Nuclear & Particle Physics of Compact Stars

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Thermal Evolution of a Neutron Star

(Spherical, non-rotating & non-magnetic)

$$\frac{dM}{dr} = 4\pi r^2 \epsilon\,; \qquad \frac{dP}{dr} = -\frac{GM\epsilon}{c^2 r^2}\left[1 + \frac{P}{\epsilon}\right]\left[1 + \frac{4\pi r^3 P}{Mc^2}\right] e^{2\Lambda}$$

$$\frac{d}{dr}\left(Te^{\Phi/c^2}\right) = -\frac{3}{16\sigma}\frac{\kappa\rho}{T^3}\frac{L_d}{4\pi r^2} e^{\Phi/c^2} e^{\Lambda}$$

$$\frac{d\Phi}{dr} = \frac{G\left(M + 4\pi r^3 P/c^2\right)}{r^2} e^{2\Lambda}$$

$$\frac{d}{dr}\left(L_\nu e^{2\Phi/c^2}\right) = \epsilon_\nu e^{2\Phi/c^2} 4\pi r^2 e^{\Lambda}$$

$$\frac{d}{dr}\left(Le^{2\Phi/c^2}\right) = -c_v \frac{dT}{dt} e^{\Phi/c^2} 4\pi r^2 e^{\Lambda}\,, \quad \text{with} \quad \Lambda = \exp(1 - 2GM/c^2 r)^{-1/2}\,.$$

(P, ϵ) : (Pressure, energy density) M : Enclosed mass

κ : Opacity of matter Φ : Gravitational potential

L_d : Luminosity (thermal conductivity & radiation)

(L_ν, ϵ_ν) : Neutrino (luminosity, emissivity)

$L = L_d + L_\nu$; Net luminosity

c_v : Specific heat/volume, Time t measured at $r = \infty$.

Boundary Conditions

Inner boundary conditions:

$$M(0) = L(0) = L_\nu(0) = 0$$

Outer boundary conditions:

$$P_s = \tfrac{2}{3} g_s/\kappa_s\,, \qquad L_s = L_d(R) = 4\pi R^2 \sigma T_s^4$$

$$e^{\Phi/c^2} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} = e^{-\Lambda}$$

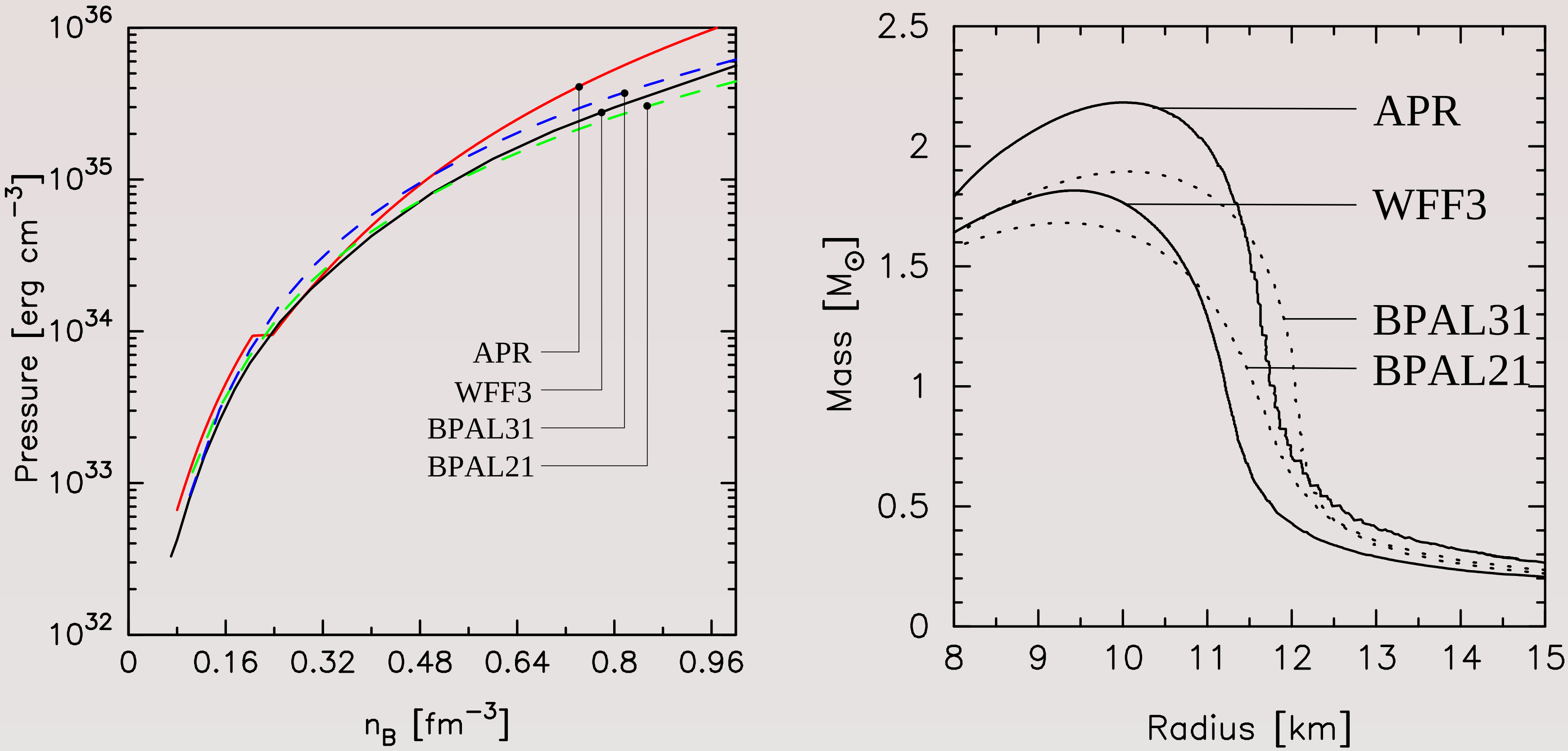
$g_s = (GM/R^2)e^{\Lambda_s}$: Surface gravity

