Nuclear & Particle Physics of Compact Stars

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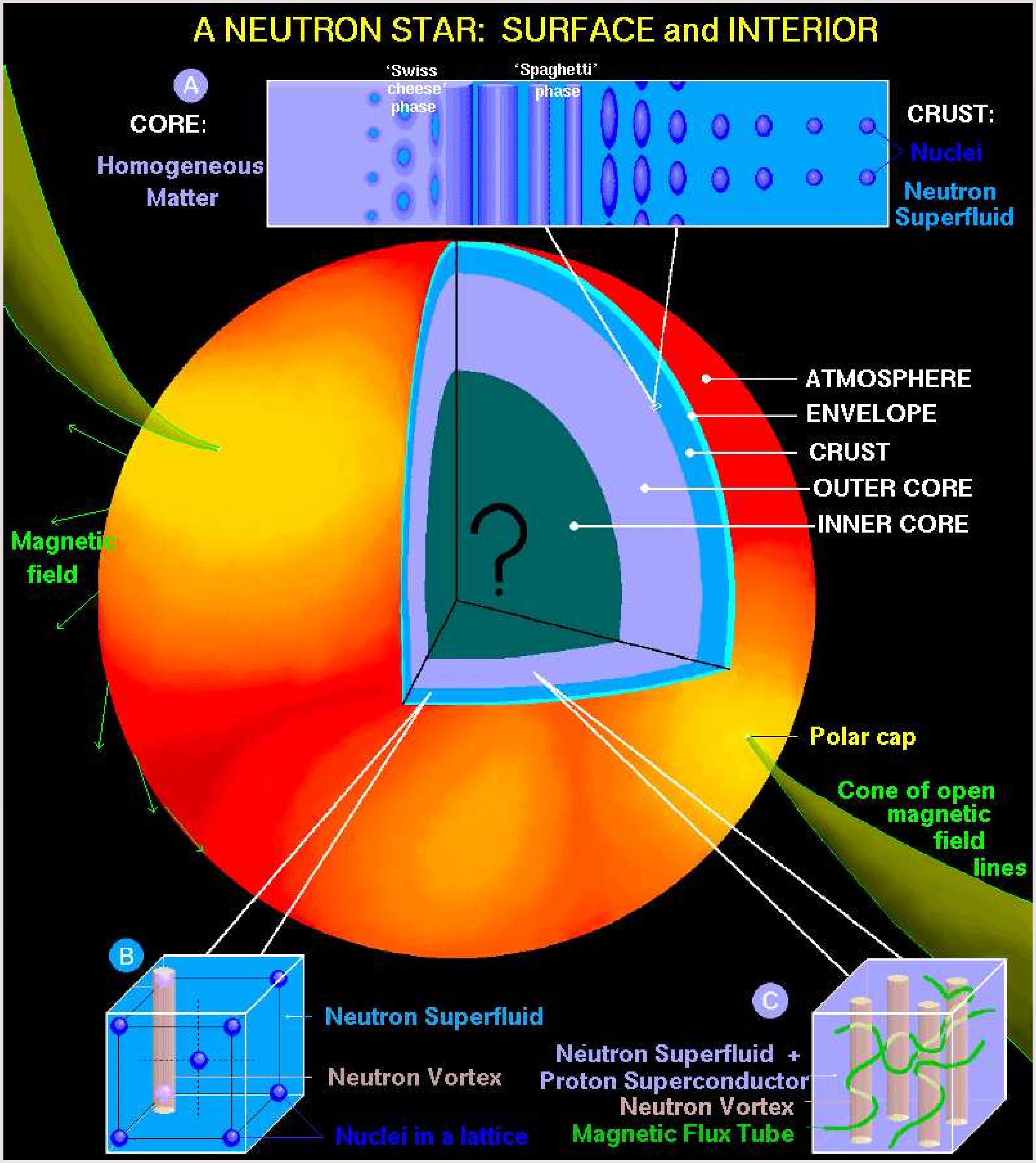
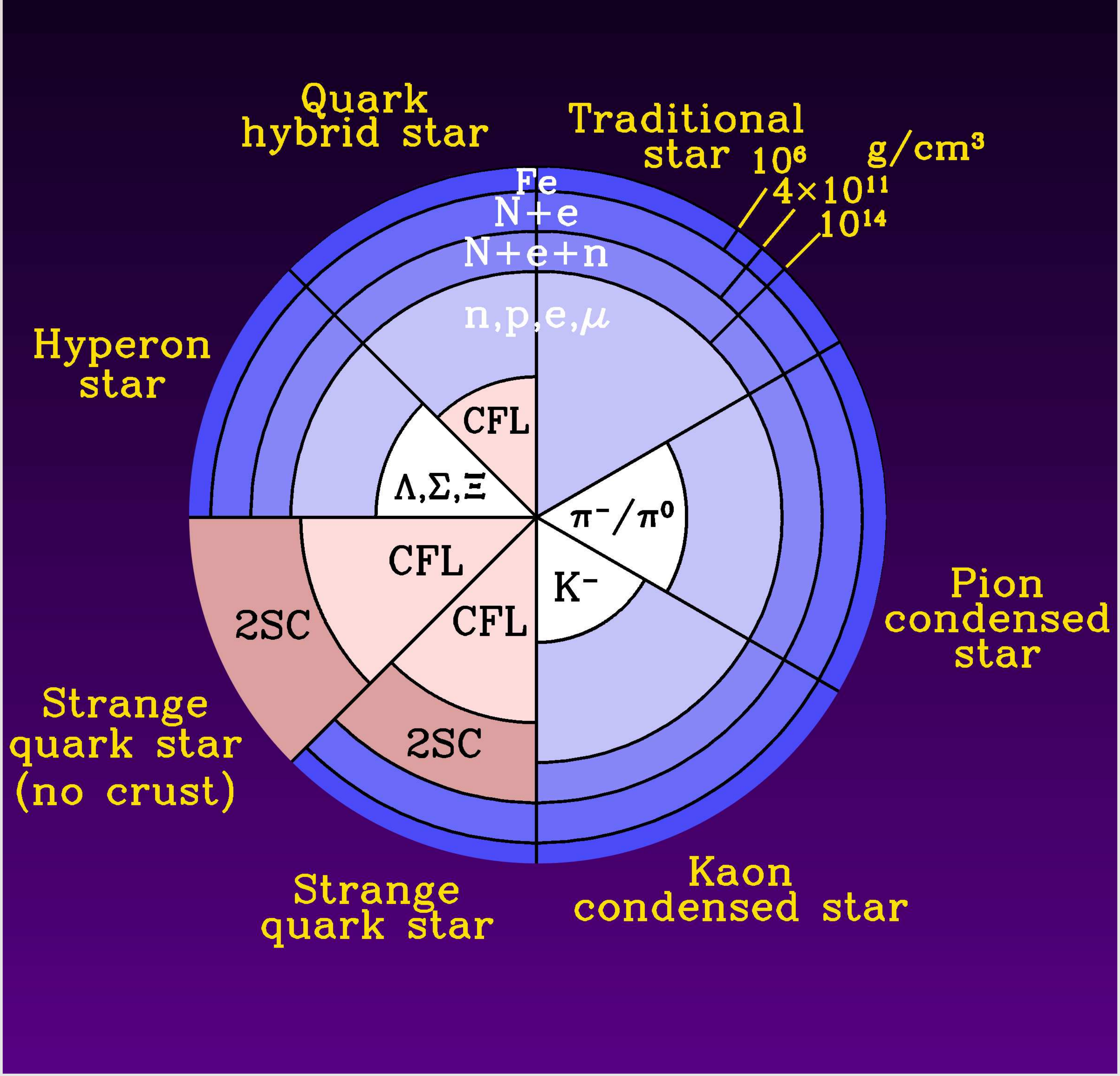


