Nuclear & Particle Physics of Compact Stars

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Charge neutral neutron-rich matter-I

- Old neutron stars are in equilibrium w.r.t. weak interactions.

- The ground state consists of strongly interacting hadrons and weakly interacting leptons generated in β-decays and inverse β-decays.

- Local charge neutrality and chemical equilibrium prevail.

First consider matter with neutrons, protons and electrons involved in

$$\begin{aligned} n\,(\text{or } + n) &\longrightarrow p\,(\text{or } + n) + e^- + \overline{\nu}_e\,, \\ p\,(\text{or } + n) + e^- &\longrightarrow n\,(\text{or } + n) + \nu_e \end{aligned}$$

- In cold catalyzed neutron star matter, neutrinos leave the system leading to

$$\widehat{\mu} = \mu_n - \mu_p = \mu_e\,.$$

(Later we will also consider the case in which neutrinos are trapped in matter, which leads instead to $\mu_n - \mu_p = \mu_e - \mu_{\nu_e}$.)

Charge neutral neutron-rich matter-II

- In beta equilibrium, one has

$$\frac{\partial}{\partial x}\left[E_b(n,x)+E_e(x)\right]=0\,.$$

- Charge neutrality implies that $n_e = n_p = nx$, or, $k_{F_e} = k_{F_p}$.

Combining these results, $\widetilde{x}(n)$ is determined from

$$4(1-2x)\;\left[S_2(n)+2S_4(n)\,(1-2x)^2+\cdots\right]=\hbar c\left(3\pi^2 nx\right)^{1/3}\,.$$

When $S_4(n) << S_2(n)$, $\widetilde{x}$ is obtained from $\beta\,\widetilde{x}-(1-2\widetilde{x})^3=0\,,$ where $\beta = 3\pi^2 n\,(\hbar c/4S_2)^3$. Analytic solution ugly!

For $u \leq 1$, $\widetilde{x} << 1$, and to a good approximation $\widetilde{x} \simeq (\beta+6)^{-1}\,.$

- Notice the high sensitivity to $S_2(n)$, which favors the addition of protons to matter.

Charge neutral neutron-rich matter-III

Muons in matter :

When $E_{F_e} \geq m_\mu c^2 \sim 105$ MeV, electrons convert to muons through

$$e^- \longrightarrow \mu^- + \overline{\nu}_\mu + \nu_e \,.$$

Chemical equilibrium implies $\mu_\mu = \mu_e$.

At threshold, $\mu_\mu = m_\mu c^2 \sim 105$ MeV.

As the proton fraction at nuclear density is small, $4S_2(u)/m_\mu c^2 \sim 1$.

Using $S_2(u=1) \simeq 30$ MeV, threshold density is $\sim n_0 = 0.16\ \mathrm{fm}^{-3}$.

Above threshold,

$$\mu_\mu = \sqrt{k_{F_\mu}^2 + m_\mu^2 c^4} = \sqrt{(\hbar c)^2 (3\pi^2 n x_\mu)^{2/3} + m_\mu^2 c^4}\,.$$

- $x_\mu = n_\mu / n_b$:= muon fraction in matter.

The new charge neutrality condition is $n_e + n_\mu = n_p$.

Muons make $x_e = n_e/n_b$ to be lower than its value without muons.

Charge neutral neutron-rich matter-IV

Total energy density & pressure :

$$\epsilon_{tot} = \epsilon_b + \sum_{\ell=e^-,\mu^-} \epsilon_\ell \quad \& \quad P_{tot} = P_b + \sum_{\ell=e^-,\mu^-} P_\ell$$

- $\epsilon_{b,\ell}$ and $P_{b,\ell}$:= energy density and pressure of baryons (leptons).

$$\epsilon_\ell = 2\int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m_\ell^2} \quad \& \quad P_\ell = \frac{1}{3}\cdot 2\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2+m_\ell^2}}$$

$$\begin{aligned} \epsilon_b &= mn_0u + \left\{\frac{3}{5}E_F^0 n_0 u^{5/3} + V(u)\right\} + n_0(1-2x)^2 u S(u)\,, \\ P_b &= \left\{\frac{2}{5}E_F^0 n_0 u^{5/3} + \left(u\frac{dV}{du} - V\right)\right\} + n_0(1-2x)^2 u^2 \frac{dS}{du}\,. \end{aligned}$$

- As $\alpha_{em} \simeq 1/137$, free gas expressions for leptons are satisfactory.