(κ_s, T_S): Opacity and temperature at the surface

Physics ingredients:

- Equation of state $P = P(\epsilon)$
- Opacity κ and specific heat c_v
- Photon and neutrino emissivites

Equation of State



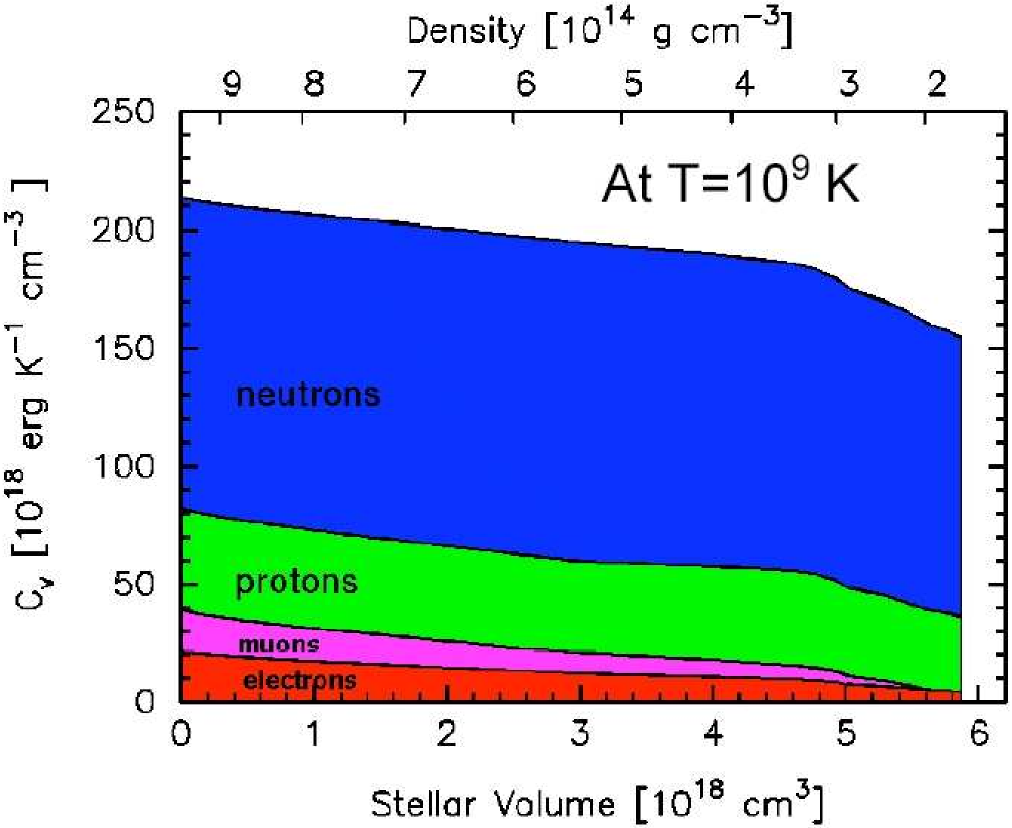
Moderate variation with nucleons-only matter.

Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

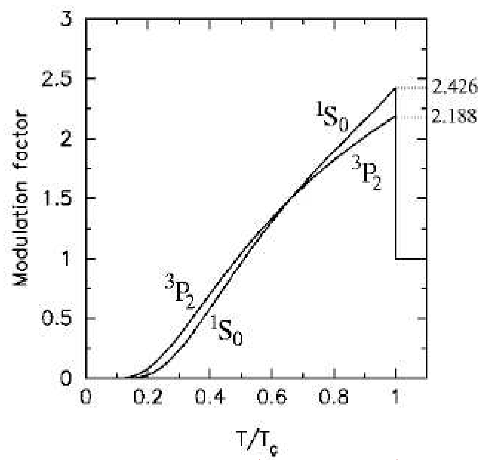
Specific Heat

Distribution of C_v in the core among constituents

$$C_V = N(0)\frac{\pi^2}{3}k_B^2 T \qquad N(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$



$$C_V^{paired} =$$
$$C_V^{normal} \times M(T/T_c)$$
$$\approx C_V^{normal} \times e^{-\Delta(T)/kT}$$



Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

Neutrino Emission Processes

- The direct Urca process :

$$n \rightarrow \; p + e^- + \bar{\nu}_e \quad \& \quad p + e^- \rightarrow \; n + \nu_e$$

cannot occur if the proton abundance is small as energy and momentum are not simultaneously conserved.

- For $T << T_F \sim 10^{12}$ K, momenta $\sim p_{F_i}$ for $i = n, p \;\&\; e$.

Neutrino and antineutrino momenta are $\sim kT/c \ll p_{F_i}$.

- Chemical equilibrium requires $\mu_n = \mu_p + \mu_e$.

Energy can be conserved for some states close to E_{F_i}.

For momentum conservation, the three triangle inequalities

$$p_{F_i} + p_{F_j} \;\; \geq \;\; p_{F_k} \, , \quad \text{where } i, j \;\&\; k \text{ are } p, \; e \;\&\; \text{and } n \, ,$$

must be satisfi ed failing which the modifi ed Urca processes, featuring a bystander particle that enables momentum conservation, occur.