- $M \sim (1-2) M_\odot$
  $M_\odot \simeq 2 \times 10^{33}$ g.
- $R \sim (8-16)$ km
- $\rho > 10^{15}$ g cm^{-3}
- $B_s = 10^9 - 10^{15}$ G.
- Tallest mountain?
- Atmospheric height?

Lattimer & Prakash , Science **304**, 536 (2004).



Quark
hybrid star
Traditional
star
10⁶
g/cm³
4×10¹¹
10¹⁴
Fe
N+e
N+e+n
n,p,e,μ
Hyperon
star
CFL
Λ,Σ,Ξ
π⁻/π⁰
K⁻
CFL
CFL
2SC
2SC
Pion
condensed
star
Strange
quark star
(no crust)
Strange
quark star
Kaon
condensed star

Traits of Compact Objects

Object	Mass ($M_\odot$)	Radius (R)	Mean Density ($\mathrm{g\ cm^{-3}}$)
Sun	$M_\odot$	$R_\odot$	~ 1
White Dwarf	$\lesssim M_\odot$	$\sim 10^{-2} R_\odot$	$\lesssim 10^7$
Neutron Star	$1 - 2\ M_\odot$	$\sim 10^{-5} R_\odot$	$\lesssim 10^{15}$
Black Hole	Arbitrary	$2GM/c^2$	$\sim M/R^3$

- $M_\odot \simeq 2 \times 10^{33}$ g , $R_\odot \simeq 7 \times 10^5$ km ,
- $M_\odot c^2 \simeq 1.8 \times 10^{54}$ erg ,
- $2GM_\odot/c^2 \simeq 2.95$ km , $R_\oplus \simeq 6.4 \times 10^3$ km .

The Depth of Gravity's Well

How much work is needed to raise a unit mass of matter through an infinite height?

$$W = \int_R^\infty f\,dr = \int_R^\infty \frac{GM}{r^2}\,dr = \frac{GM}{R}$$

Object	Surface Potential GM/Rc^2
Sun	$\sim 10^{-6}$
White Dwarf	$\sim 10^{-4}$
Neutron Star	$\sim 10^{-1}$
Black Hole	~ 1

G=6.67×10^{-11} m^3 kg^{-1} s^{-2}

The Strength of Gravity

What is the kinetic energy that can surmount the gravitational energy?

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \Rightarrow v = \sqrt{\frac{2GM}{R}}$$

Object	Escape Speed (in km/sec) estimated by $\sqrt{2GM/R}$
Moon	2.4
Earth	11.2
Jupiter	61
Sun	620
White Dwarf	5000
Neutron Star	130,000
Black Hole	3×10^5 (c)

Gravitational Binding Energies

- What is the Binding Energy (B.E.) of our Earth if it had a uniform density distribution?

$$\begin{aligned} \text{B.E.} &= \frac{3}{5}\frac{GM_{\oplus}^2}{R_{\oplus}} = 2.4 \times 10^{32} \text{ joules} \\ &= 6.6 \times 10^{25} \text{ kwh} \end{aligned}$$

Object	Binding energy (in joules) estimated by $3GM^2/5R$
Moon	1.2×10^{29}
Earth	2.4×10^{32}
Sun	2.4×10^{41}
White Dwarf	2.4×10^{43}
Neutron Star	10^{46}
Our Galaxy	5×10^{52}

Neutron Star Curiosities

What is the tallest mountain that can be supported on a neutron star?

$$h < h_{max} \sim \frac{E_{liq}}{A m_p g}$$

A: Molecular weight of the planetary material
g: Surface gravity
E_{liq}: Liquefaction energy per molecule

- For Earth, $h_{max} \simeq 10$ km
- For a neutron star, $h_{max} \simeq 1$ cm

Neutron Star Curiosities

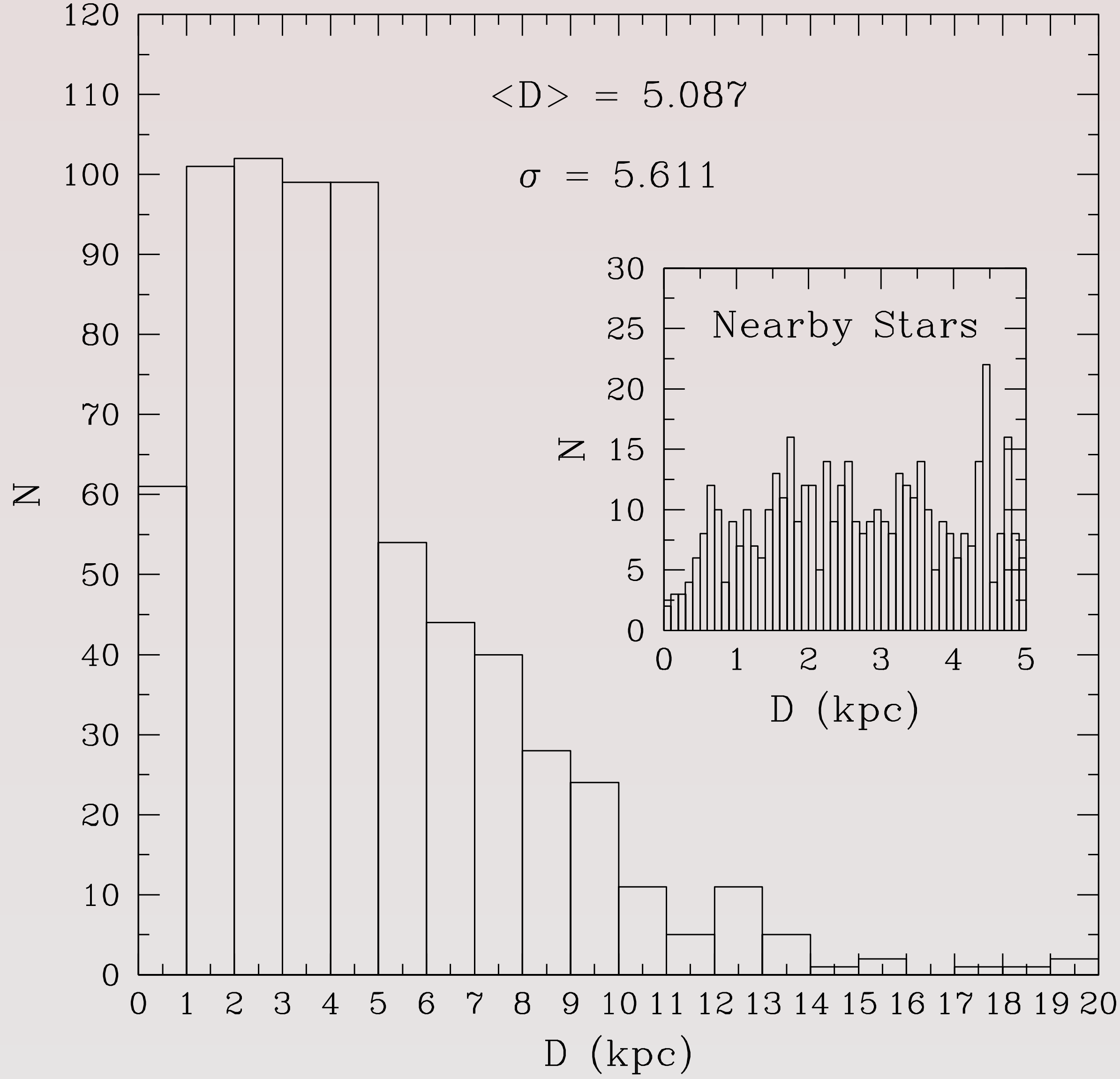
What is the height of the atmosphere of a neutron star?