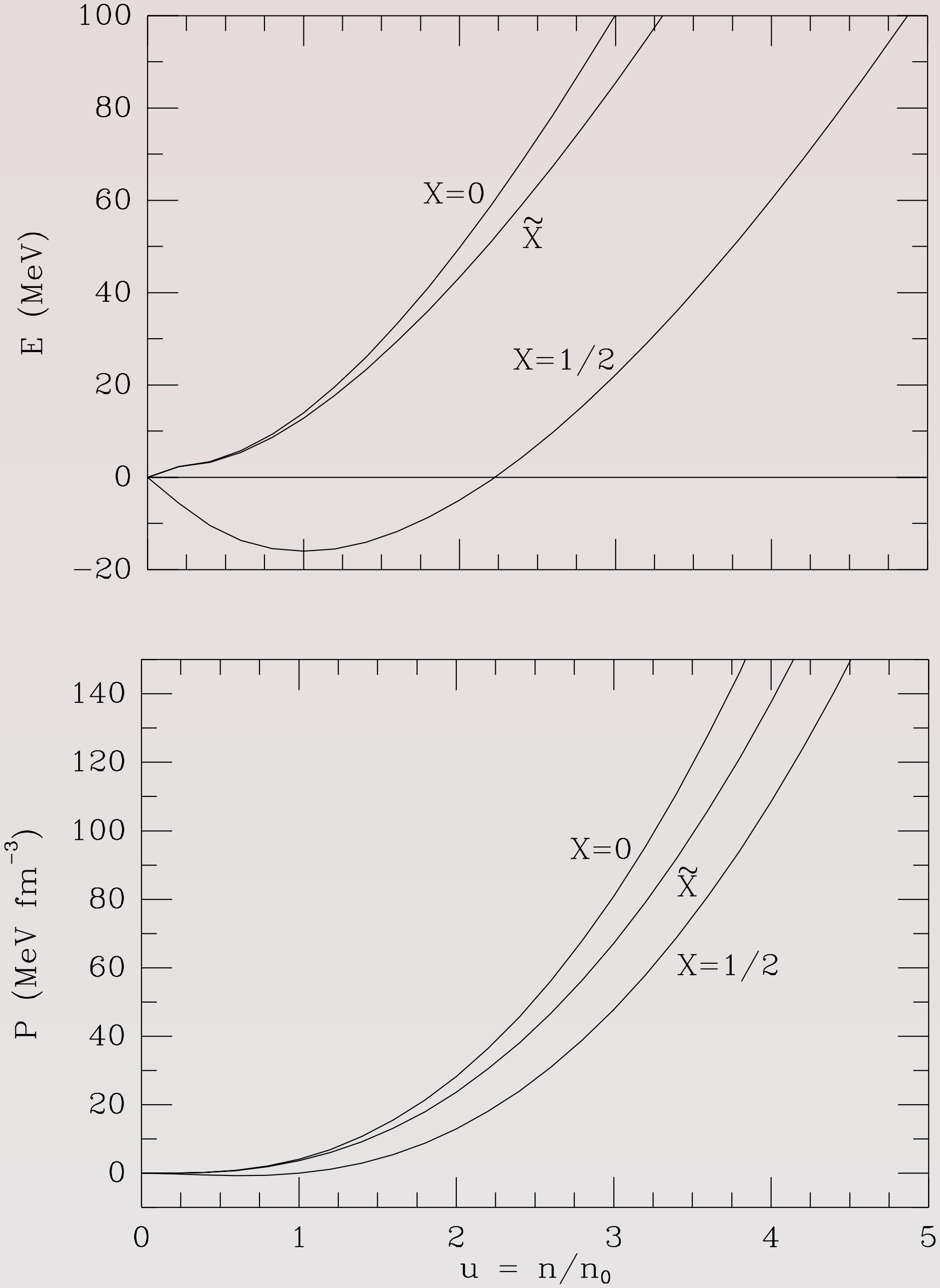
Charge neutral neutron-rich matter-V

STATE VARIABLES AT NUCLEAR DENSITY

Quantity	Nuclear matter	Stellar matter
$\tilde{x}$	0.5	0.037
$\epsilon_b/n - m$	-16	9.6
ϵ_e/n	0	3.18
P_b	0	3.5
P_e	0	0.17
$\mu_n - m$	-16	35.74
$\mu_p - m$	-16	-75.14
$\mu_e = \mu_n - \mu_p$	0	110.88

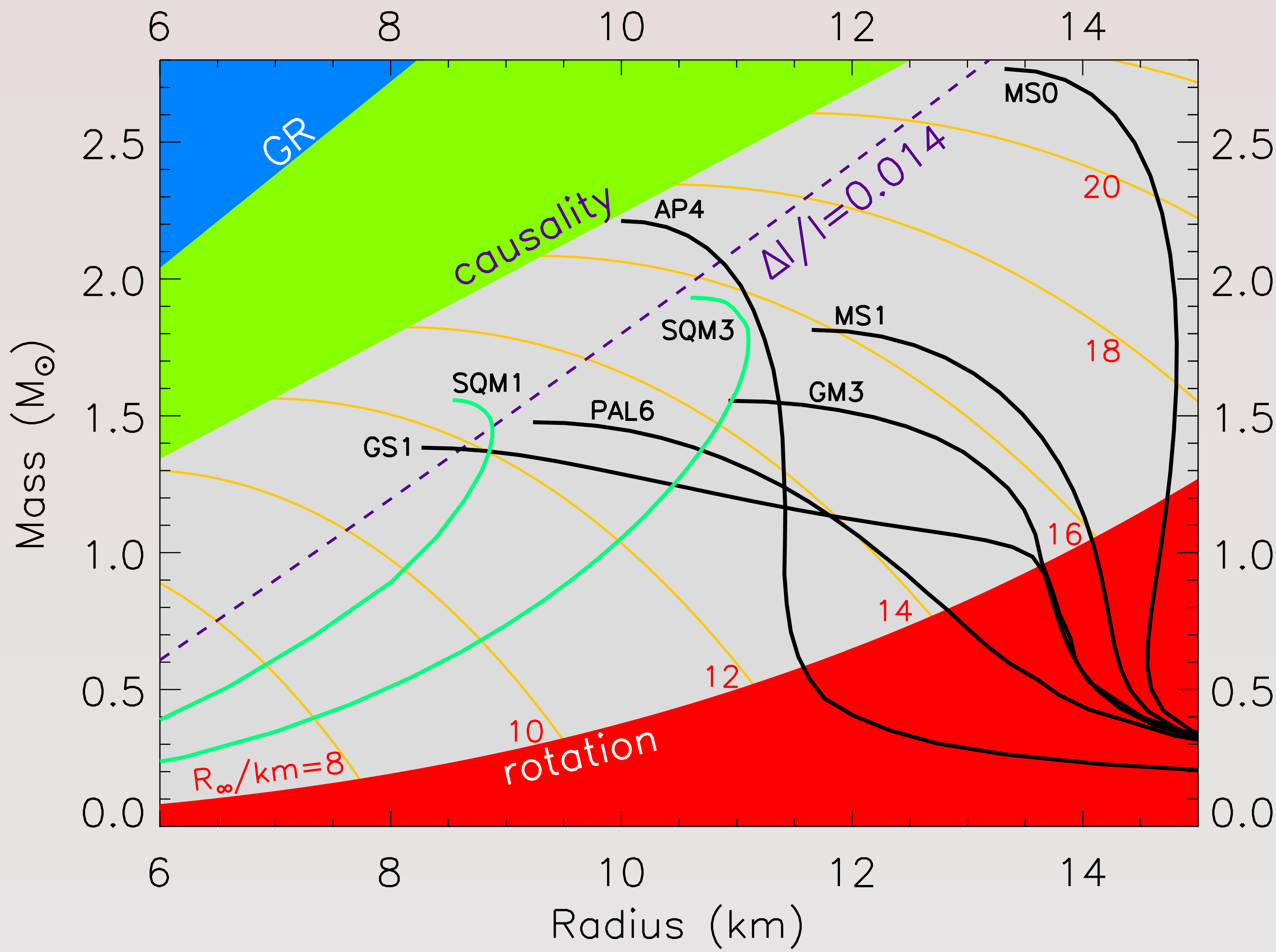
Energies in MeV and pressure in MeV fm^{-3}. The numerical estimates are based on an assumed symmetry energy $S_2(u) = 13u^{2/3} + 17u$, where $u = n/n_b$.