Threshold proton fraction

- Number densities : $n_i = k_{Fi}^3/(3\pi^2)$ for i $(n, p$ or $e)$.
- Proton fraction : $x = n_p/(n_p + n_n)$.

At threshold, momentum conservation implies $k_{Fn} = k_{Fp} + k_{Fe}$.

$$x_c = \frac{k_{Fp}^3}{k_{Fp}^3 + (k_{Fp} + k_{Fe})^3} = \frac{1}{1 + (1 + k_{Fe}/k_{Fp})^3} \, .$$

In charge neutral n, p & e matter, $n_p = n_e$, or $k_{Fp} = k_{Fe}$.

- Hence, the proton fraction at threshold is $x_c = 1/9$.

In charge neutral n, p, e & μ matter, $n_e + n_\mu = n_p$ and $\mu_e = \mu_\mu$.

$$k_{Fe}^3 + \left(k_{Fe}^2 - m_\mu^2\right)^{3/2} = k_{Fp}^3 \, .$$

For $\mu_e = k_{Fe} >> m_\mu$, one has $k_{Fe} = k_{F\mu} = (1/2)^{1/3} k_{Fp}$, which gives

$$x_c = \frac{1}{1 + (1 + 1/2^{1/3})^3} \simeq 0.148 \, .$$

DUrca Threshold Density-I

- Energy per baryon :

$$E(n,x) = E(n,1/2) + S_v(n)(1-2x)^2 + \cdots \,,$$

where S_v is the density dependent bulk symmetry energy; at $n_s \simeq 0.16\ \mathrm{fm}^{-3}$, $S_v(n_s) \equiv S_o \approx 27 - 36$ MeV.

- Beta equilibrium :

$$\mu_e = \mu_n - \mu_p = -(\partial E/\partial x)\,.$$

- Equilibrium proton fraction :

$$\hbar c(3\pi^2 nx)^{1/3} = 4S_v(n)(1-2x)\,,$$

The density, n_c, at which $x = x_c = 1/9$, from

$$S_v(n_c) = 51.2 \left(\frac{S_o}{30\ \mathrm{MeV}}\right)\left(\frac{n_c}{n_s}\right)^{1/3}\ \mathrm{MeV}.$$

DUrca Threshold Density-II

Urca threshold densities

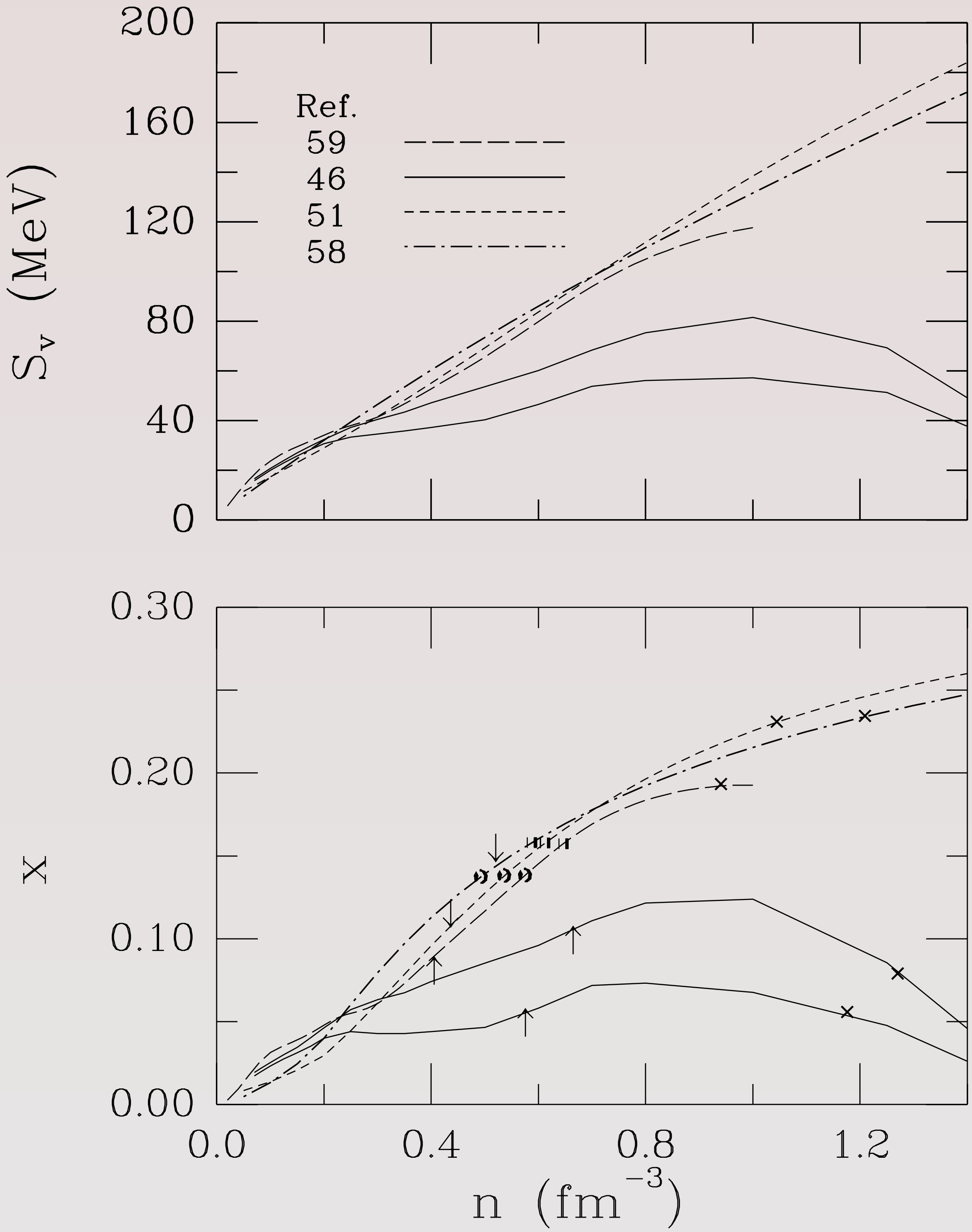
q	1/3	2/3	1	4/3
n_c/n_s	25(9.7)	5.0(3.1)	2.2(1.8)	1.71(1.46)

The quantity n_c/n_s was calculated for power law symmetry energies $S_v \propto n^q$ using $S_o = 30(35)$ MeV.