$$h = \frac{RT}{\mu g}$$

R: Gas constant
T: Temperature
μ: The mean molecular weight
g: Surface gravity

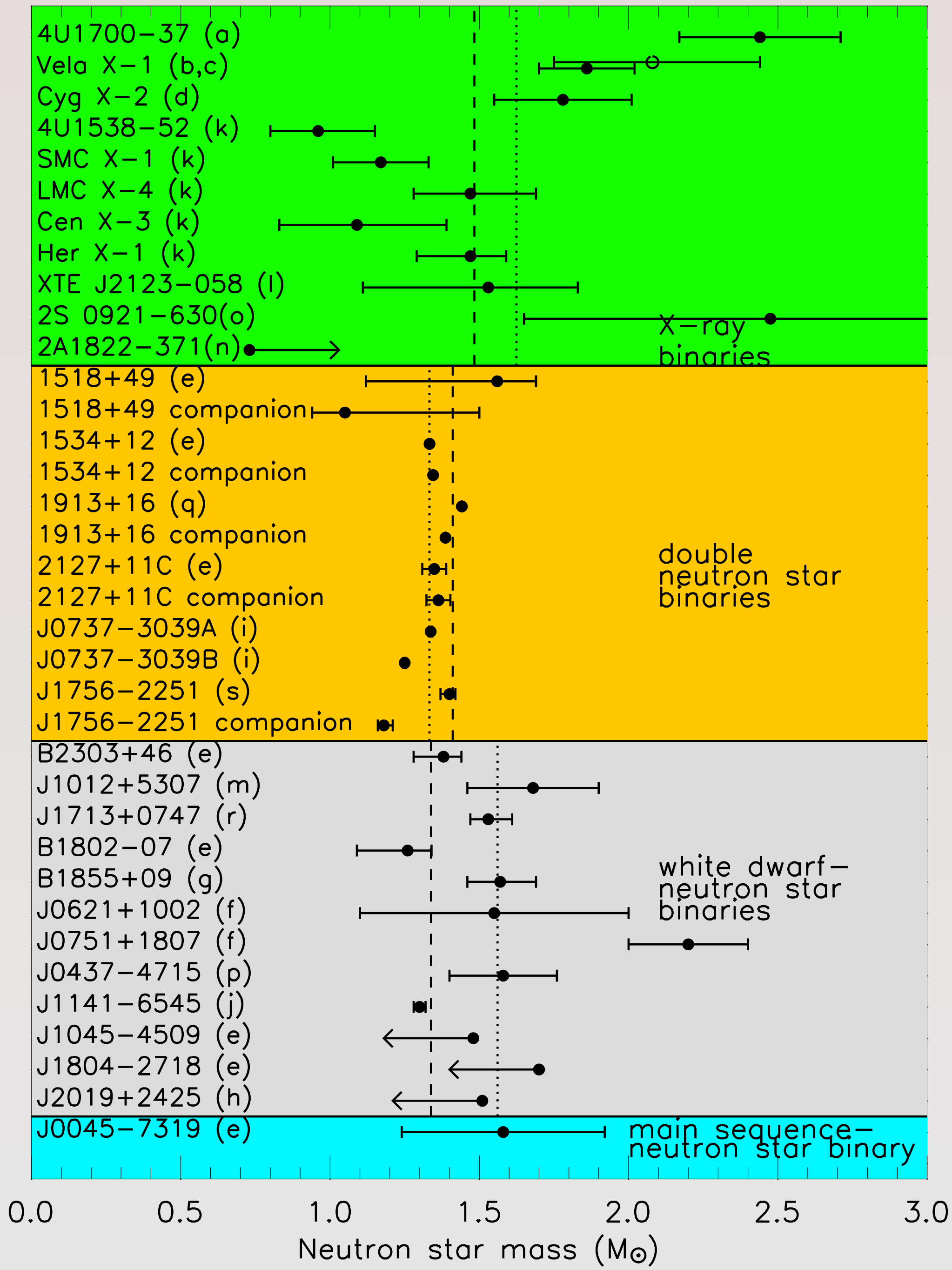
- For Earth, $h = 8$ km
- For a neutron star, $h = 1$ mm

Where Are They?