Nucleonic Equation of State

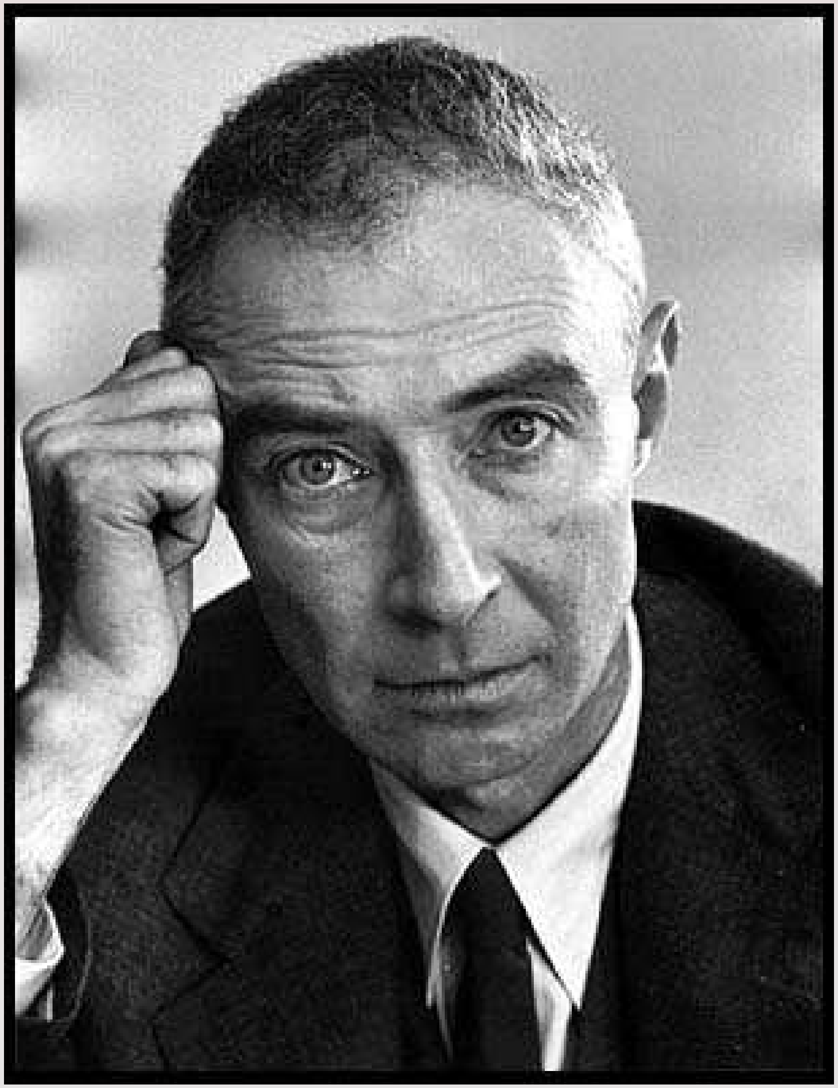


- Energy (E) & Pressure (P) vs. scaled density ($u = n/n_0$).
- Nuclear matter equilibrium density $n_0 = 0.16\ \mathrm{fm}^{-3}$.
- Proton fraction $x = n_p/(n_p + n_n)$.
- Nuclear matter : $x = 1/2$.
- Neutron matter : $x = 0$.
- Stellar matter in $\beta-$ equilibrium : $x = \tilde{x}$.

Mass Radius Relationship



Lattimer & Prakash , Science **304**, 536 (2004).