- The critical density is sensitive to interactions and the magnitude of the symmetry energy.

- The case $q = 2/3$ and $S_0 = (1/3)(\hbar^2 k_{F_s}^2/2m) \cong 12.28$ MeV corresponds to free non–relativistic nucleons for which $n_c/n_s \simeq 73$!

Models of Dense matter



- (a) Nuclear symmetry energy vs. density for different EOS's.
- (b) Equilibrium proton fractions including muons.
- Solid circles (squares): critical density for DUrca for electrons (muons).
- Arrows(crosses): central density of $1.4 M_{\odot}$ (maximum–mass) neutron stars.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Neutrino Emissivities-I

The $\bar{\nu}$ energy emission rate from neutron beta decay:

$$\epsilon_\beta = \frac{2\pi}{\hbar} 2 \sum_i G_F^2 (1+3g_A)^2 \;\; n_1(1-n_2)(1-n_3)$$

$$E_4 \;\; \delta^{(4)}(p_1 - p_2 - p_3 - p_4)\,,$$

- Including neutrinos from electron capture, $\epsilon_{Urca} = 2\epsilon_\beta$.

$$\begin{aligned} \epsilon_{Urca} &= \frac{457\pi}{10080} \frac{G_F^2(1+3g_A^2)}{\hbar^{10}c^5} m_n m_p \mu_e (kT)^6 \Theta_t \\ &= 4.00 \times 10^{27} (Y_e n/n_s)^{1/3} T_9^6 \Theta_t \text{ erg cm}^{-3}\text{ s}^{-1}\,, \end{aligned}$$

T_9 : temperature in units of 10^9K,
n_s : 0.16 fm^{-3},
$Y_e = n_e/n$: electron fraction, and
$\Theta_t = \theta(p_{Fe} + p_{Fp} - p_{Fn})$:

- If the muon Urca process can occur, we gain another factor of 2.

Neutrino Emissivities-II

Effects of strong & weak interactions

- n & p density of states at $p_{F_{n,p}}$ renormalized: $m_{n,p} \rightarrow m^*_{n,p}$.

- In-medium quenching of g_A : $|g_A| \rightarrow 1$

- Final state interaction modifi cations of weak interaction ME's: small, since $n - p$ interactions small at momentum transfers $\sim p_F$.

- At best a reduction of $\sim 5 - 10$.

- Similar corrections for other $\nu-$ emissivities involving nucleons.

The time for a star's center to cool:

$$\Delta t = -\int \frac{c_v}{\epsilon_{urca}} dT \simeq 30\, T_9^{-4}\, \mathrm{s}\,,$$

where T is the temperature and c_v is the specifi c heat per unit volume.

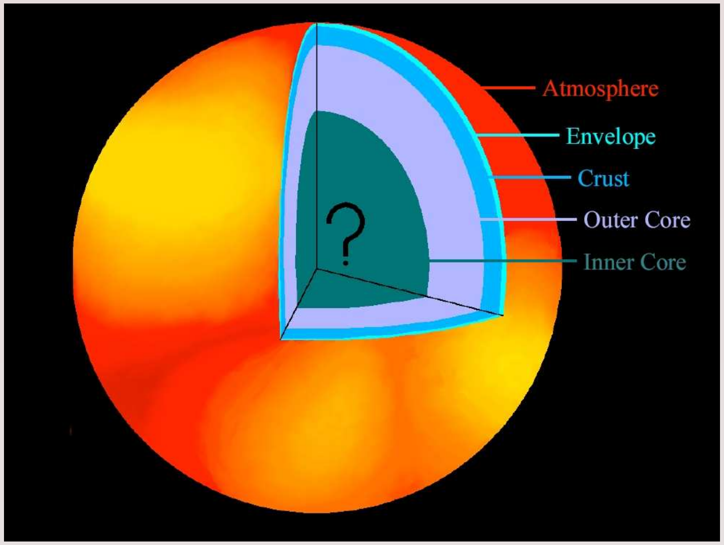
Rule of thumb for surace T is $T_s/10^6\ \mathrm{K} \simeq (T/10^8\ \mathrm{K})^{1/2}$.