$1\ \text{pc} \simeq 3.1 \times 10^{16}\ \text{m}$

Measured Neutron Star Masses



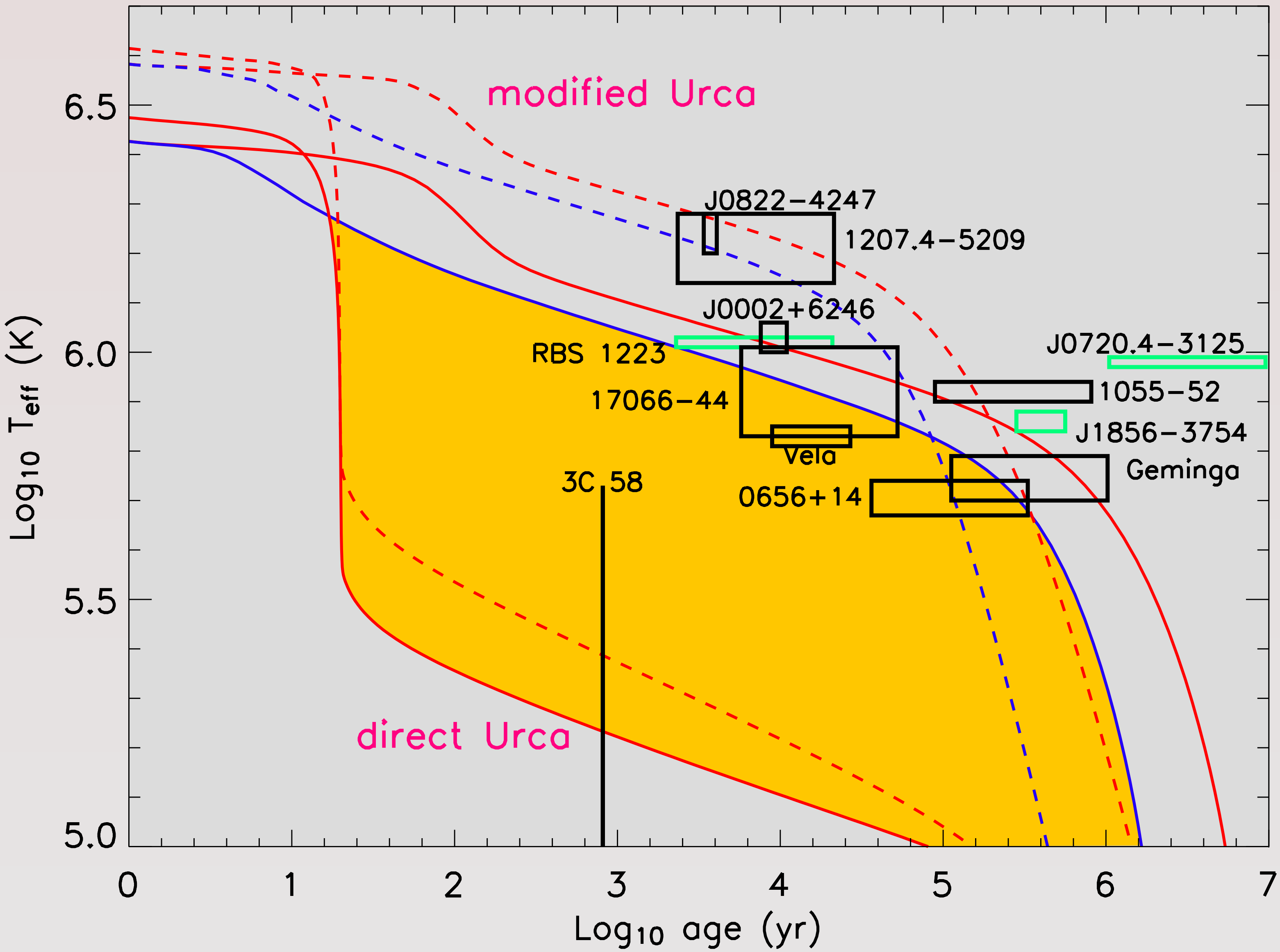
- Mean & weighted means in $M_\odot$
- X-ray binaries: 1.62 & 1.48
- Double NS binaries: 1.33 & 1.41
- WD & NS binaries: 1.56 & 1.34
- Lattimer & Prakash, PRL 94 (2005) 111101

Who're they? & why so happy?

Neutron star radius measurements

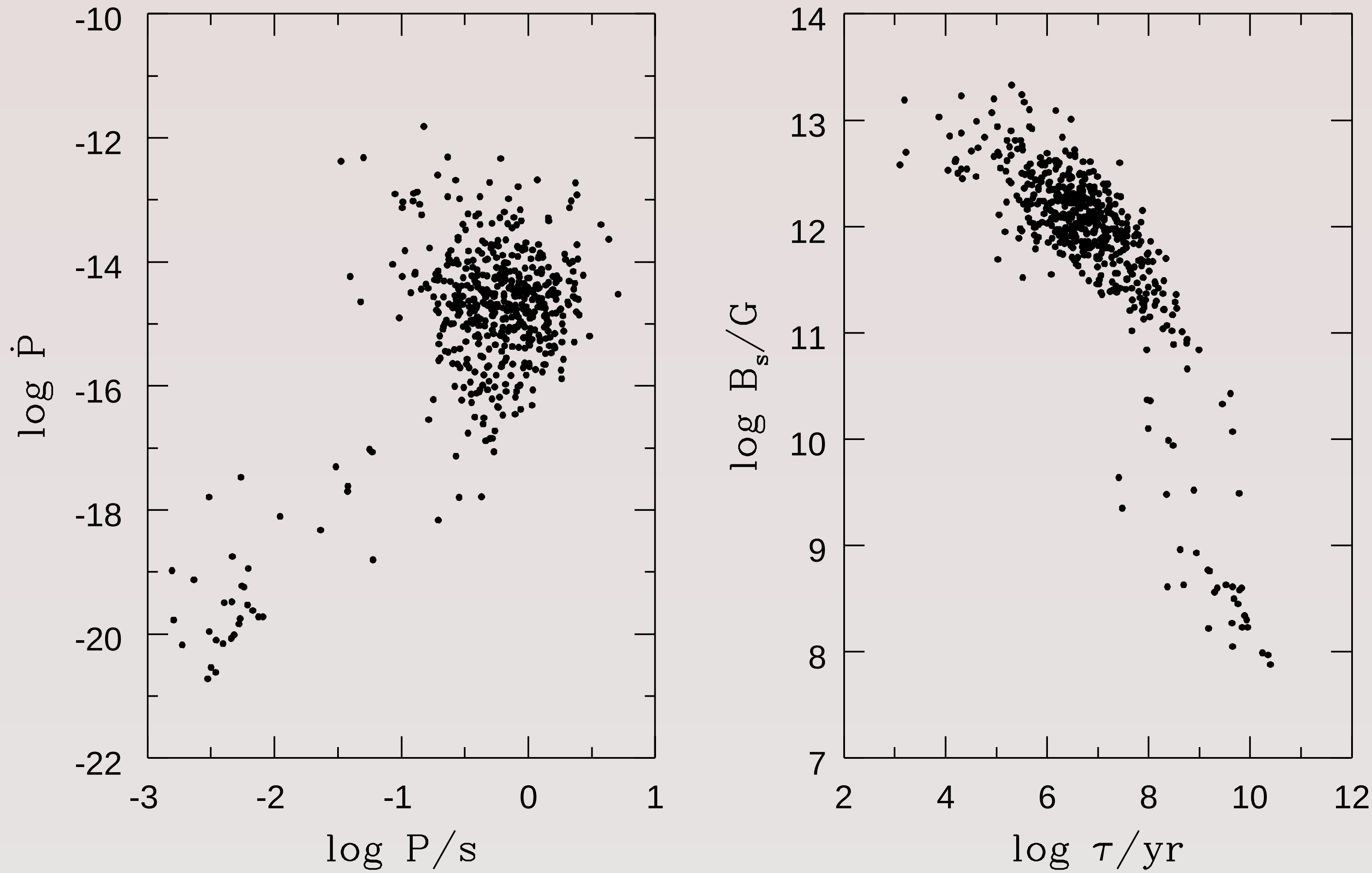
Object	R (km)	D (kpc)	Ref
Omega Cen Chandra	13.5 ± 2.1	$5.36 \pm 6\%$	Rutledge et al. ('02)
Omega Cen (XMM)	13.6 ± 0.3	$5.36 \pm 6\%$	Gendre et al. ('02)
M13 (XMM)	12.6 ± 0.4	$7.80 \pm 2\%$	Gendre et al. ('02)
47 Tuc X7 (Chandra)	$14.5^{+1.6}_{-1.4}$ $(1.4\ M_\odot)$	$5.13 \pm 4\%$	Rybicki et al. ('05)
M28 (Chandra)	$14.5^{+6.9}_{-3.8}$	$5.5 \pm 10\%$	Becker et al. ('03)
EXO 0748-676 (Chandra)	13.8 ± 1.8 $(2.10 \pm 0.28\ M_\odot)$	9.2 ± 1.0	Ozel ('06)