Evolution of a Nascent Neutron Star

$$\frac{dP}{dr} = -G\frac{(m+4\pi r^3 P)(\rho+P/c^2)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi r^2\rho\,; \qquad \frac{d\mu}{dr} = 4\pi r^2\rho_0 e^{\Lambda}$$

$$\frac{d(N_\nu/\rho_0)}{d\tau} + e^{-\phi}\frac{\partial(L_n e^{\phi})}{\partial\mu} = S_n\,, \qquad L_n = 4\pi r^2 F_n$$

$$\frac{d(N_e/\rho_0)}{d\tau} = -S_n$$

$$\frac{d(E/\rho_0)}{d\tau} + P\frac{(1/\rho_0)}{d\tau} + e^{-2\phi}\frac{\partial(L_e e^{2\phi})}{\partial\mu} = S_e\,; \qquad L_e = 4\pi r^2 F_e$$

- m := enclosed grav. mass ;
- μ := enclosed rest mass
- ρ := mass-energy density ;
- ρ_0 := baryon rest mass density
- N_ν := ν_e number density ;
- L_n := lepton luminosity
- F_n := lepton flux ;
- S_n := ν_e source term

- Metric: $ds^2 = -e^{2\phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\Omega$
- t := universal coordinate time;
- τ := local proper time

$$e^{\phi} = \sqrt{-g_{00}} \qquad e^{2\Lambda} = 1/(1-2Gm/rc^2) \qquad e^{-\phi}\, d/dt = d/d\tau$$

Composition of Dense Stellar Matter

- Crustal Surface :
  electrons, nuclei, dripped neutrons, $\cdots$ set in a lattice
  new phases with lasagna, sphagetti, $\cdots$ like structures
- Liquid (Solid?) Core :
  $n, p, \Delta, \cdots$ leptons: $e^{\pm}, \mu^{\pm}, \nu_e' s, \nu_\mu' s$
  $\Lambda, \Sigma, \Xi, \cdots$
  $K^-, \pi^-, \cdots$ condensates
  $u, d, s, \cdots$ quarks
- Constraints :
  1. $n_b = n_n + n_p + n_\Lambda + \cdots$: baryon # conservation
  2. $n_p + n_{\Sigma^+} + \cdots = n_e + n_\mu$: charge neutrality
  3. $\mu_i = b_i \mu_n - q_i \mu_\ell$: energy conservation

$$\Rightarrow$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \quad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e$$

$$\Rightarrow$$

$$\mu_{K^-} = \mu_e = \mu_\mu = \mu_n - \mu_p$$

$$\Rightarrow$$

$$\mu_d = \mu_u + \mu_e = \mu_s = (\mu_n + \mu_e)/3$$
$$\mu_u = (\mu_n - 2\mu_e)/3$$

NEUTRINO TRAPPED STARS

(Newly-Born neutron stars)

- Entropy/baryon $\sim 1 - 2$
- Leptons/baryons $Y_{L\ell} = Y_\ell + Y_{\nu\ell}$, $(\ell = e, \mu$ and $\tau)$
  conserved on dynamical timescales of collapse
- Neutrinos trapped !

- Chemical equilibrium

$$\Rightarrow \boxed{\mu_i = b_i \mu_n - q_i(\mu_\ell - \mu_{\nu\ell})}$$

b_i : baryon #
q_i : baryon charge
$\mu_{\nu\ell}$: neutrino chemical potential

- Collapse calculations

$$\Rightarrow Y_{Le} = Y_e + Y_{\nu_e} \simeq 0.4$$
$$Y_{L\mu} = Y_\mu + Y_{\nu_\mu} \simeq 0.0$$

Prakash et al., Phys. Rep. **280**, 1 (1997).