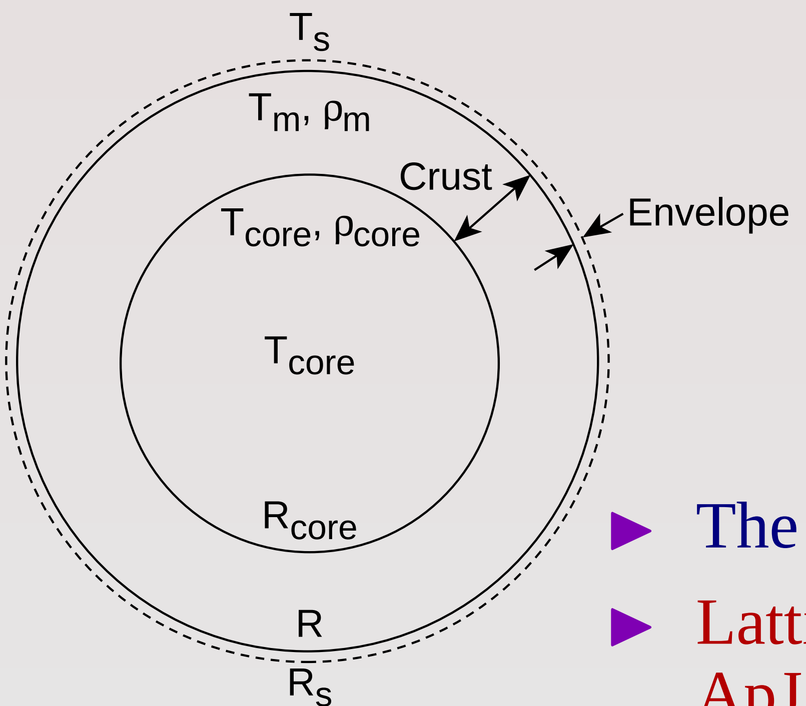
Neutrino Emissivities-III

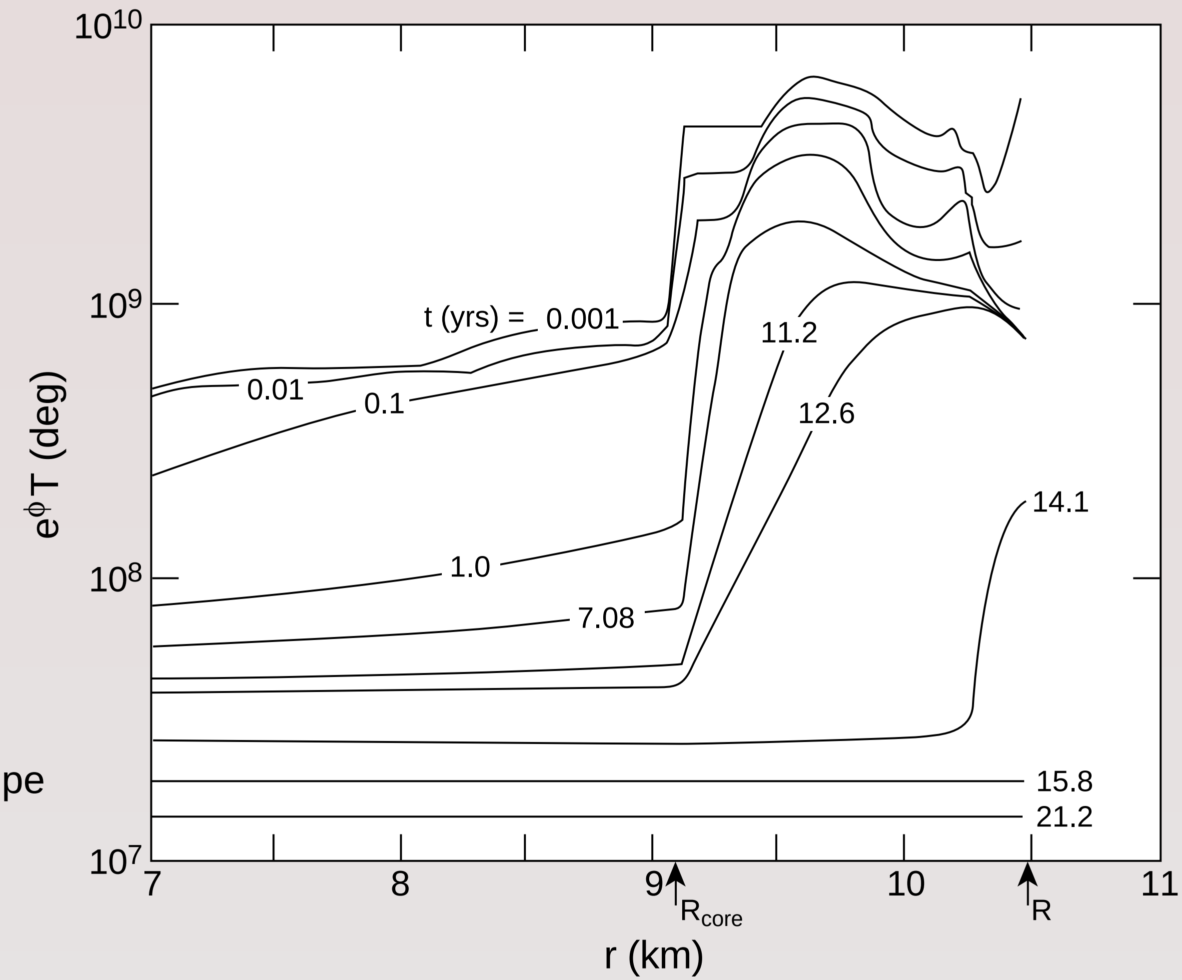
Name	Process	Emissivity (erg s^{-1} cm^{-3})	References
Modifi ed Urca	$n + n' \rightarrow n + p + e^- + \bar{\nu}_e$ $n' + p + e^- \rightarrow n' + n + \nu_e$	$\sim 10^{20} T_9^8$	Friman & Maxwell 1979
Kaon Condensate	$n + K^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + K^- + \nu_e$	$\sim 10^{24} T_9^6$	Brown et al., 1988
Pion Condensate	$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + \pi^- + \nu_e$	$\sim 10^{26} T_9^6$	Maxwell et al., 1977
Direct Urca	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} T_9^6$	Lattimer et al., 1991
Hyperon Urca	$B_1 \rightarrow B_2 + l + \bar{\nu}_l$ $B_2 + l \rightarrow B_1 + \nu_l$	$\sim 10^{26} T_9^6$	Prakash et al., 1992
Quark Urca	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$	$\sim 10^{26} \alpha_c T_9^6$	Iwamoto 1980

T_9: Temperature in units of 10^9 K.

