## QCD Phenomenology and Nucleon Structure



Stan Brodsky, SLAC

Lecture V



#### National Nuclear Physics Summer School



QCD Phenomenology

#### QCD Lagrangían



QCD: 
$$N_C = 3$$
 Quarks:  $3_C$  Gluons:  $8_C$ .  
 $\alpha_s = \frac{g^2}{4\pi}$  is dimensionless

Classical Lagrangian is scale invariant for massless quarks

If 
$$\beta = \frac{d\alpha_s(Q^2)}{d\log Q^2} = 0$$
 then QCD is invariant under conformal trans-  
formations:

Parisi

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## Conformal symmetry: Template for QCD

- Initial approximation to PQCD; then correct for non-zero beta function and quark masses
- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation
- Arguments for Infrared fixed-point for  $\alpha_s$

Alhofer, et al.

- Effective Charges: analytic at quark mass thresholds, finite at small momenta
- Eigensolutions of Evolution Equation of distribution amplitudes



QCD Phenomenology

### The Renormalization Scale Problem

## $\rho = C_1 \alpha_s(\mu_R) + C_2 \alpha_s^2(\mu R) + C_3 \alpha_s^3(\mu R) + \cdots$

# How does one set the renormalization scale $\mu_R$ ?



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### Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \to ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

- No renormalization scale ambiguity!
- Two separate physical scales.
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one must sum an infinite number of graphs -- but then recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds



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 $e^+e^- \rightarrow \mu^+\mu^-$ 



 $\mu_B^2 = s$ 

Scale of  $\alpha(\mu_r)$  unique !

 $M \propto \alpha(s)$ 

### The QED Effective Charge

- Complex
- Analytic through mass thresholds
- Distinguishes between timelike and spacelike momenta

Analyticity essential!



QCD Phenomenology

## The Renormalization Scale Problem

- M. Binger, sjb
- No renormalization scale ambiguity in QED
- Gell Mann-Low-Dyson QED Coupling defined from physical observable;
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion



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## Lessons from QED : Summary

- Effective couplings are complex analytic functions with the correct threshold structure expected from unitarity
- Multiple "renormalization" scales appear
- The scales are unambiguous since they are physical kinematic invariants
- Optimal improvement of perturbation theory



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#### **BLM Scale Setting**

Use  $n_f$  dependence at NLO to identify  $A_{VP}$ 

by

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q^*) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} C_1^* + \cdots \right],$$

where

Conformal Coefficient

 $Q^* = Q \exp(3A_{VP})$ ,  $C_1^* = \frac{33}{2}A_{VP} + B$ .

The term  $33A_{VP}/2$  in  $C_1^*$  serves to remove that part of the constant *B* which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by  $\beta_0 = 11 - \frac{2}{3}n_f$ . Use s

Use skeleton expansion Gardi, Rathsman, sjb

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$$V(Q^{2}) = -\frac{C_{F}4\pi\alpha_{\overline{\mathrm{MS}}}(Q)}{Q^{2}} \left[ 1 + \frac{\alpha_{\overline{\mathrm{MS}}}}{\pi} (\frac{5}{12}\beta_{0} - 2) + \cdots \right]$$
(19a)  
$$\rightarrow -\frac{C_{F}4\pi\alpha_{\overline{\mathrm{MS}}}(Q^{*})}{Q^{2}} \left[ 1 - \frac{\alpha_{\overline{\mathrm{MS}}}(Q^{*})}{\pi} 2 + \cdots \right],$$
(19b)

where  $Q^* = e^{-5/6}$ ,  $Q \cong 0.43Q$ . This result shows that the effective scale of the  $\overline{\text{MS}}$  scheme should generally be about half of the true momentum transfer occurring in the interaction. In parallel to QED, the effective potential  $V(Q^2)$  gives a particularly intuitive scheme for defining the QCD coupling constant

$$V(Q^2) \equiv -\frac{4\pi C_F \alpha_v(Q)}{Q^2} \tag{20}$$



QCD Phenomenology

## Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

#### Lepage, Mackenzie, sjb

#### Phys.Rev.D28:228,1983

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q\* sets the number of active flavors
- Only n<sub>f</sub> dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q\* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- Resulting series identical to conformal series!
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants



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Deep-inelastic scattering. The moments of the nonsinglet structure function  $F_2(x,Q^2)$  obey the evolution equation

$$Q^{2} \frac{d}{dQ^{2}} \ln M_{n}(Q^{2})$$

$$= -\frac{\gamma_{n}^{(0)}}{8\pi} \alpha_{\overline{\mathrm{MS}}}(Q) \left[ 1 + \frac{\alpha_{\overline{\mathrm{MS}}}}{4\pi} \frac{2\beta_{0}\beta_{n} + \gamma_{n}^{(1)}}{\gamma_{n}^{(0)}} + \cdots \right]$$

$$\to -\frac{\gamma_{n}^{(0)}}{8\pi} \alpha_{\overline{\mathrm{MS}}}(Q_{n}^{*}) \left[ 1 - \frac{\alpha_{\overline{\mathrm{MS}}}(Q_{n}^{*})}{\pi} C_{n} + \cdots \right],$$

where, for example,

$$Q_2^* = 0.48Q, \quad C_2 = 0.27,$$
  
 $Q_{10}^* = 0.21Q, \quad C_{10} = 1.1.$ 

For *n* very large, the effective scale here becomes  $Q_n^* \sim Q/\sqrt{n}$ 

#### BLM scales for DIS moments



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### Three-Jet Rate

Kramer & Lampe

The scale  $\mu/\sqrt{s}$  according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and  $\sqrt{y}$  (dotted) procedures for the three-jet rate in  $e^+e^-$  annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of  $\mu$  as the jet invariant mass  $\mathcal{M} < \sqrt{(ys)}$  decreases.

#### Rathsman

#### Other Jet Observables:

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$$V(Q^{2}) = -\frac{C_{F}4\pi\alpha_{\overline{\mathrm{MS}}}(Q)}{Q^{2}} \left[ 1 + \frac{\alpha_{\overline{\mathrm{MS}}}}{\pi} (\frac{5}{12}\beta_{0} - 2) + \cdots \right]$$
$$\rightarrow -\frac{C_{F}4\pi\alpha_{\overline{\mathrm{MS}}}(Q^{*})}{Q^{2}} \left[ 1 - \frac{\alpha_{\overline{\mathrm{MS}}}(Q^{*})}{\pi} 2 + \cdots \right],$$

where  $Q^* = e^{-5/6} Q \cong 0.43Q$ . This result shows that the effective scale of the  $\overline{\text{MS}}$  scheme should generally be about half of the true momentum transfer occurring in the interaction. In parallel to QED, the effective potential  $V(Q^2)$  gives a particularly intuitive scheme for defining the QCD coupling constant

$$V(Q^2) \equiv -\frac{4\pi C_F \alpha_v(Q)}{Q^2}$$

Application of BLM to Multi-Scale Threshold Production Hoang, Kuhn, Tuebner, SJB

Phys.Lett.B359:355-361,1995

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Perturbative Qcd Effects In Heavy Meson Decays. <u>Adam Szczepaniak</u>, <u>Ernest M. Henley</u> (<u>Washington U., Seattle</u>), <u>Stanley J. Brodsky</u> (<u>SLAC</u>). SLAC-PUB-5228, Mar 1990. 14pp. Published in Phys.Lett.B243:287-292,1990





QCD Phenomenology

## Features of BLM Scale Setting

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- Conformal series preserved
- BLM Scale Q\* sets the number of active flavors
- Correct analytic dependence in the quark mass
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- Result is scheme independent: Q\* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit!