Why Neutrinos Get Trapped

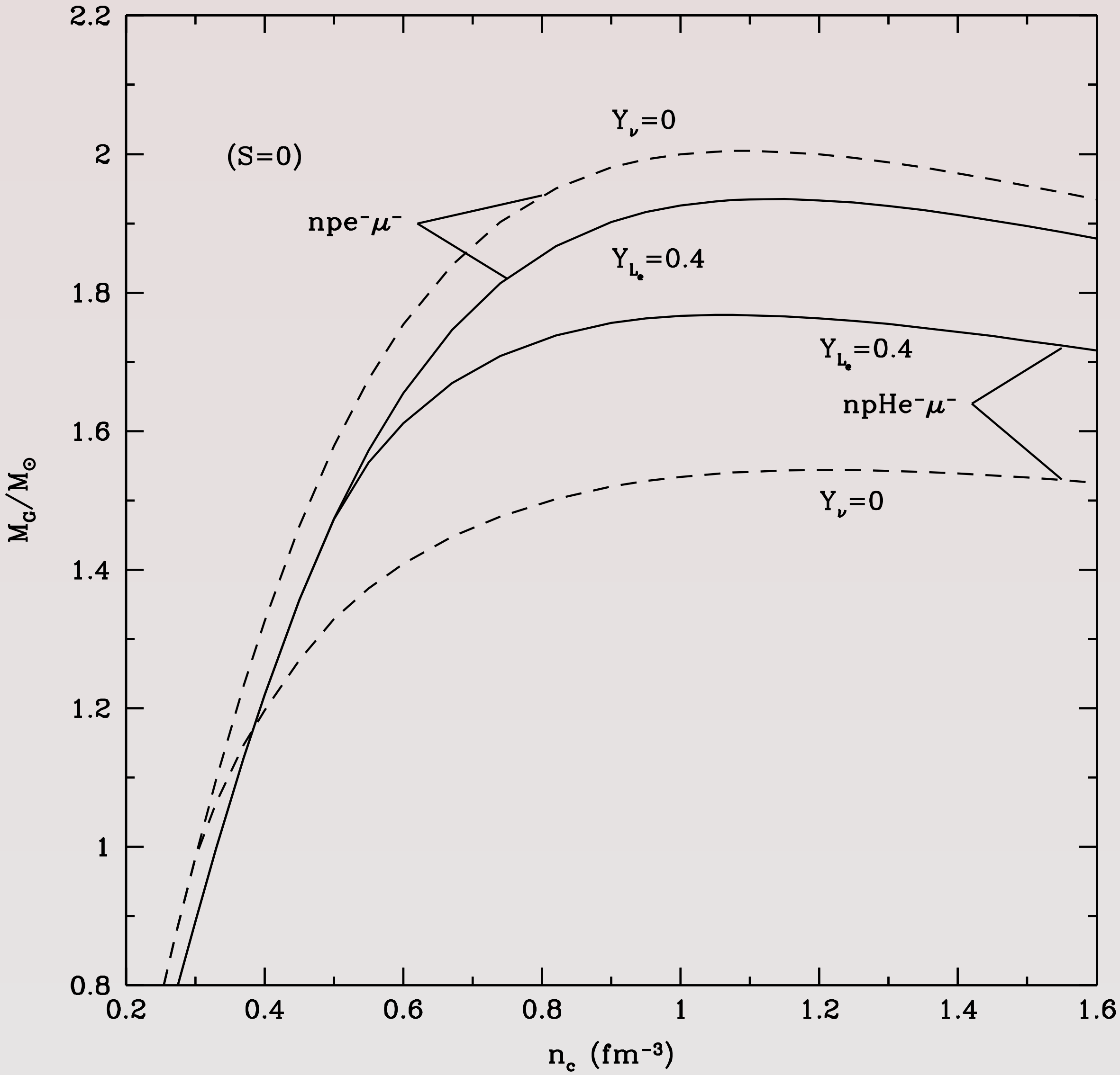
- Roughly, the scattering mean free path of ν's in dilute matter is

$$\begin{aligned} \lambda &= \left(n_n G_F^2 E_\nu^2\right)^{-1} \\ &\simeq 2 \times 10^5 \left(\frac{\mathrm{MeV}}{E_\nu}\right)^2 \mathrm{cm} \qquad \text{for} \quad n_n = 0.16 \ \mathrm{fm}^{-3} \end{aligned}$$

$$\begin{aligned} \text{For} \qquad E_\nu &\sim 200 \ \mathrm{MeV}\,, \\ \text{Mean free path } \lambda &\simeq 5 \ \mathrm{cm} << R \sim (10 - 100) \ \mathrm{km} \end{aligned}$$

- Effects of degeneracy (Pauli exclusion) and interactions increase λ by an order of magnitude and are important, but affect trapping only quantitatively.
- Even the elusive neutrinos are trapped in matter, albeit transiently, in the supernova environment!