Thermal Evolution

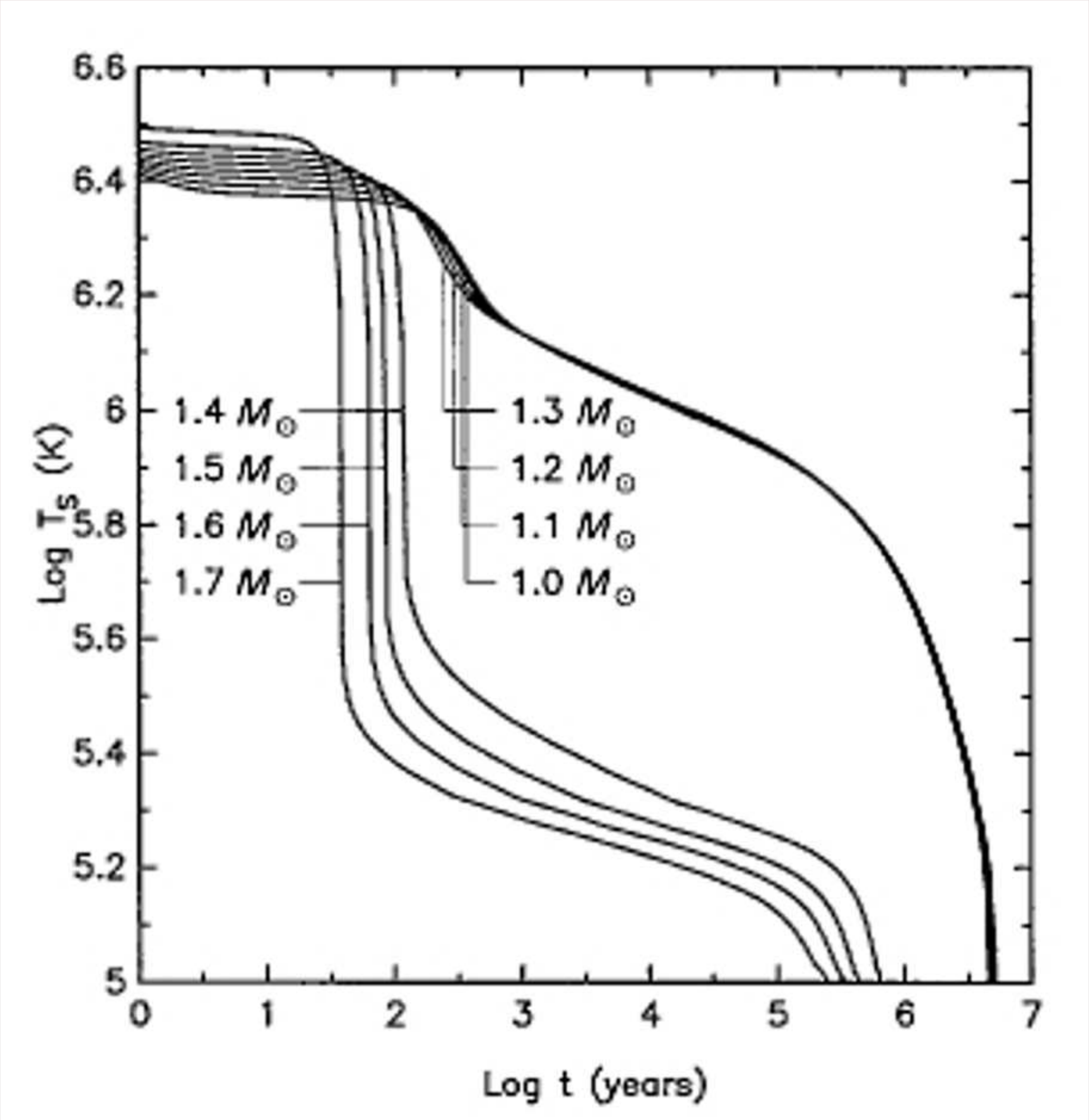






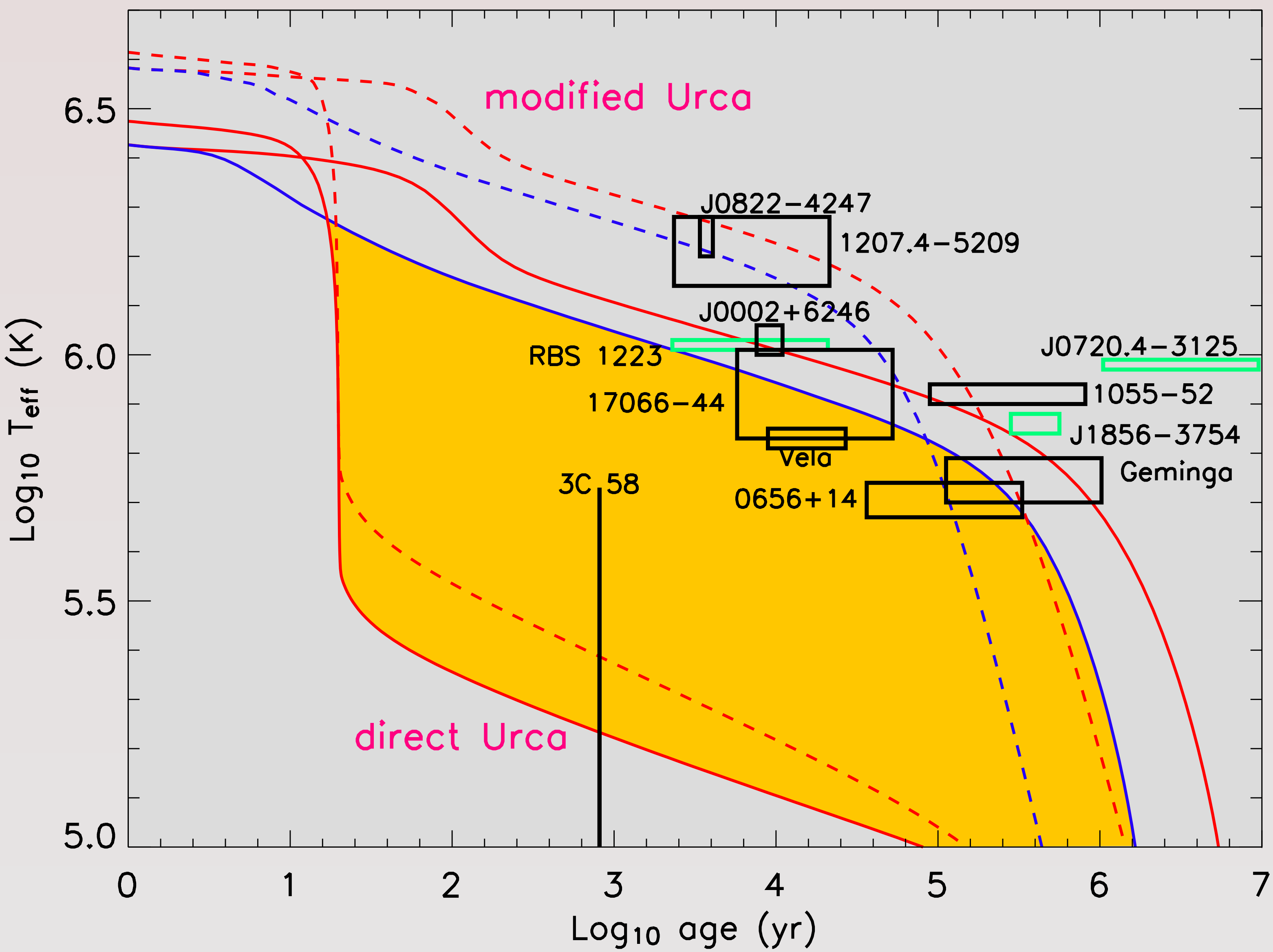
- The star becomes isothermal in tens of years.
- Lattimer, Van Riper, Prakash & Prakash, ApJ **425**, 802 (1993).

Direct versus Modified Urca



- Unlike MUrca, Durca exhibits threshold effects.
- Superfluidity abates DUrca cooling.
- Page & Applegate, ApJ **394**, L17 (1992).
- Cooper pair breaking & reformation affects both DUrca & MUrca.