Inferred Surface Temperatures



Lattimer & Prakash , Science **304**, 536 (2004).

Periods & Magnetic Fields



$$B_s = \left(\frac{3c^3 I}{8\pi^2 R^6 \sin\alpha} P\dot{P} \right)^{1/2}$$

Physics & Astrophysics of Neutron Stars

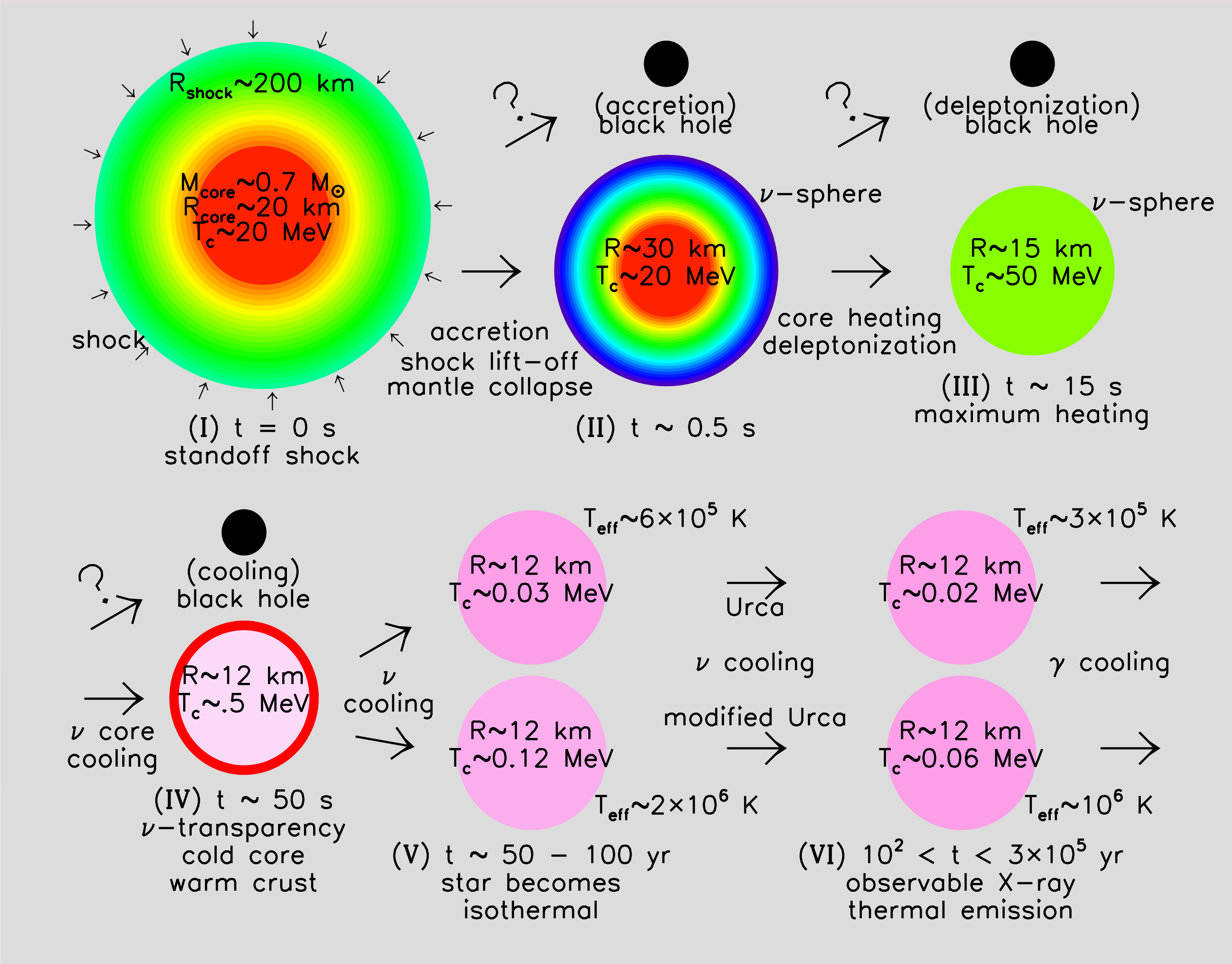
- Cores of neutron stars may contain hyperons, Bose condensates, or quarks (*Exotica*)
- *Can* observations of $M, \ R$ & B.E (composition & structure) & $P, \ \dot{P}, \ T_S$ & B etc., (evolution) reveal *Exotica* ?
- Neutron stars implicated in x-ray & γ-ray bursters, mergers with other neutron stars & black holes, etc.
- **Observational Programs :**

  SK, SNO, LVD's, AMANDA ... (ν's)
  HST, CHANDRA, XMM, ASTROE ... (γ's)
  LIGO, VIRGO, GEO600, TAMA ... (Gravity Waves)

Connections:
Atomic, Cond. Matter, Nucl. & Part., Grav. Physics

- Theory : Many-body theory of strongly interacting systems, Dynamical response (ν- & γ- propagation & emissivities)
- Experiment : h, e^- and ν- scattering experiments on nuclei, masses of neutron-rich nuclei, heavy-ion reactions, etc.