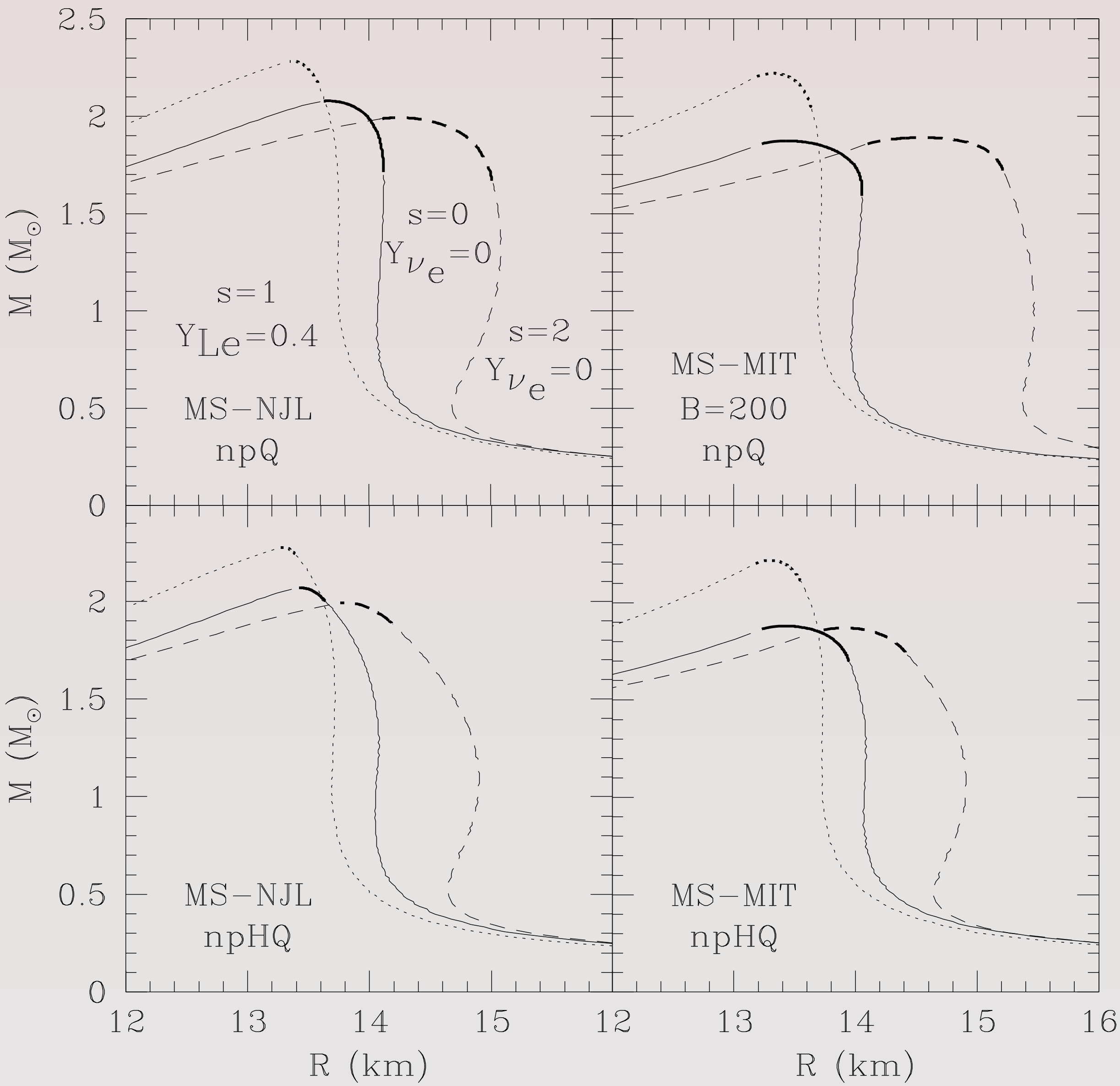
Neutrino Trapping in Hyperonic Matter



- $npe^-\mu^-$: Trapped ν's decrease the maximum mass
- $npHe^-\mu^-$: Trapped ν's increase the maximum mass
- ν-trapping delays the appearence of H's till larger densities are reached

Prakash et al., Phys. Rep. **280**, 1 (1997).

Neutrino Trapping in Quark Matter

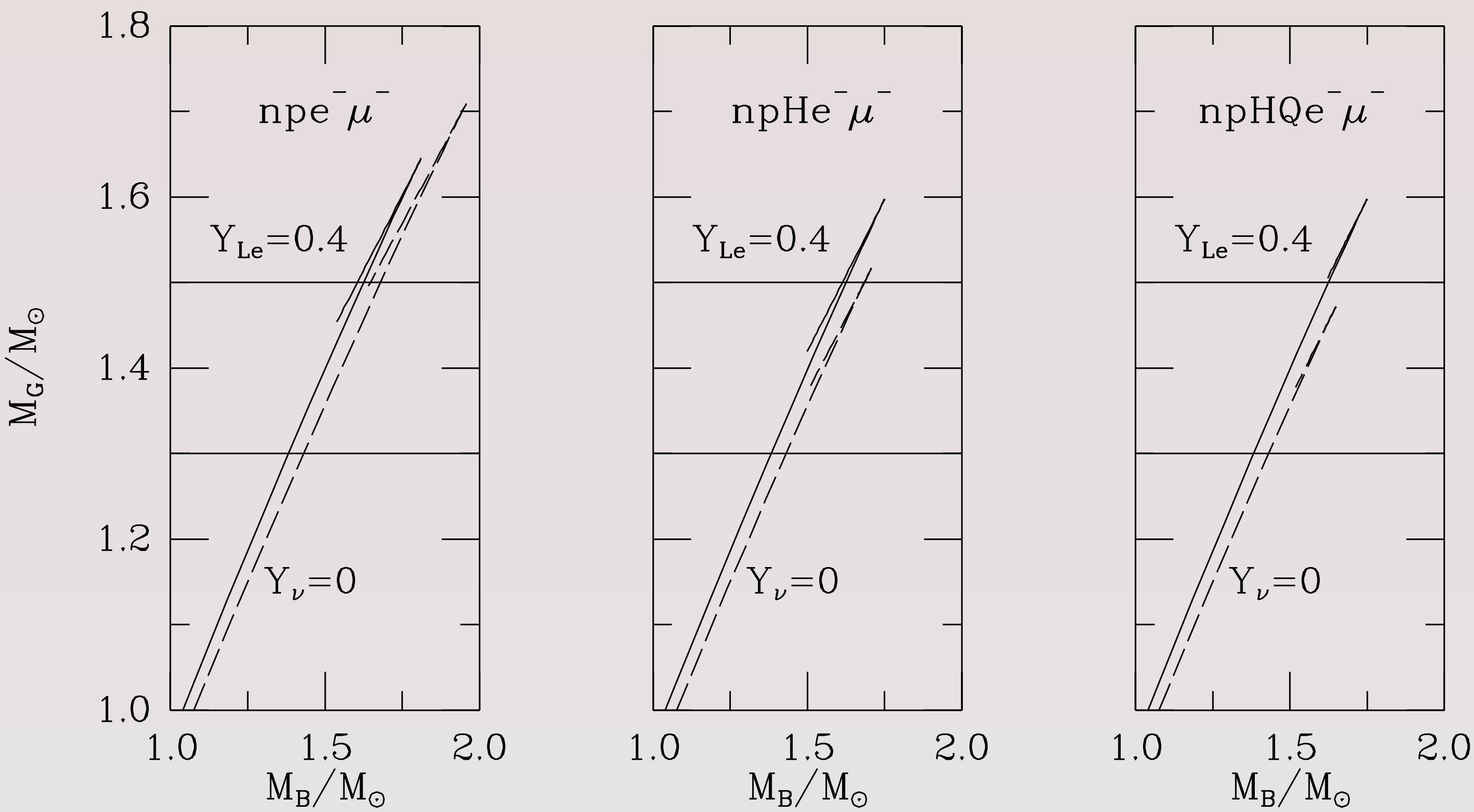


- Dark portions show mixed phase regions
- ν-free stars with $s = 2$ have smaller maxium masses than those with $s = 0$
- With ν trapping a range of masses are metastable

Steiner, Prakash & Lattimer, Phys. Lett. **B486**, 239 (2000).