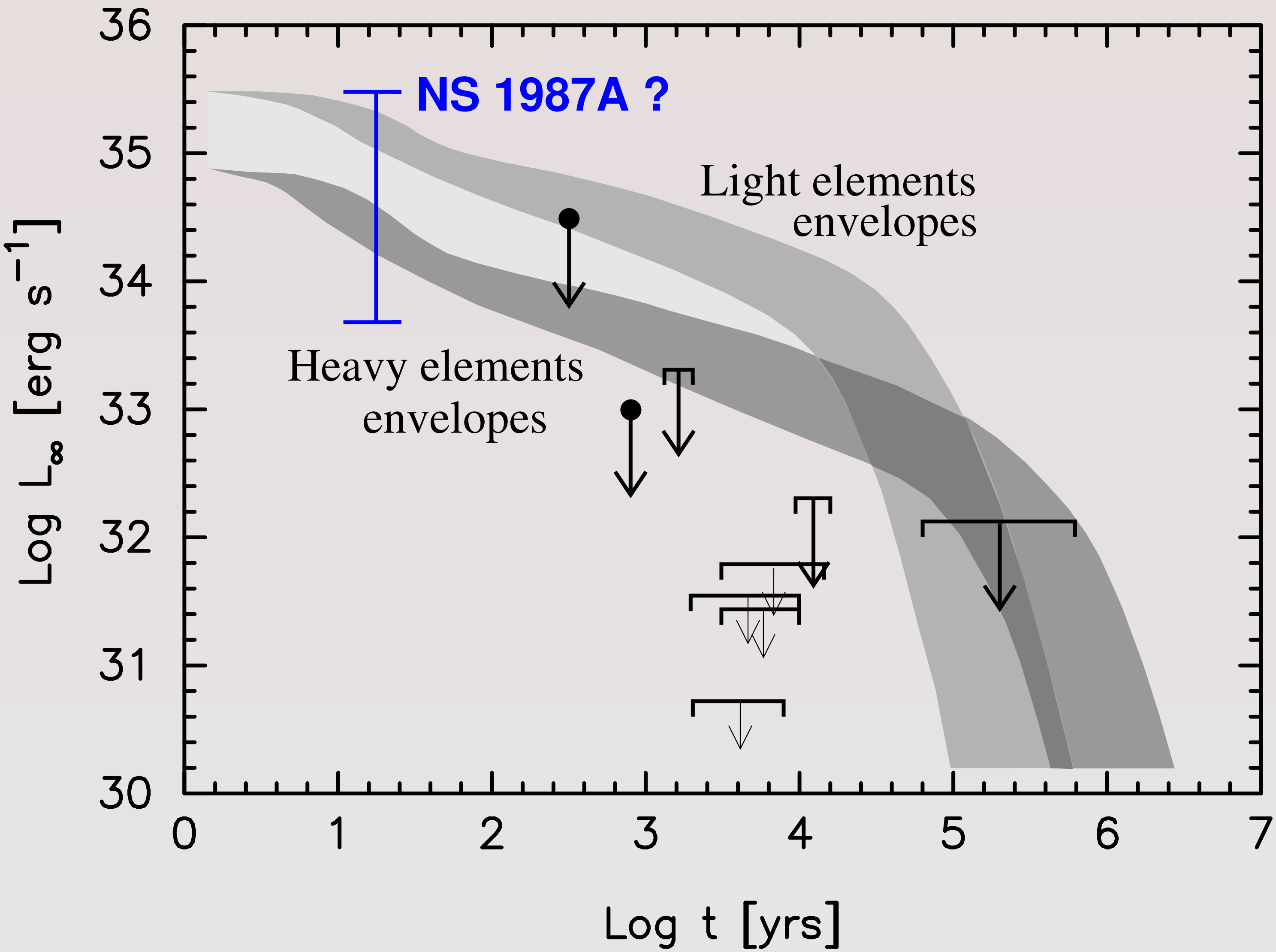
Inferred Surface Temperatures



Lattimer & Prakash , Science **304**, 536 (2004).

New Cold Objects

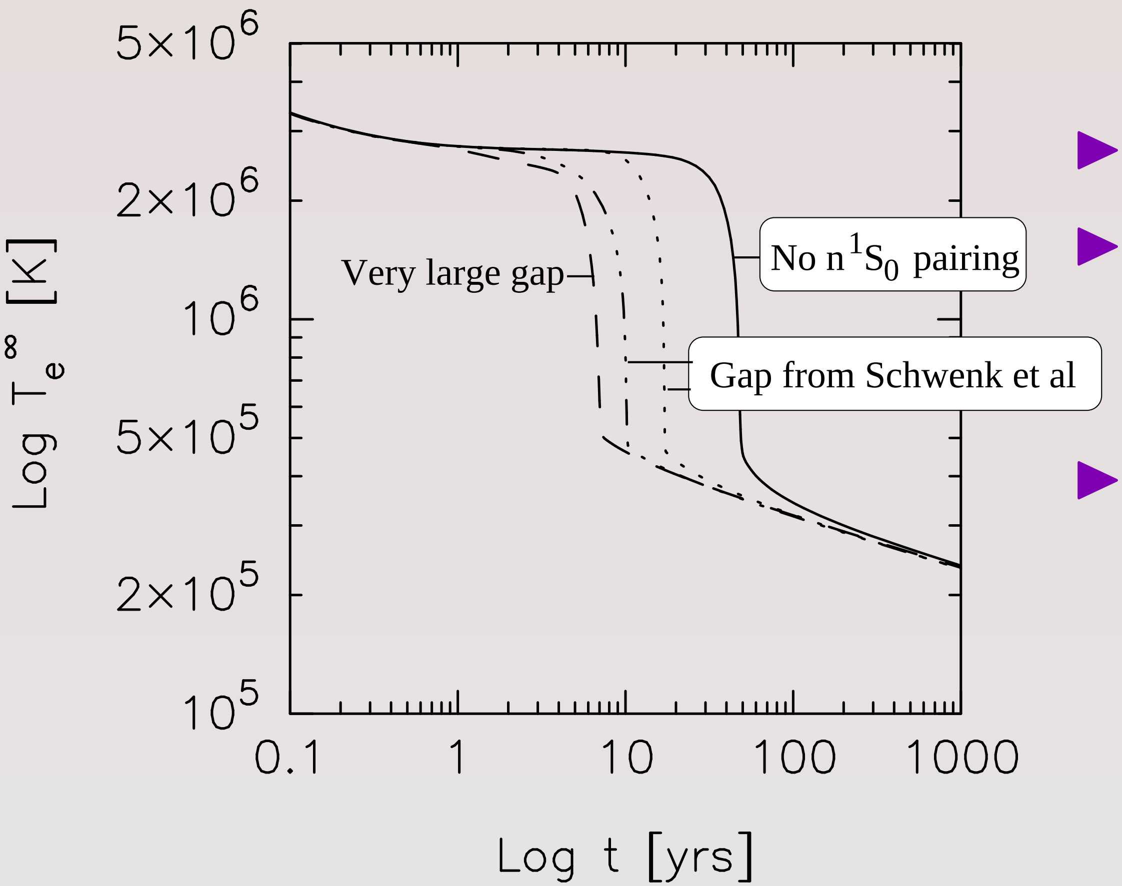
Several cases fall below the “Minimal Cooling” paradigm & point to enhanced cooling, if these objects correspond to neutron stars.



Page, Lattimer, Prakash & Steiner, ApJS **155**, 623 (2004).

Ongoing Work

Preparing to interpret the detection of really cold objects.



- SN 1987A is being monitored regularly (astro-ph/0501561).
- A NS is yet to be seen!
- If rapid cooling occurs, when can thermal emission begin?
- W/O Cooper pair breaking & formation (CBF): $t_w \propto (R_{sh}/1\ \mathrm{km})^2 \times (1-r_G)^{-3/2}$ yr $\simeq 10'$s of yr (Lattimer et al., ApJ **425** 802 (1994)).
- Time scales with CBF?

Page, Prakash & Lattimer (2006).