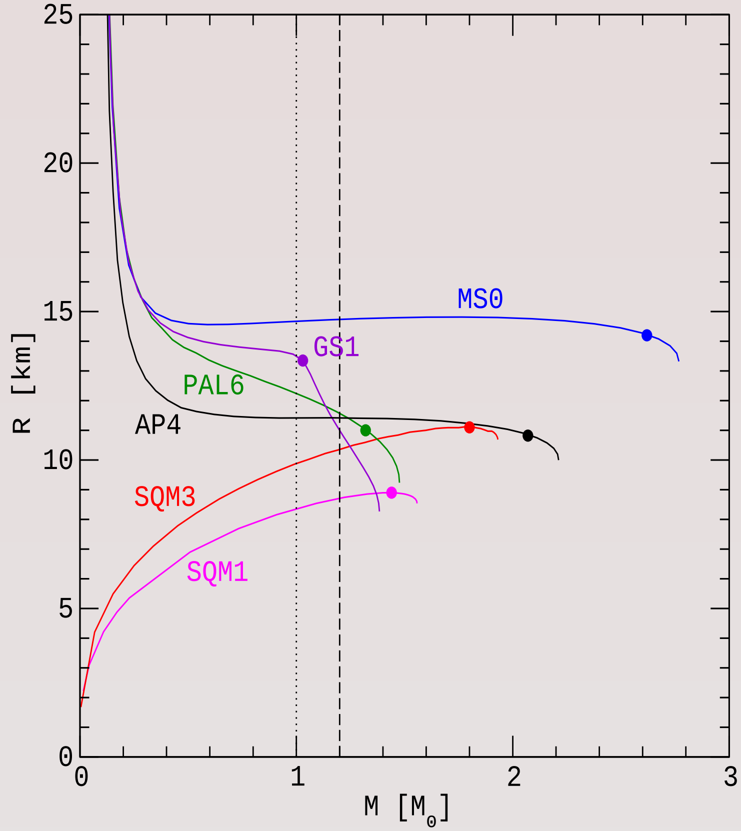


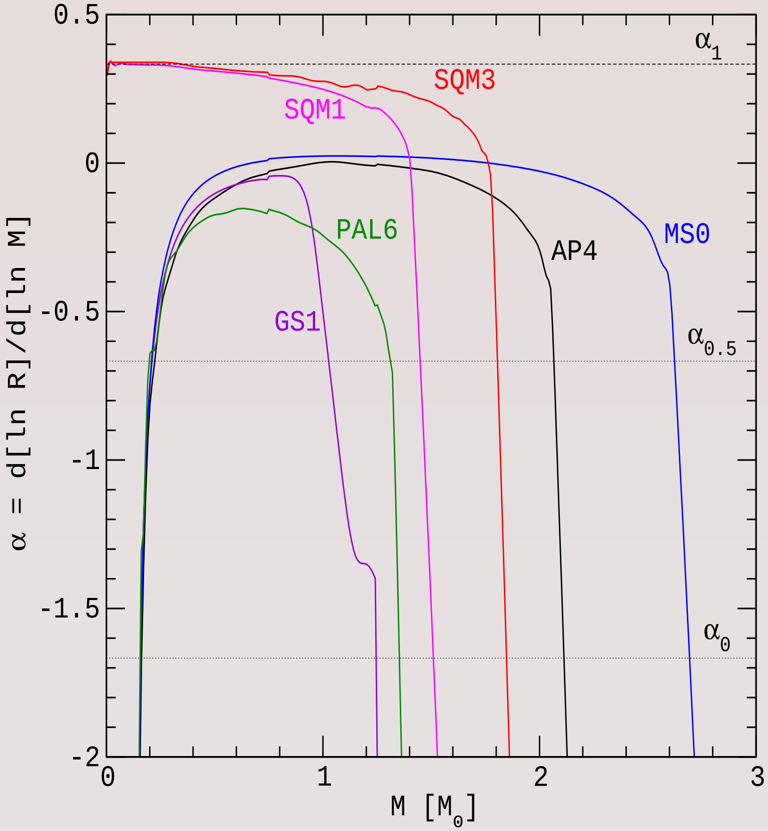
- $M \sim (1-2)M_\odot$
  $M_\odot \simeq 2\times 10^{33}$ g.
- $R \sim (8-16)$ km
- $\rho > 10^{15}$ g cm^{-3}
- $B_s = 10^9 - 10^{15}$ G.
- Tallest mountain:
  $\sim \frac{E_{liq}}{Am_p g_s} \sim 1$cm
- Atmospheric height:
  $\sim \frac{RT}{\mu g_s} \sim 1$cm

Lattimer & Prakash , Science **304**, 536 (2004).


Quark hybrid star
Traditional star
10⁶ g/cm³
4×10¹¹
10¹⁴
Fe
N+e
N+e+n
n,p,e,μ
Hyperon star
CFL
Λ,Σ,Ξ
π⁻/π⁰
K⁻
CFL
CFL
2SC
2SC
Pion condensed star
Strange quark star (no crust)
Strange quark star
Kaon condensed star