How Neutron Stars are Formed



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Equations of Stellar Structure-I

• In hydrostatic equilibrium, the structure of a spherically symmetric neutron star from the Tolman-Oppenheimer-Volkov (TOV) equations:

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

$$\frac{dP(r)}{dr} = -\frac{GM(r)\epsilon(r)}{c^2 r^2} \frac{\left[1 + \frac{P(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right]}{\left[1 - \frac{2GM(r)}{c^2 r}\right]}$$

• G := Gravitational constant
• P := Pressure
• ϵ := Energy density
• $M(r)$:= Enclosed gravitational mass
• $R_s = 2GM/c^2$:= Schwarzschild radius

Equations of Stellar Structure-II

- The gravitational and baryon masses of the star:

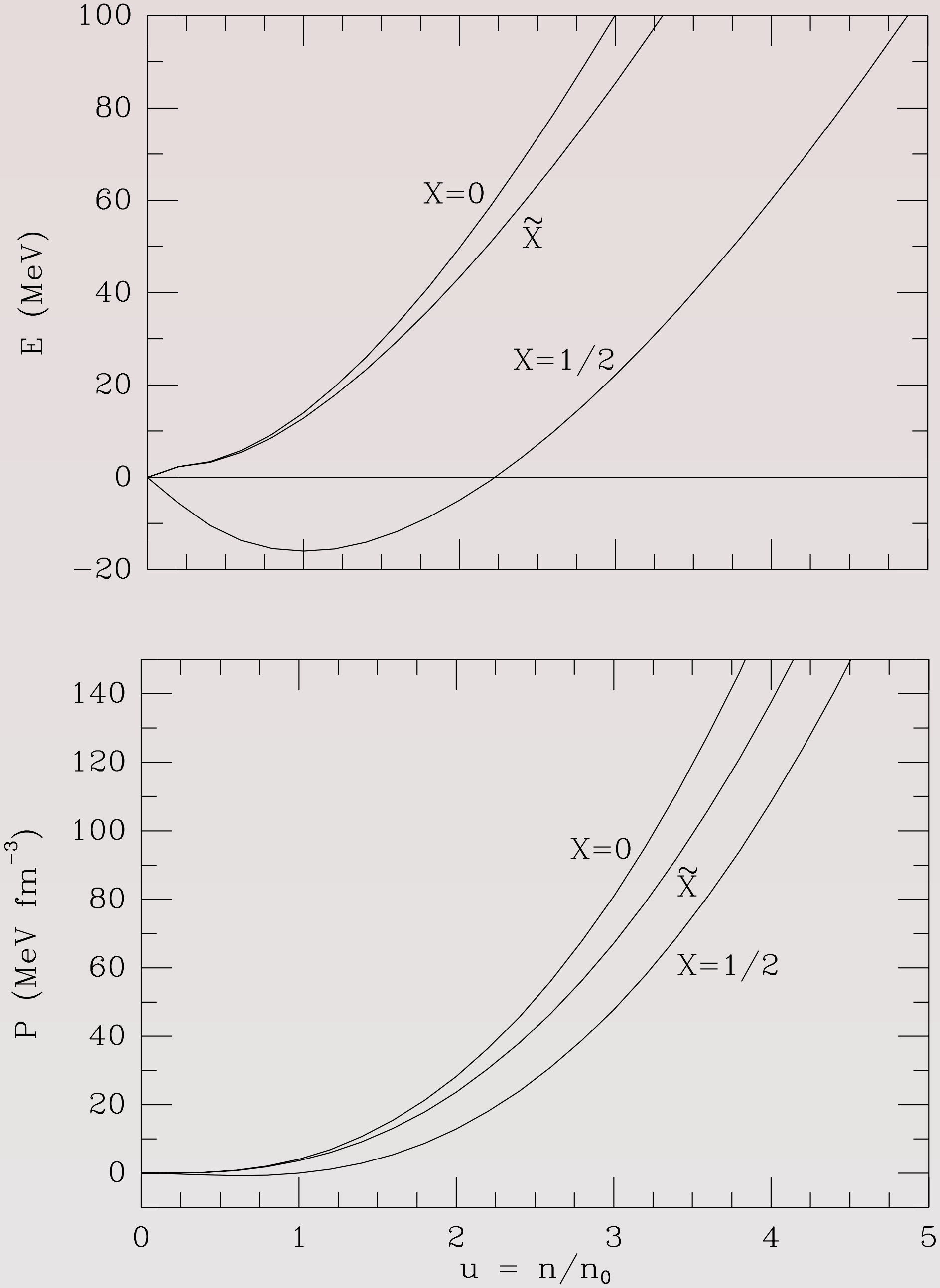
$$
\begin{aligned}
M_G c^2 &= \int_0^R dr\, 4\pi r^2\, \epsilon(r) \\
M_A c^2 &= m_A \int_0^R dr\, 4\pi r^2\, \frac{n(r)}{\left[1 - \frac{2GM(r)}{c^2 r}\right]^{1/2}}
\end{aligned}
$$

- m_A := Baryonic mass
- $n(r)$:= Baryon number density
- The binding energy of the star $B.E. = (M_A - M_G)c^2$.

To determine star structure :