Neutrino Trapping & Metastability

If the initial protoneutron star maximum mass is greater than that of the final configuration, collapse to a black hole is inevitable.



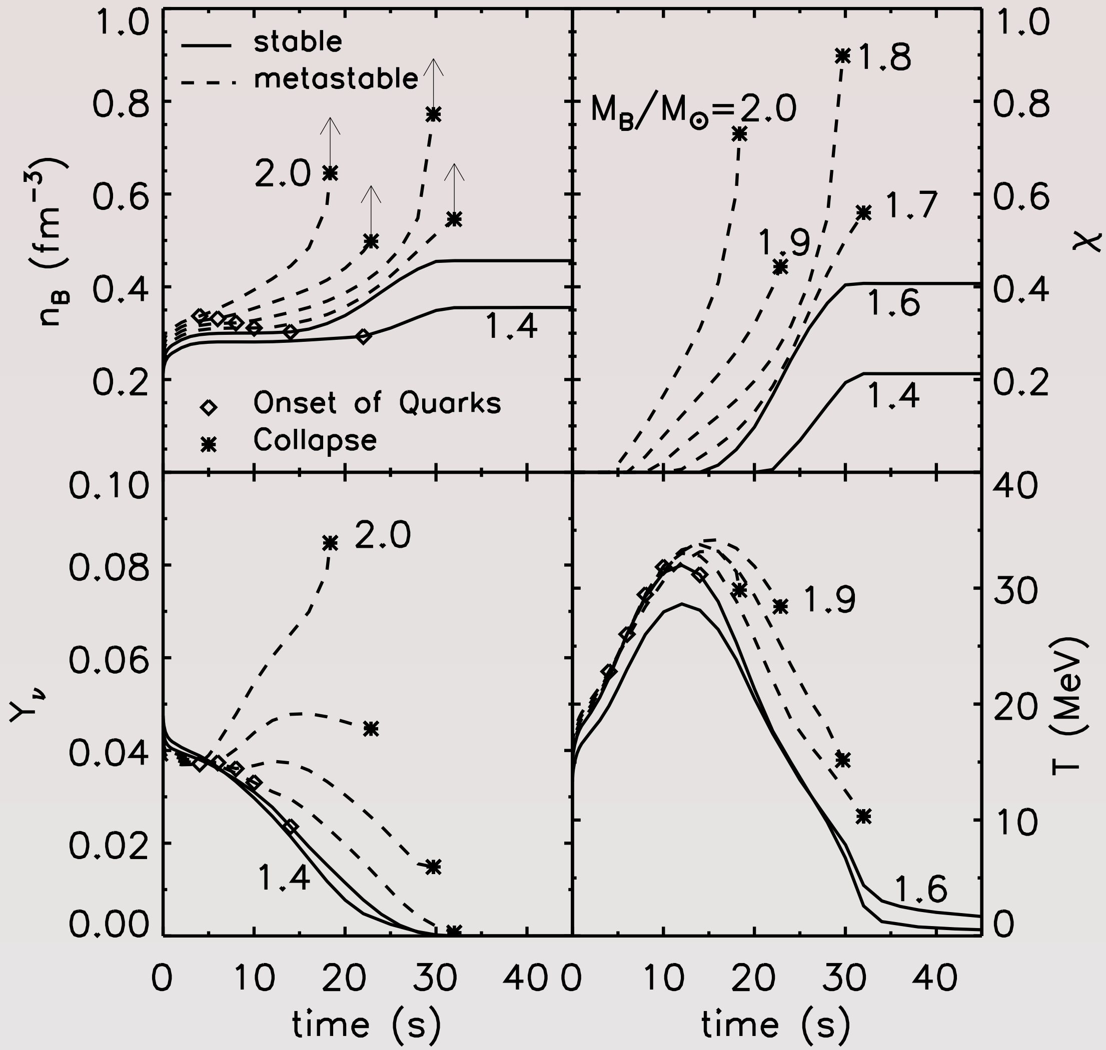
Ellis, Lattimer & Prakash, Comm. Nucl. & Part. Phys. **22**, 63 (1996).

Metastability

1. Appearance of hyperons, bosons and/or quarks is delayed to higher density in neutrino-trapped (lepton-rich) matter.
2. Nascent neutron stars, with negatively charged strongly interacting particles, have larger maximum masses than their cold catalyzed counterparts; a reversal in behavior from matter containing only neutrons, protons and leptons.
3. Above permits existence of metastable young stars that could collapse to black holes during deleptonization.
4. In all cases, effects of entropy (of order 1 or 2) on the maximum mass are small in comparison to effects of neutrino trapping.

Prakash et al., Phys. Rep. **280**, 1 (1997).

Evolution of Matter with Quarks



- As ν's leave, the central density & mass of nucleons-only stars stabilize
- Massive enough stars with quarks end up as black holes upon deleptonization

Pons, Steiner, Prakash & Lattimer, Phys. Rev. Lett. **86**, 10 (2001).