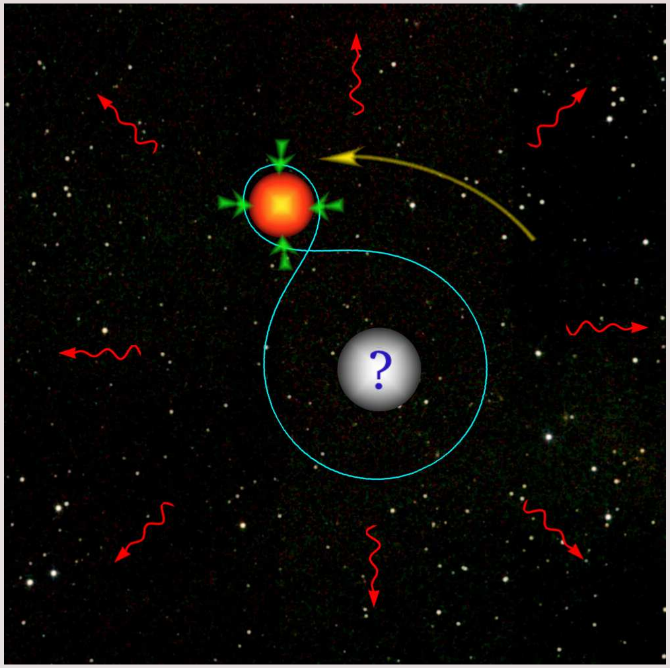
Equation of State: $\alpha(M)$

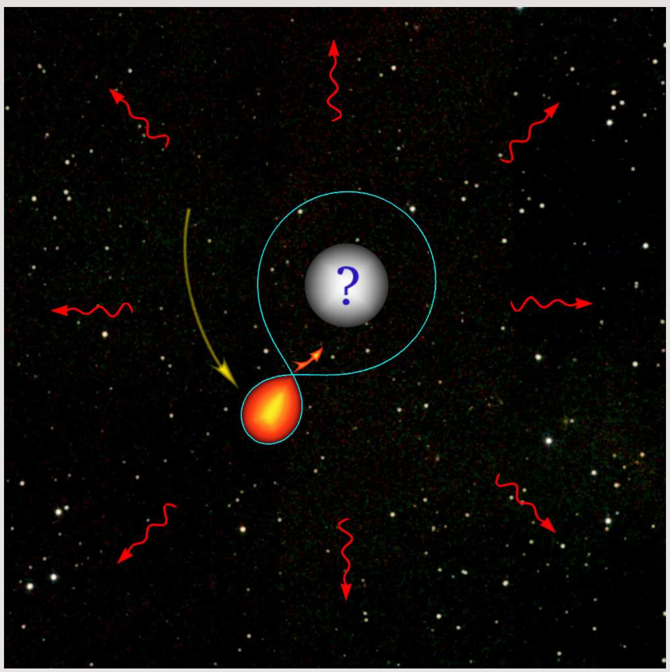




- $\alpha_{NS} \leq 0$
- $\alpha_{SQM} \geq 0$
  $(\approx 1/3)$

Roche Lobe Overflow





- Energy Loss

$$L_{GW} = \frac{1}{5}\langle \dddot{I}_{jk}\dddot{I}_{jk}\rangle = \frac{32}{5}a^4\mu^2\omega^6$$

- Angular Momentum Loss

$$\left(\dot{J}_{GW}\right)_i = \frac{2}{5}\epsilon_{ijk}\langle \ddot{I}_{jm}\dddot{I}_{km}\rangle = \frac{32}{5}a^4\mu^2\omega^5$$

- $a(t)$ and V_{Roche} shrink!
- $R_1 = r_{Roche}$

  $\Rightarrow$ Mass transfer begins!
- To merge or not to merge?

Pseudo-GR Potentials

- Paczyński-Wiita (accretion disks)

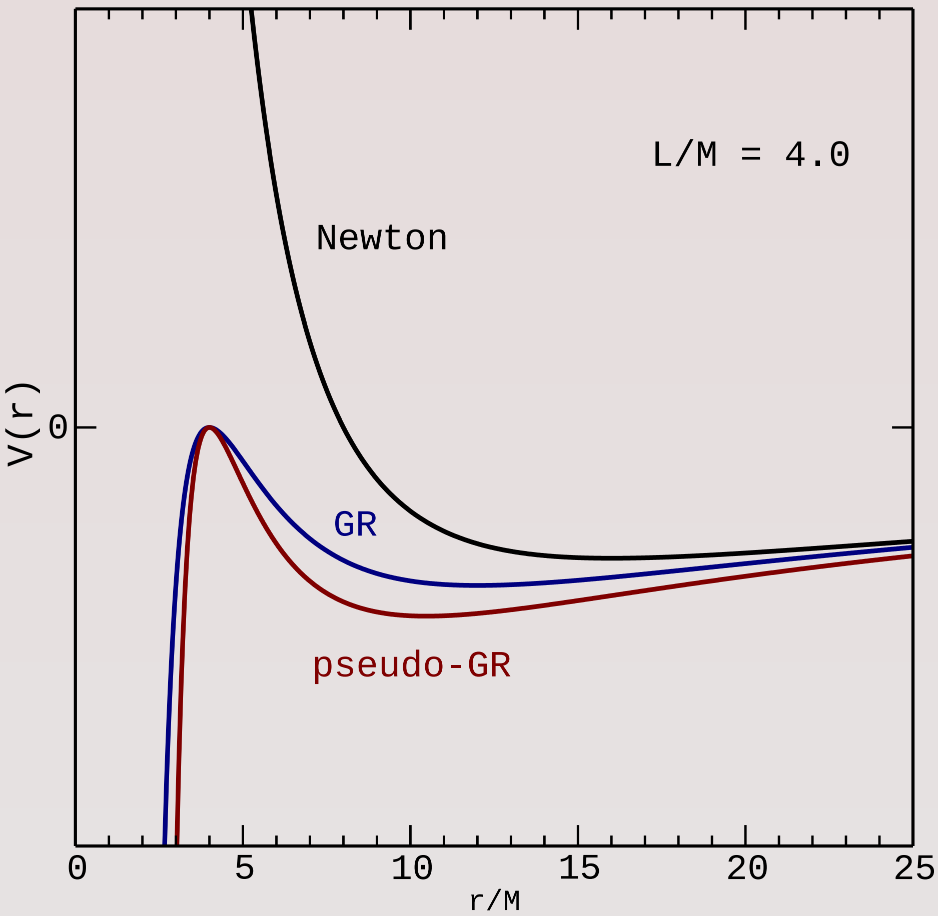
$$\phi_N(r) = -\frac{M}{r} \qquad \longrightarrow \qquad \phi_{PW}(r) = -\frac{M}{r - r_G}$$

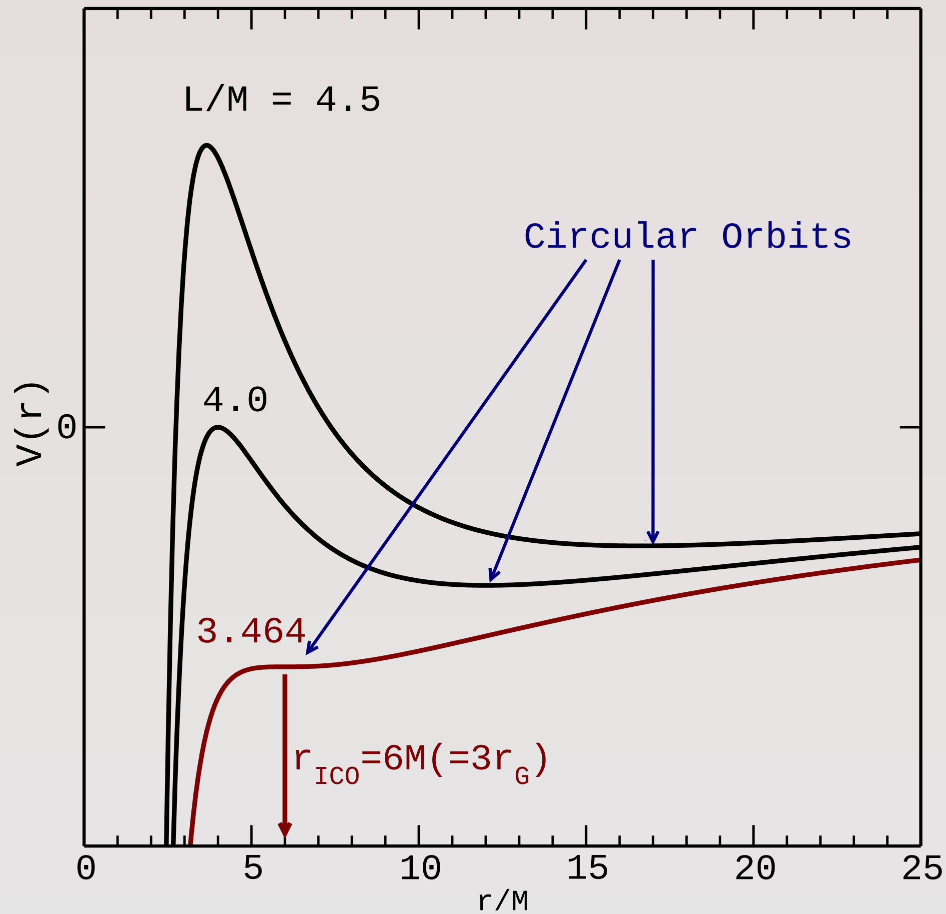
- Innermost Circular Orbit (ICO) at $r_{ICO} = 3r_G$; $r_G = 2M$
- Post-Newtonian (PN) : $r_{ICO} < 3r_G$ for $q \neq 0$
- Pseudo-GR or Hybrid Potential :