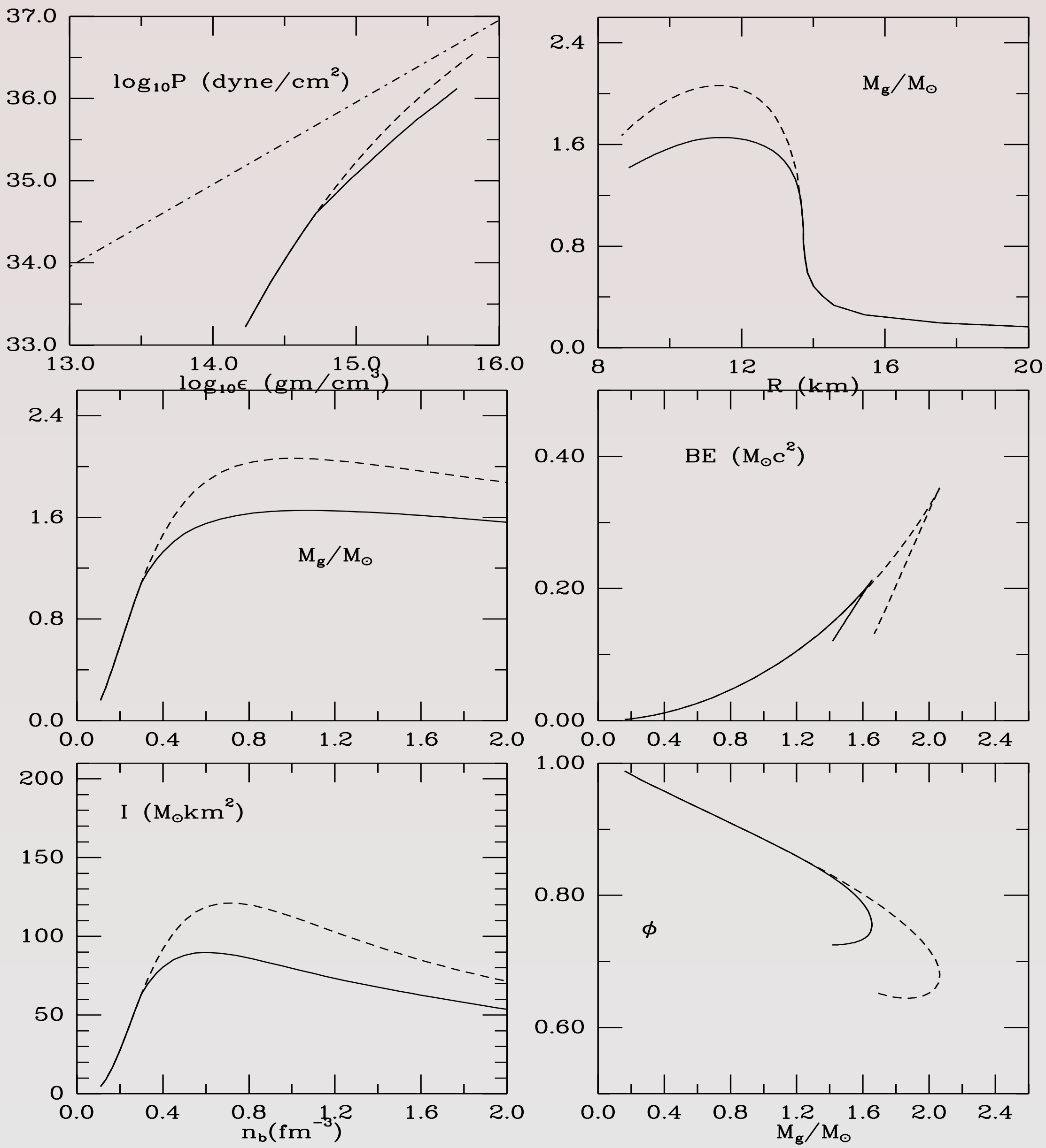
- Specify equation of state, $P = P(\epsilon)$
- Choose a central pressure $P_c = P(\epsilon_c)$ at $r = 0$
- Integrate the 2 DE's out to surface $r = R$, where $P(r = R) = 0$.

Nucleonic Equation of State



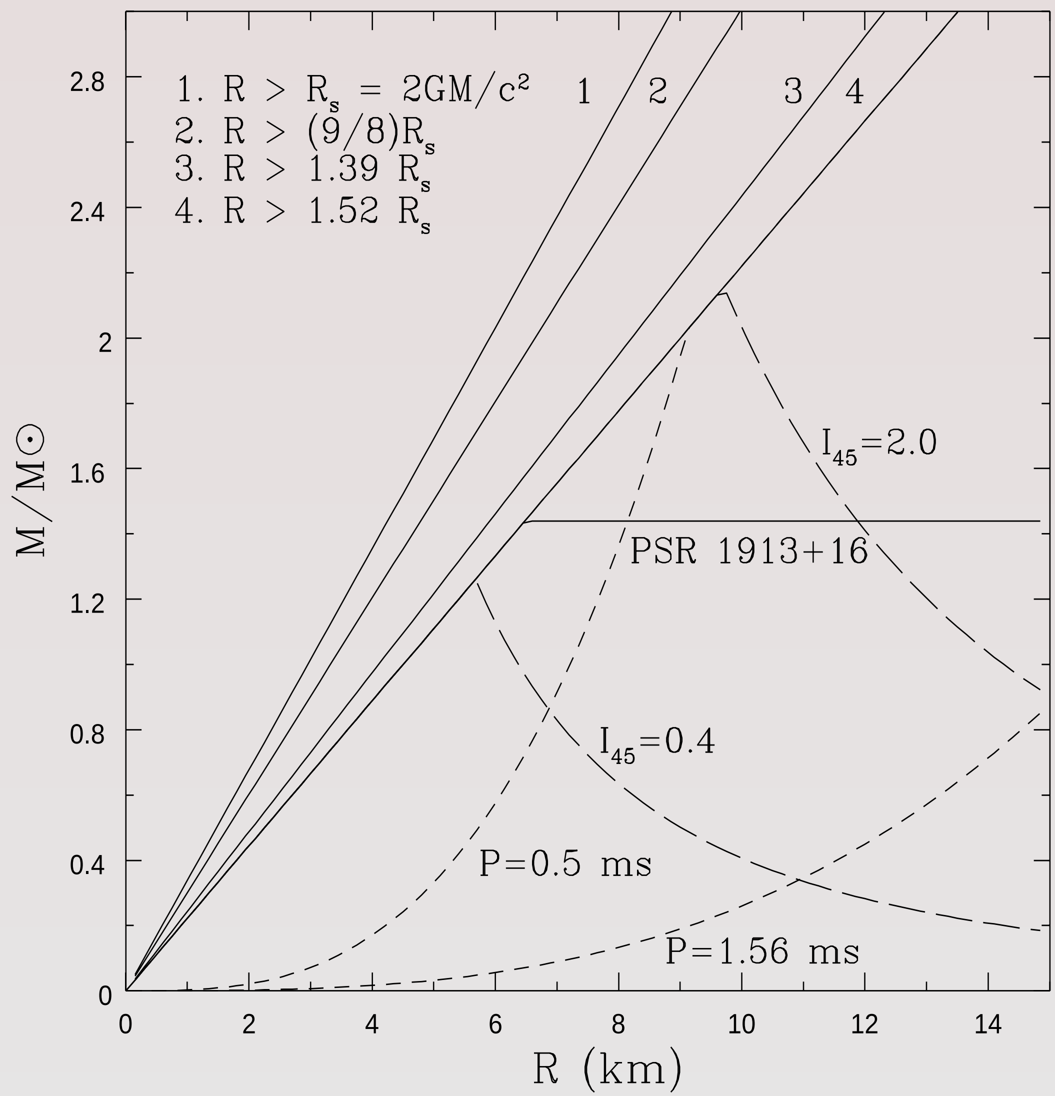
- Energy (E) & Pressure (P) vs. scaled density ($u = n/n_0$).
- Nuclear matter equilibrium density $n_0 = 0.16$ fm^{-3}.
- Proton fraction $x = n_p/(n_p + n_n)$.
- Nuclear matter : $x = 1/2$.
- Neutron matter : $x = 0$.
- Stellar matter in $\beta-$ equilibrium : $x = \tilde{x}$.