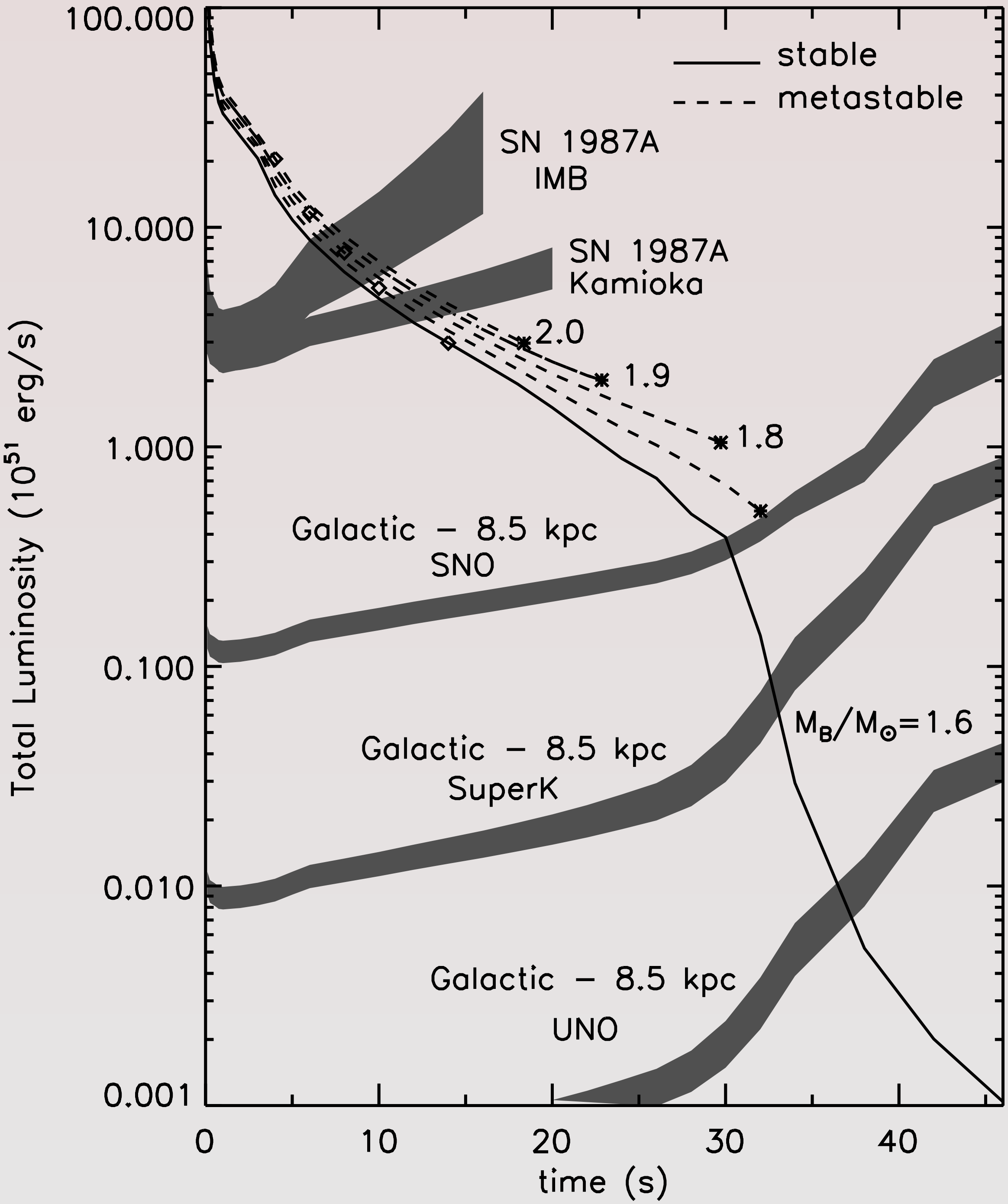
Signals in Terrestrial Detectors

(Bare-Bone Approach)

$$\frac{dN}{dt} = \left(\frac{R_\nu^\infty}{D}\right)^2 \frac{c\sigma_0 n_p}{8\pi(\hbar c)^3}\mathcal{M}\int_{E_{th}}^{\infty} E_\nu^4 f(E_\nu, T_\nu^\infty) W(E_\nu) dE_\nu$$

- $R_\nu^\infty = e^{-\phi_s} R_\nu$: Radius of the neutrinosphere
- D: Distance to supernova
- $\sigma_0 = 9.3 \times 10^{-44}$ cm^2 : $\bar{\nu}_e\, p$ crossection
- $n_p = 6.7 \times 10^{28}$: Free protons per kiloton of water
- $\mathcal{M}$: Detector mass in kilotons
- $T_\nu^\infty = e^{\phi_s} T_\nu$: Neutrino temperature at freezeout
- $e^{\phi_s} = \left(1 - \frac{2GM_G}{R_\nu c^2}\right)^{1/2}$: Redshift factor
- E_{th} & $W(E_\nu)$: Detector threshold & efficiency

Neutrino Luminosities



- Early detectors lacked sensitivity to test if SN 1987A ended up as a black hole.
- Current & future detectors can do better in the case of a future event.
- Prakash et al., Ann. Rev. Nucl. & Part. Sci. **51**, 295 (2001).
- Future work: Luminosities in different ν-flavors.

Time to Instability

- Observation of metastability would signal the presence of exotica
- Frequency of galactic SN: 1 per (30-50) yr