$$\phi_H(r) = -\frac{M}{r - \zeta(q) r_G} \; ; \qquad q = M_1/M_2$$

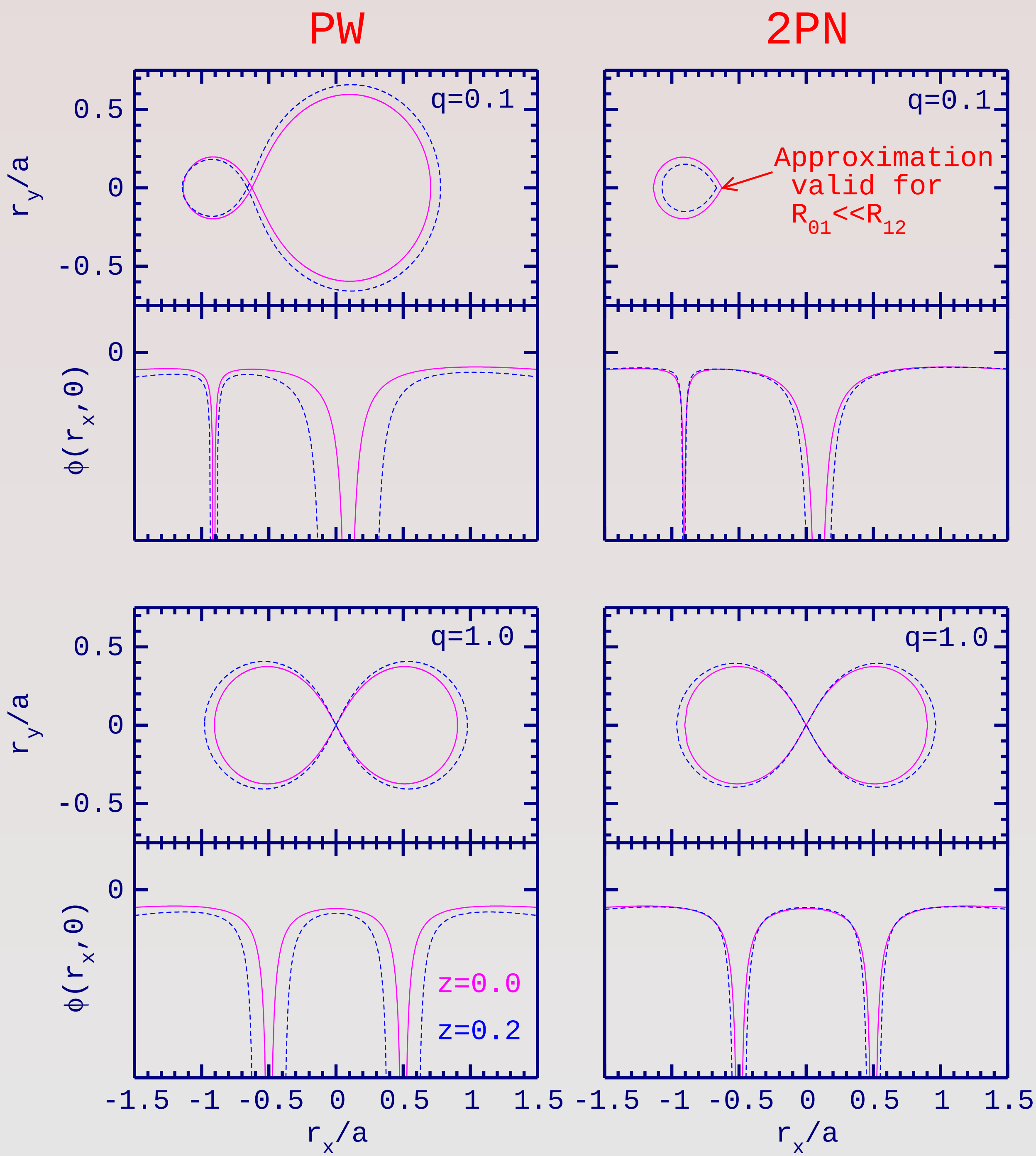
- $\zeta(q)$ - Mimics 2PN, 3PN Corrections to ICO