Results of Star Structure



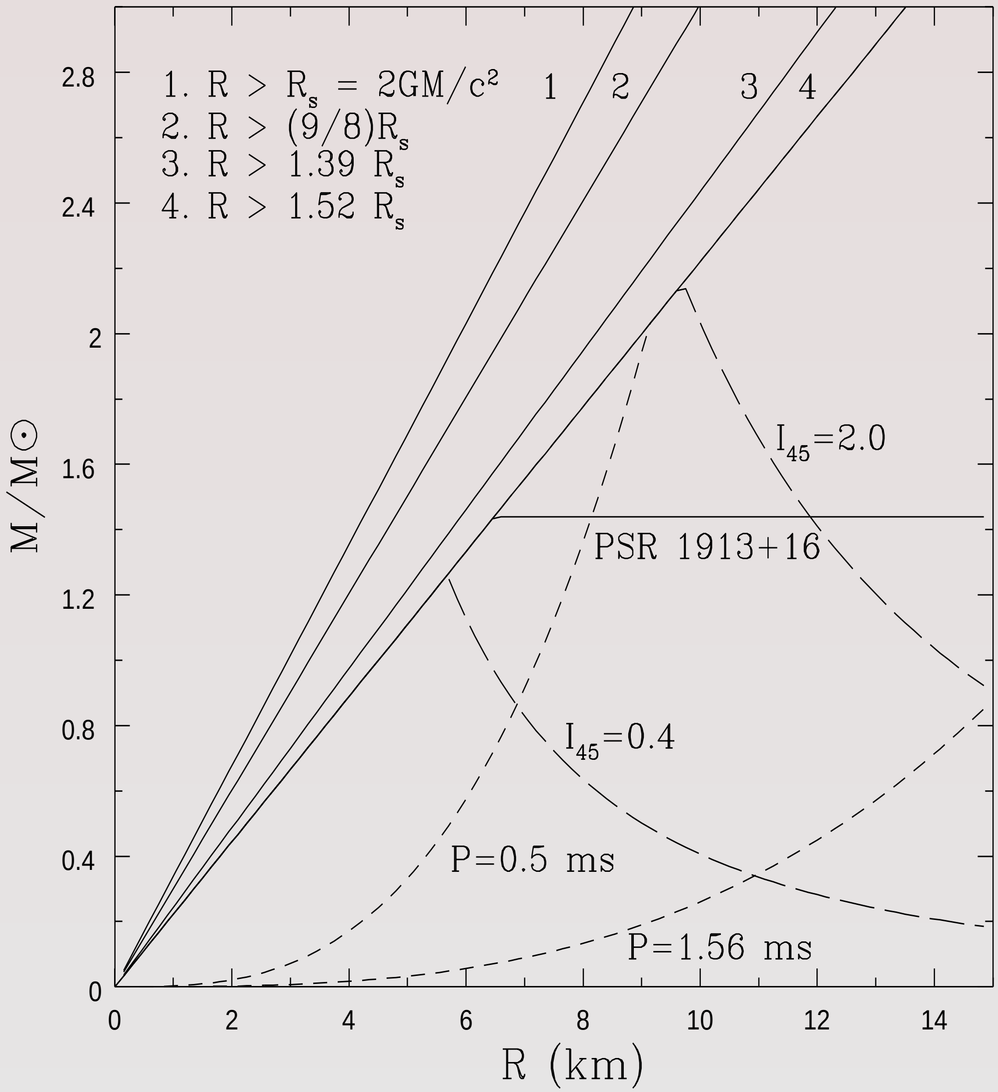
- Stellar properties for soft & stiff (by comparison) EOS's.
- Causal limit : $P = \epsilon$.
- M_g : Gravitational mass
- R : Radius
- BE : Binding energy
- n_b : Central density
- I : Moment of inertia
- ϕ : Surface red shift , $e^{\phi/c^2} = (1 - 2GM/c^2R)^{-1/2}$.

Constraints on the EOS-I



- $R > R_s = 2GM/c^2 \Rightarrow$ $M/M_\odot \geq R/R_{s\odot}$; $R_{s\odot} = 2GM_\odot/c^2$ $\simeq 2.95$ km .
- $P_c < \infty$ $\Rightarrow R > (9/8)R_s$ $\Rightarrow M/M_\odot \geq$ $(8/9)R/R_{s\odot}$.
- Sound speed c_s : $c_s = (dP/d\epsilon)^{1/2} \leq c$ $\Rightarrow R > 1.39R_s$ $\Rightarrow M/M_\odot \geq$ $R/(1.39R_{S\odot})$.
- If $P = \epsilon$ above $n_t \simeq 2n_0$, $R > 1.52R_s \Rightarrow$ $M/M_\odot R/(1.52R_{s\odot})$.

Constraints on the EOS-II



- $M_{max} \geq M_{obs}$;
  In PSR 1913+16,
  $M_{obs} = 1.44\ \mathrm{M}_\odot$.
- In PSR 1957+20,
  $P_K = 1.56$ ms :
  $\Omega_K \simeq 7.7 \times 10^3$
  $\left(\frac{M_{max}}{\mathrm{M}_\odot}\right)^{1/2} \left(\frac{R_{max}}{10\ \mathrm{km}}\right)^{-3/2}\ \mathrm{s}^{-1}$
- Mom. of Inertia I :
  $I_{max} = 0.6{\times}10^{45}\ \mathrm{g\ cm}^2$
  $\left(\frac{M_{max}}{\mathrm{M}_\odot}\right) \left(\frac{R_{max}}{10\ \mathrm{km}}\right)^2$
  $f(M_{max}, R_{max})$
- In SN 1987A
  $B.E. \simeq (1-2)$
  $\times 10^{53}$ ergs.

Composition of Dense Stellar Matter

- Crustal Surface :
  electrons, nuclei, dripped neutrons, $\cdots$ set in a lattice
  new phases with lasagna, sphagetti, $\cdots$ like structures
- Liquid (Solid?) Core :
  $n, p, \Delta, \cdots$ leptons: $e^{\pm}, \mu^{\pm}, \nu_e' s, \nu_\mu' s$
  $\Lambda, \Sigma, \Xi, \cdots$
  $K^-, \pi^-, \cdots$ condensates
  $u, d, s, \cdots$ quarks
- Constraints :
  1. $n_b = n_n + n_p + n_\Lambda + \cdots$: baryon # conservation
  2. $n_p + n_{\Sigma^+} + \cdots = n_e + n_\mu$: charge neutrality
  3. $\mu_i = b_i \mu_n - q_i \mu_\ell$: energy conservation

$$\Rightarrow$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \quad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e$$

$$\Rightarrow$$

$$\mu_{K^-} = \mu_e = \mu_\mu = \mu_n - \mu_p$$

$$\Rightarrow$$

$$\mu_d = \mu_u + \mu_e = \mu_s = (\mu_n + \mu_e)/3$$

$$\mu_u = (\mu_n - 2\mu_e)/3$$