Test Particle Effective Potentials and ICO

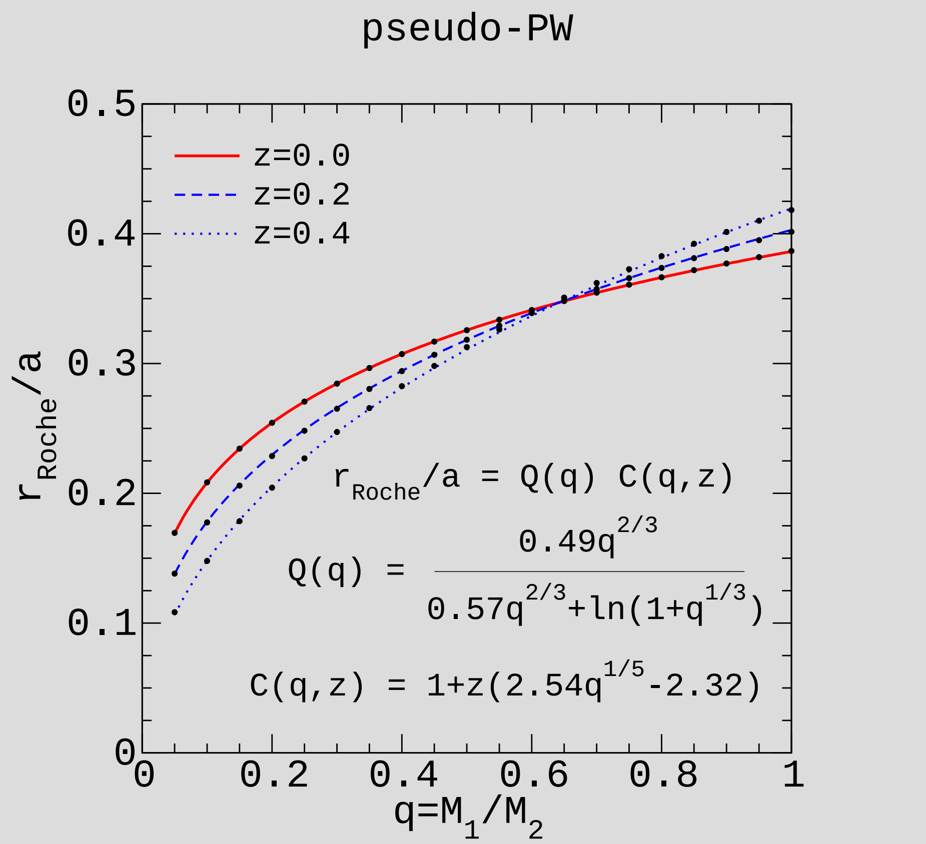




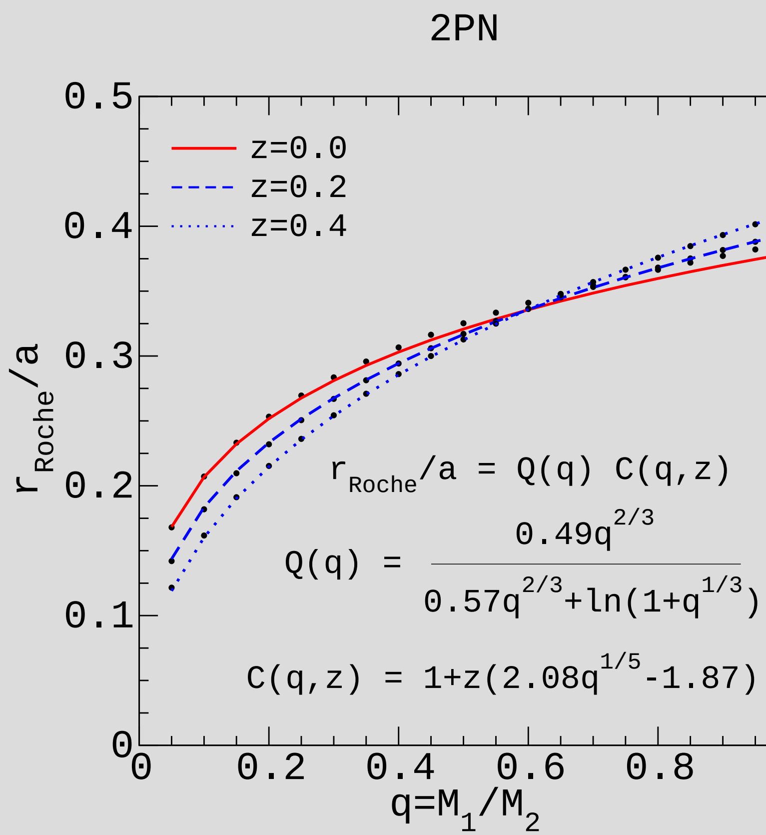
Roche Lobes: PW vs. 2PN



Effective Roche Lobe Radii



Ratković, Prakash, & Lattimer (2005)



Orbital Evolution

- Angular Momentum Loss :

$$\left[\frac{1-q}{1+q} + \frac{r_G\, q\, \zeta'(q)}{a-\zeta(q) r_G}\right]\frac{\dot{q}}{q} + \frac{a - 3\zeta(q) r_G}{2(a-\zeta(q) r_G)}\frac{\dot{a}}{a} \quad = \quad -\frac{\dot{J}_{GW}}{J_{BS}} = -\frac{32}{5} a^2 \mu \omega^4$$

- Roche Lobe :

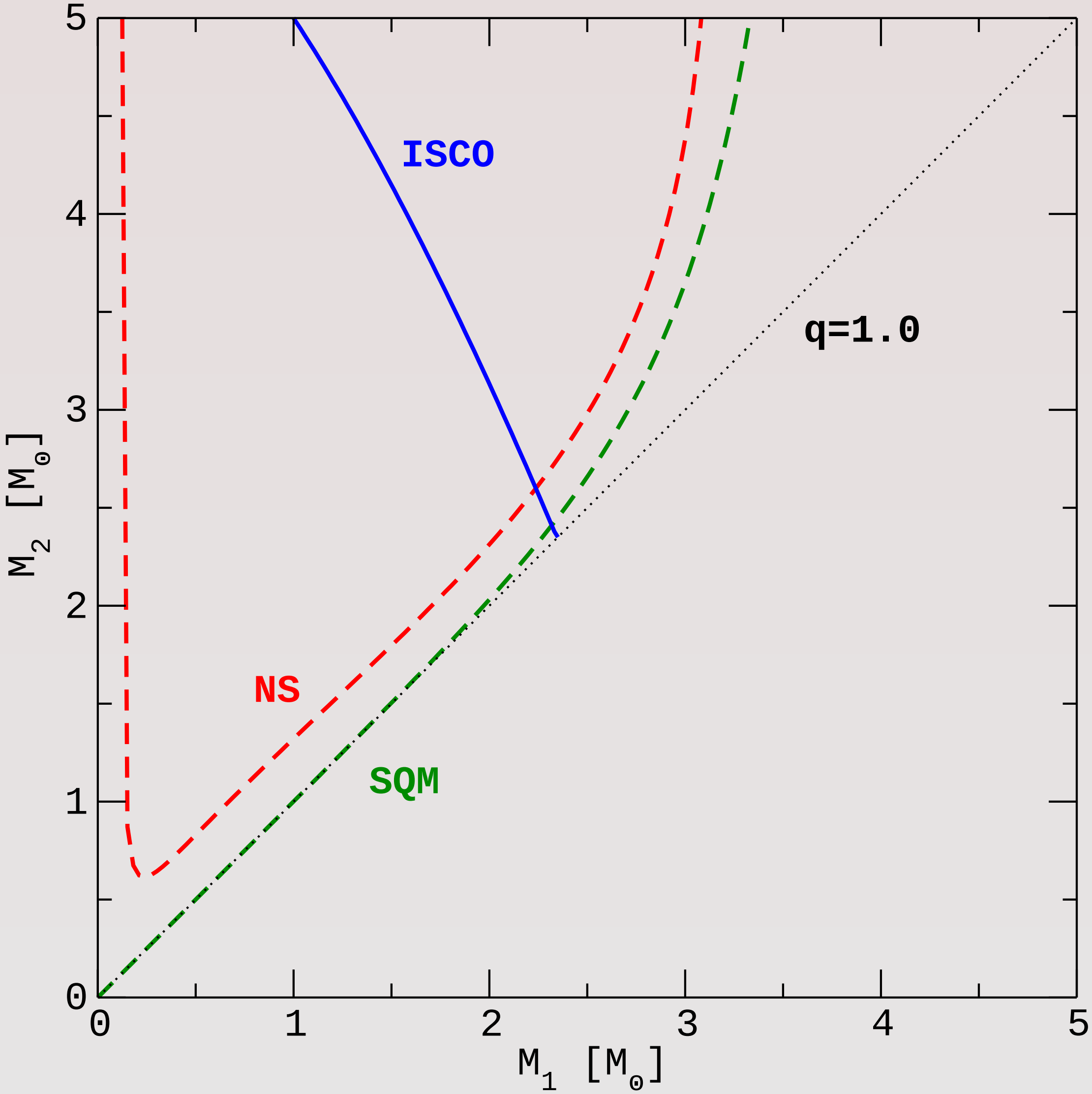
$$\frac{\dot{q}}{q} \quad = \quad \frac{1 - \dfrac{\partial \ln C(q,z)}{\partial \ln z}}{\dfrac{\alpha(M_1)}{1+q} - \dfrac{\partial \ln Q(q) C(q,z)}{\partial \ln q}} \quad \times \quad \frac{\dot{a}}{a}$$

- Connection to the dense matter EOS through

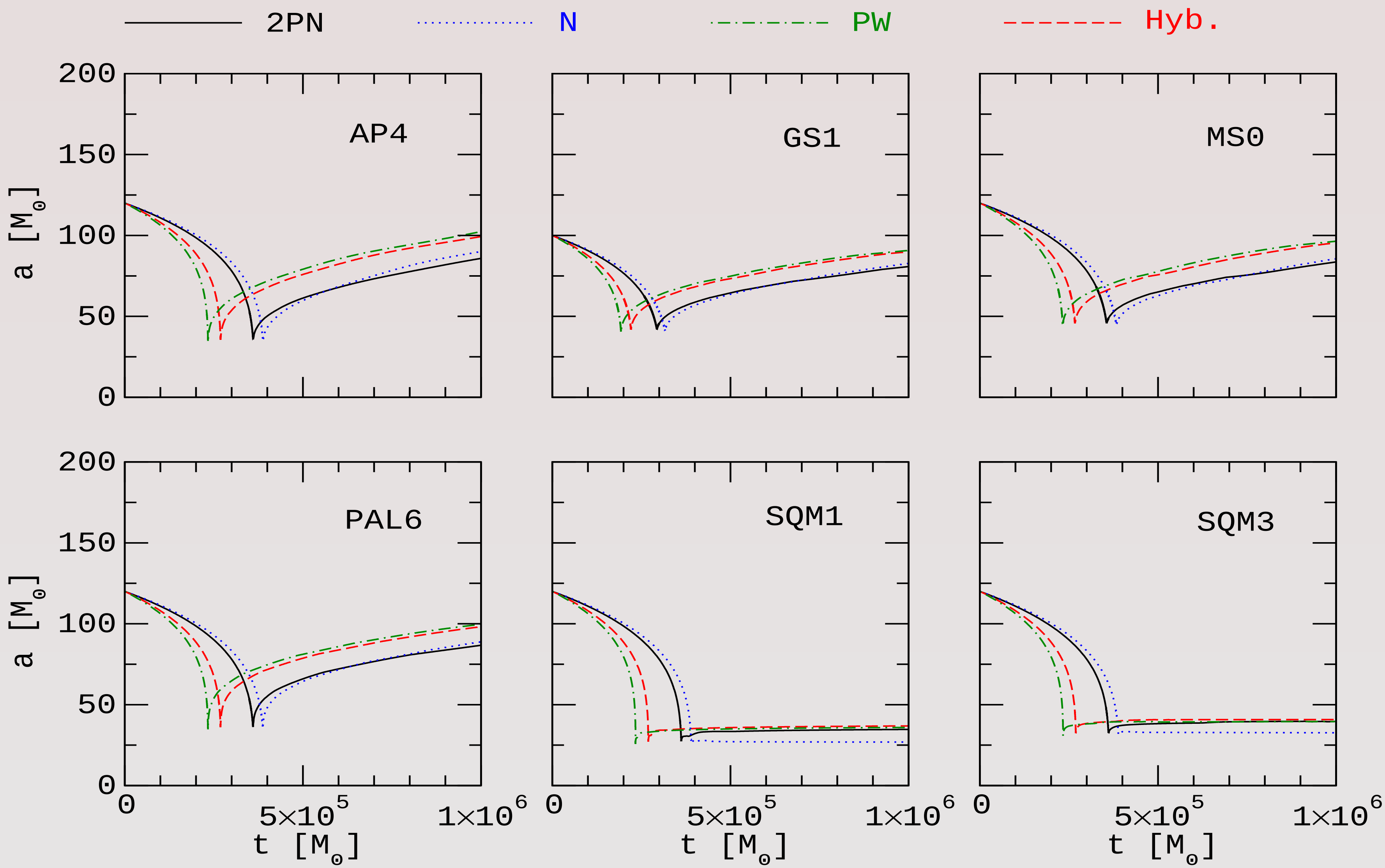
$$\alpha(M_1) \quad \equiv \quad \frac{d\ln(R_1)}{d\ln(M_1)}$$

Regions of stable mass transfer

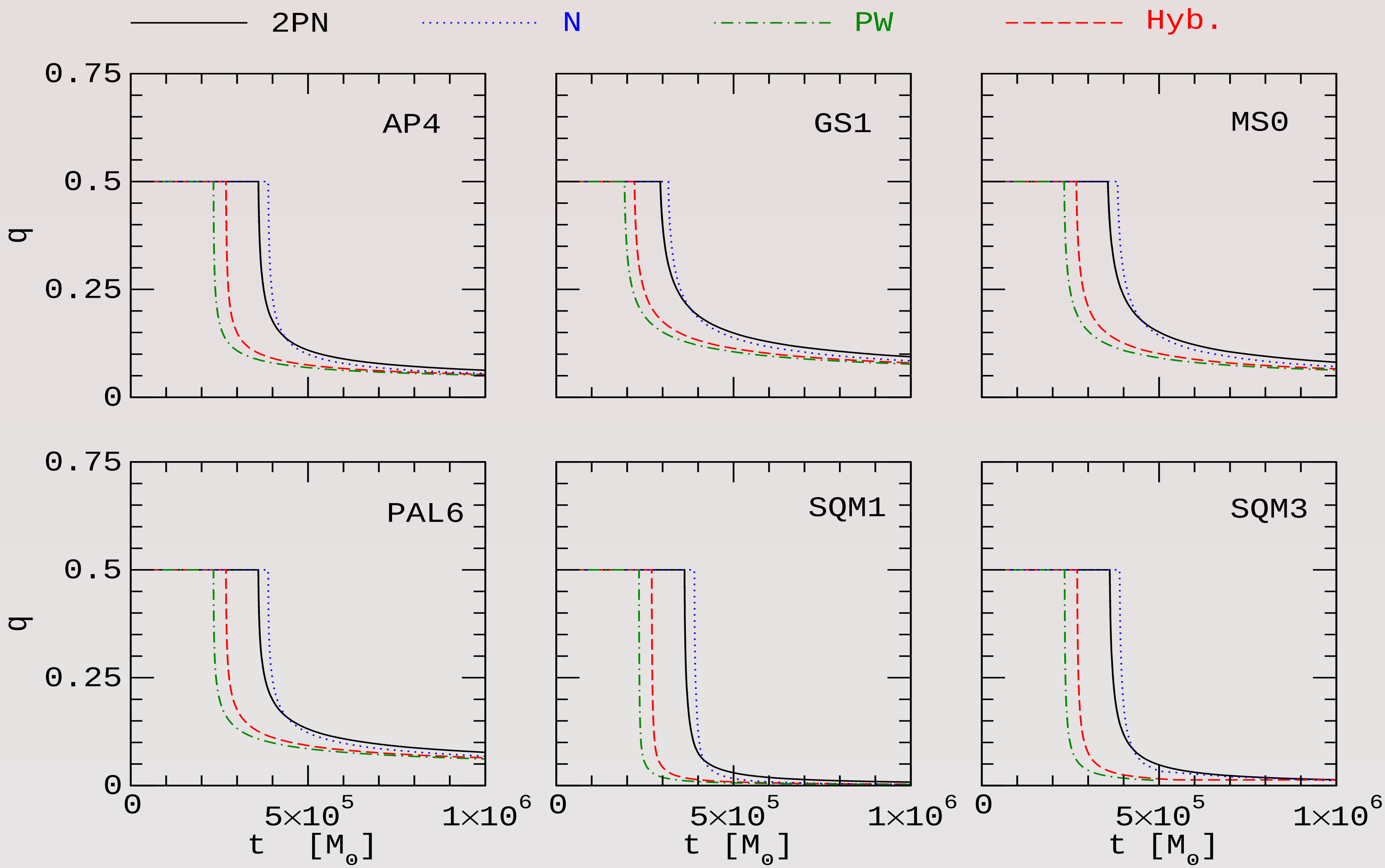
Dashed curves: Lower mass limit to M_2 for stable mass transfer.
Solid curve: Upper boundary for transfer beginning outside the ISCO.



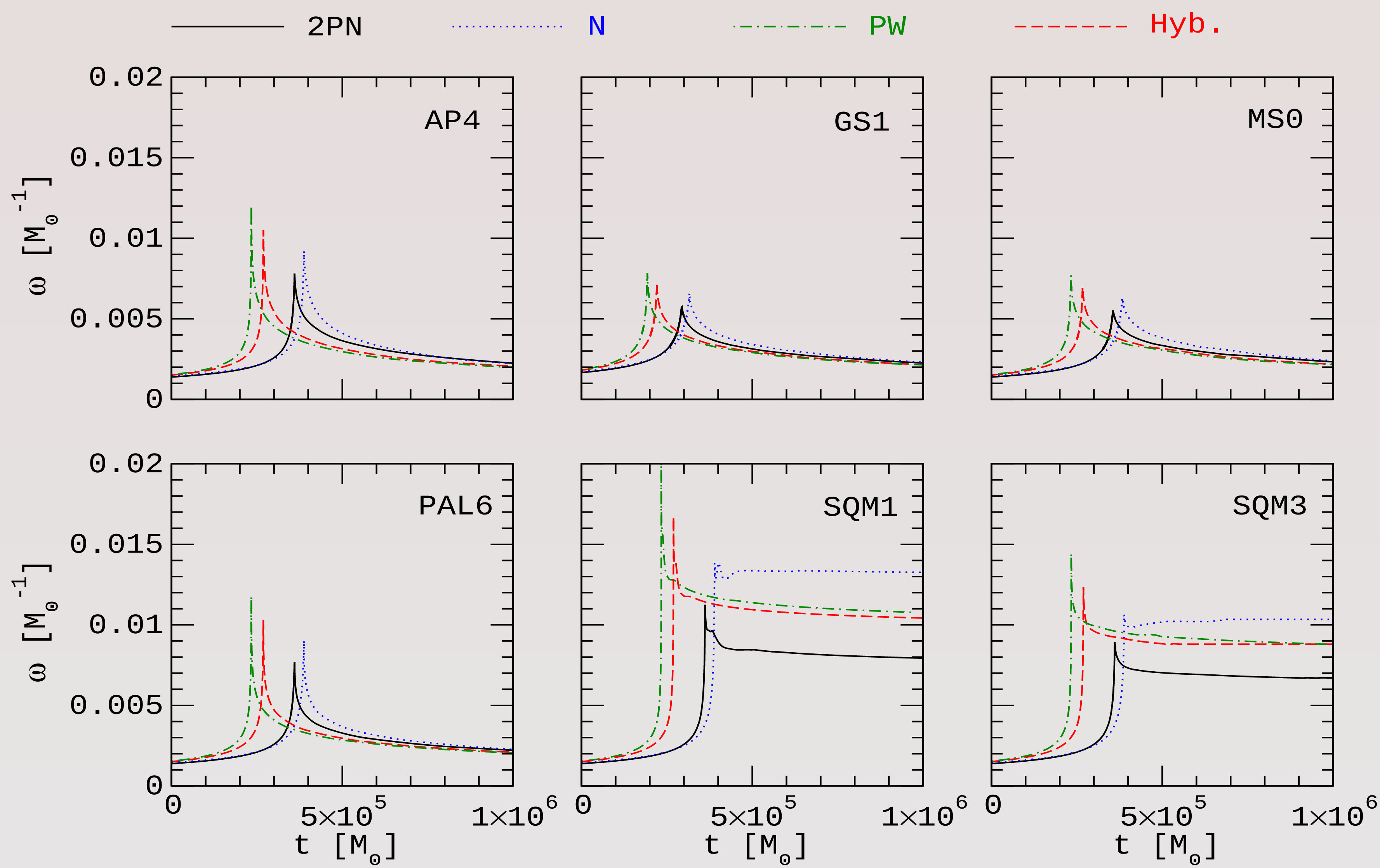
Evolution: Orbit Separation a



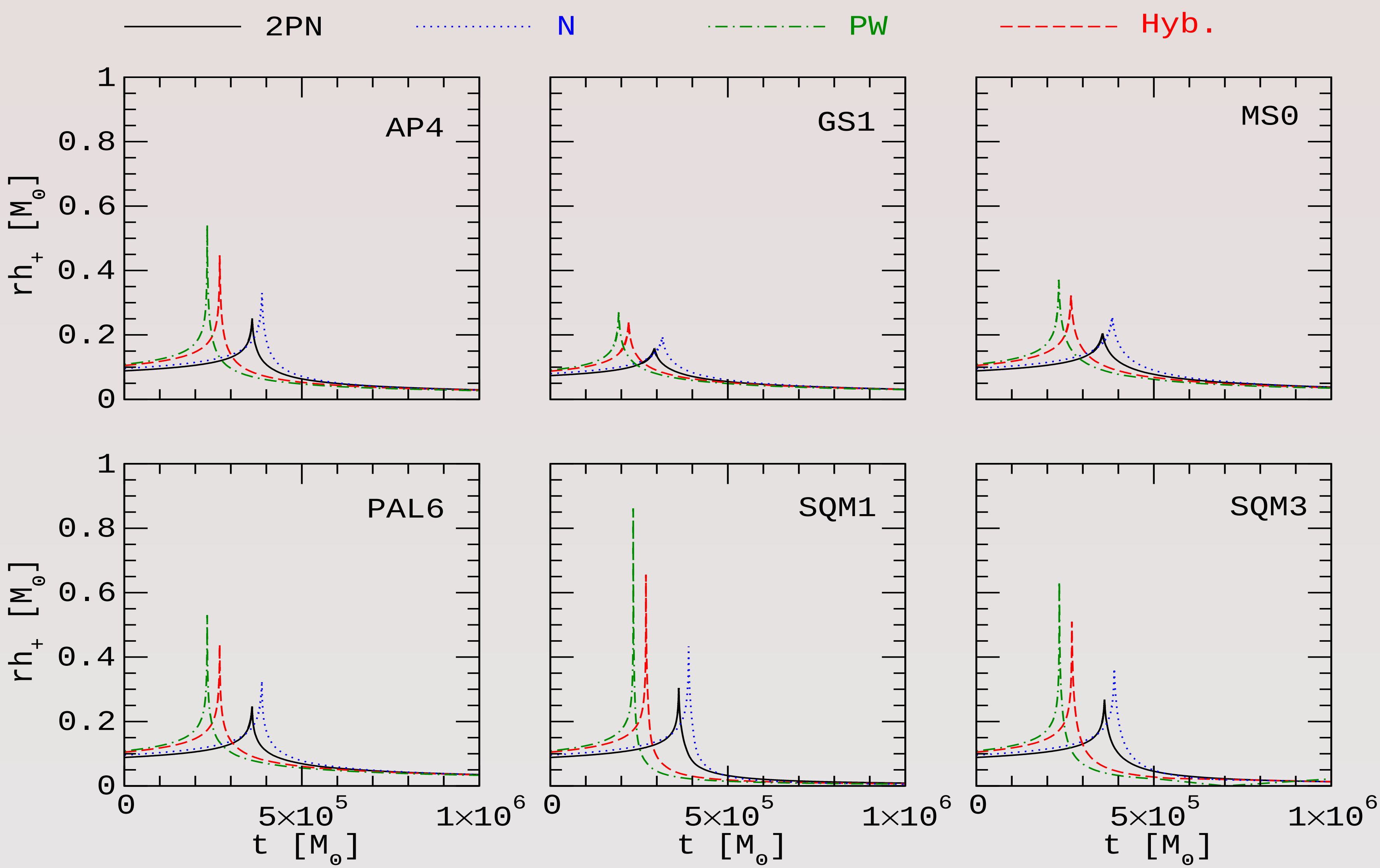
Evolution: Mass Ratio q



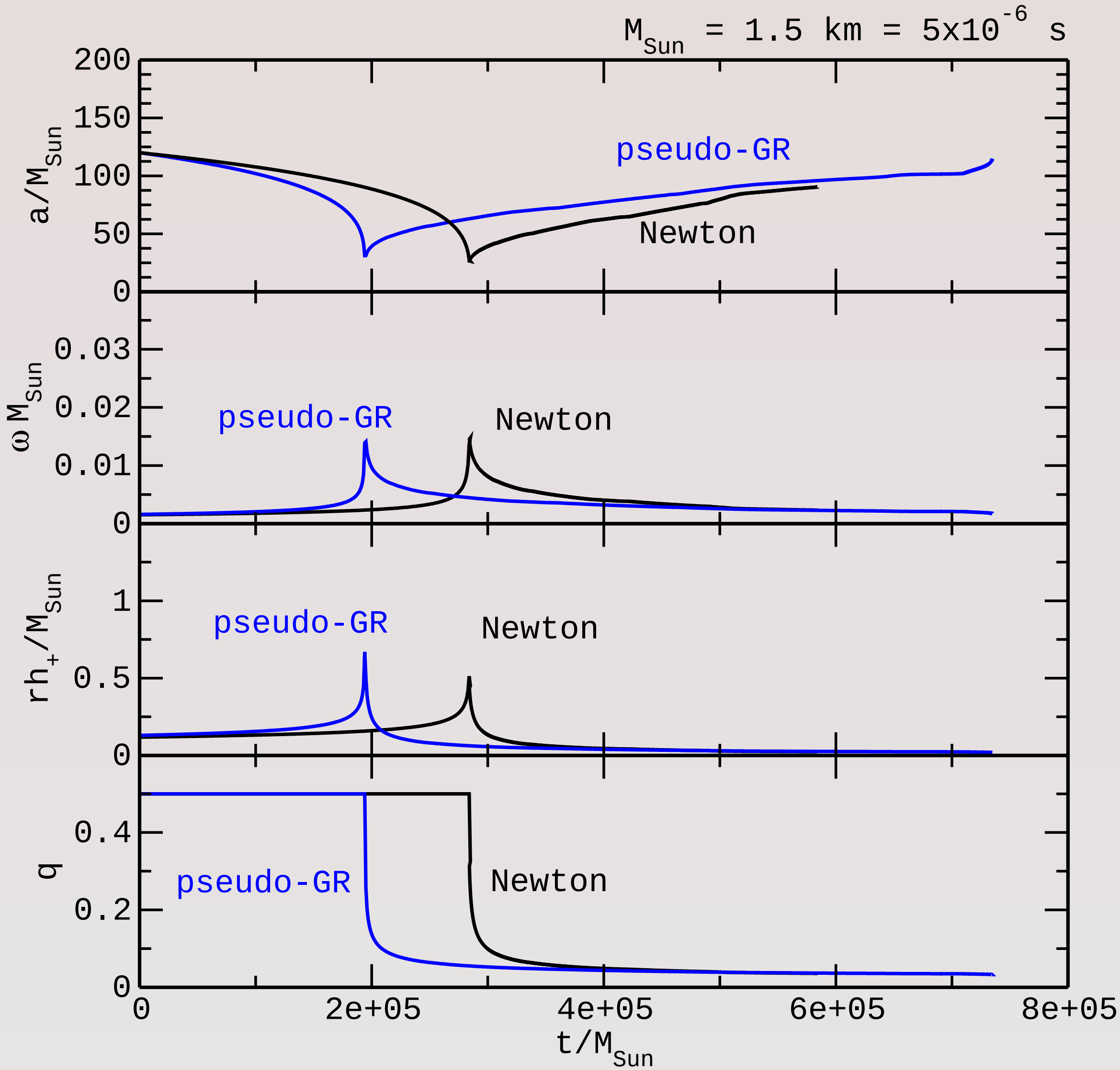
Evolution: Angular Frequency ω



Evolution: Distance × Gravitational Amplitude rh_+



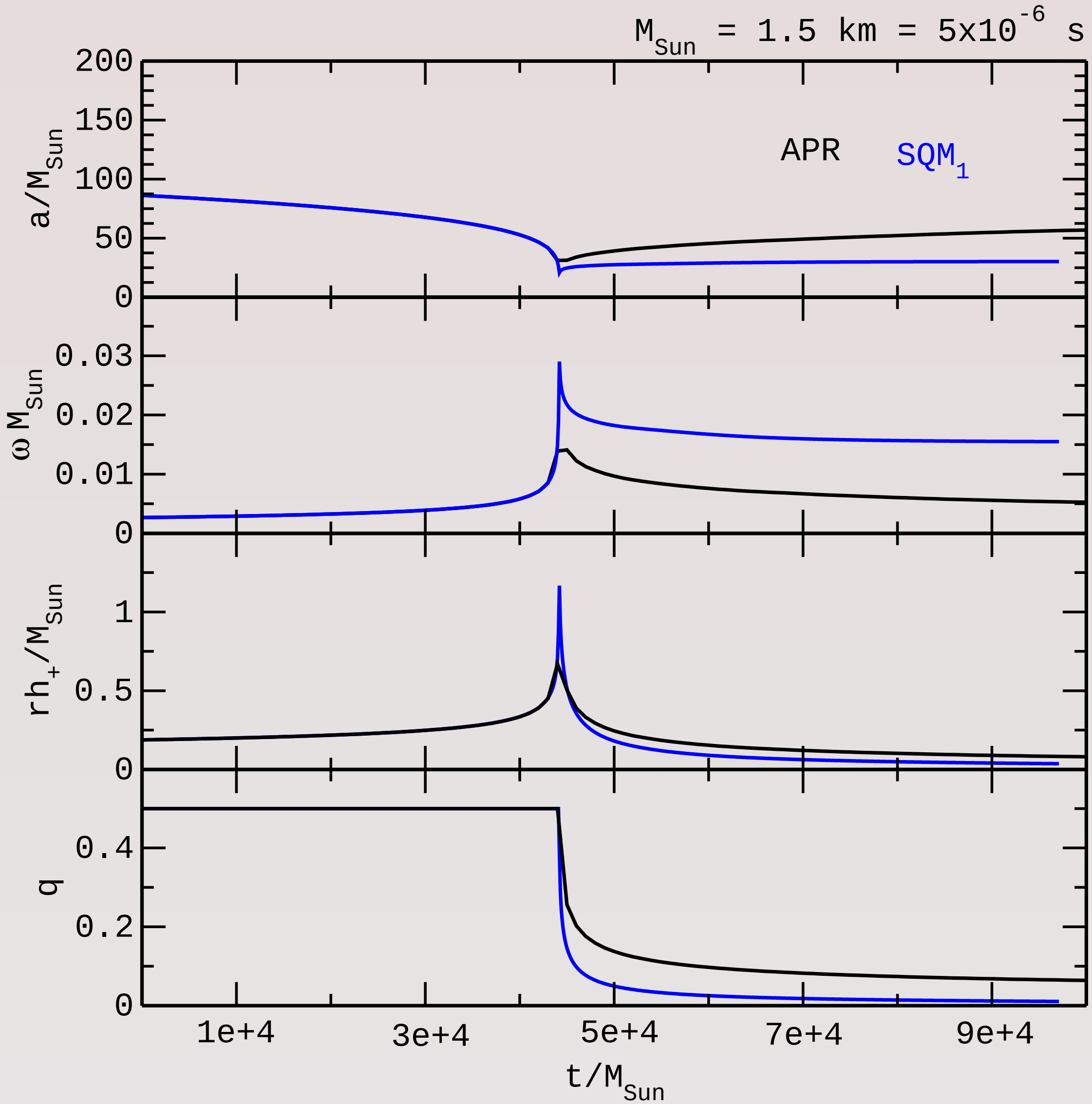
Evolution: Normal Star (APR)



$$h_+ \;=\; \frac{4}{r}\,\omega^2\,a^2\,\mu\,\cos(2\omega t)$$

- $M = 4\mathrm{M}_\odot, \quad q_{ini} = 1/3$
- GR speeds up evolution
- $a(t)$ increases after "touchdown"
- $\omega(t)$ stabilizes at long times
- Little variation among EOS's of normal stars.
- **M_1 approaches the NS minimum mass; subsequent plunge (timescale $\sim$ a few minutes) yields a second spike in the GW signal !**

Evolution: SQM Star



$$h_+ = \frac{4}{r} \omega^2 a^2 \mu \cos(2\omega t)$$

- $M = 4\mathrm{M}_\odot, \quad q_{ini} = 1/3$
- $a(t)$: "hovers" after "touchdown"
- $\omega(t)$: relaxes to $>> \omega_{initial}$
- $h_{+/\times}(t)$ & $q(t)$: exponential decay unlike for a NS
- $M_{1,final} \rightarrow M^{SQM}_{nugget}$ **unlike for a normal star; time to tiny $M_{1,final}$ is very long !**

Major Results

- **Incorporating GR into orbital dynamics leads to an evolution that is faster than the Newtonian evolution.**
- **Large differences exist between mergers of "normal" and "self-bound (SQM)" stars.**
  - SQM stars penetrate to smaller orbital radii; stable mass transfer is more difficult than for normal stars.
  - For stable mass transfer, $q = M_1/M_2$ and $M = M_1 + M_2$ limits on SQM stars are more restrictive than for normal stars.
  - The SQM case has exponentially decaying signal and mass, while normal star evolution is slower.

Future Tasks

- Evolution of normal & self-bound star-black hole mergers including the effects of
  - non-conservative mass transfer,
  - tidal synchronization,
  - the presence accretion disk, etc.
- Calculation of templates of expected GW signals


That's All Folks!