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 $\lim N_C \to 0$  at fixed  $\alpha = C_F \alpha_s, n_\ell = n_F/C_F$ 

### $QCD \rightarrow Abelian Gauge Theory$

Analytic Feature of SU(Nc) Gauge Theory

Huet, sjb



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## Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation



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$$\begin{split} \frac{\alpha_R(Q)}{\pi} &= \frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi}\right)^2 \left[ \left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f \right] \\ &\quad + \left(\frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2 \right. \\ &\quad + \left[ \left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F \right] f \\ &\quad + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2} \right\}. \end{split}$$

$$\begin{split} \frac{\alpha_{g_1}(Q)}{\pi} &= \frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right] \\ &+ \left(\frac{\alpha_{\overline{\mathrm{MS}}}(Q)}{\pi}\right)^3 \left\{ \left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right)C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right)C_A C_F + \frac{1}{32}C_F^2 \right. \\ &+ \left[ \left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right)C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right)C_F \right]f + \frac{115}{648}f^2 \right\}. \end{split}$$

#### Apply BLM, Eliminate MSbar, Find Amazing Simplification



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$$\int_0^1 dx \left[ g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi}\right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi}\right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

add Light-by-Light

Lu, Kataev, Gabadadze, Sjb

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### Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

# Conformal relation true to all orders in perturbation theory



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Transtiut property - Renormalization Group  $A \Rightarrow C \Rightarrow B$  $A \Rightarrow B$ same es inder of C Relation between observable ACB Independent y choice of G Independent y Scheme or Trepretical convertion!

PMS violates transitivity

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### Leading Order Commensurate Scales



Translate between schemes at LO



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### Production of four heavy-quark jets



#### Conventional renormalization scale-setting method:

- Guess arbitrary renormalization scale and take arbitrary range. Wrong for QED and Precision Electroweak.
- Prediction depends on choice of renormalization scheme
- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms
- Conventional procedure has no scientific basis.
- FAC and PMS give unphysical results; have no validity.
- Renormalization scale not arbitrary! Sets # active flavors



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## **The Pinch Technique**

(Cornwall, Papavassiliou)



## Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification



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### **Analyticity and Mass Thresholds**

 $M\!S$  does not have automatic decoupling of heavy particles



Must define a set of schemes in each desert region and match  $\alpha_s^{(f)}(M_O) = \alpha_s^{(f+1)}(M_O)$ 

- The coupling has discontinuous derivative at the matching point
- At higher orders the coupling itself becomes discontinuous!
- Does not distinguish between spacelike and timelike momenta

"AN ANALYTIC EXTENSION OF THE MS-BAR RENORMALIZATION SCHEME" S. Brodsky, M. Gill, M. Melles, J. Rathsman. **Phys.Rev.D58:116006,1998** 



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## **Unification in Physical Schemes**

- Smooth analytic threshold behavior with automatic decoupling
- More directly reflects the unification of the forces
- Higher "unification" scale than usual



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### BLM and Non-Abelian QCD General Structure of the Three-Gluon Vertex



3 index tensor  $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$  built out of  $\mathcal{G}_{\mu\nu}$  and  $p_1, p_2, p_3$ with  $p_1 + p_2 + p_3 = 0$ 

14 basis tensors and form factors



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## The Gauge Invariant **Three Gluon Vertex**



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## Summary of Supersymmetric Relations

Massless	Massive
$F_G + 4F_Q + (10 - d)F_S = 0$	$F_{MG} + 4F_{MQ} + (9 - d)F_{MS} = 0$
$\Sigma_{QG}(F) \equiv \frac{d-2}{2}F_Q + F_G$	$\Sigma_{MQG}(F) \equiv \frac{d-1}{2}F_{MQ} + F_{MG}$
= simple	= simple



QCD Phenomenology

### Multi-scale Renormalization of the Three-Gluon Vertex



### **3 Scale Effective Charge**

$$\widetilde{\alpha}(a,b,c) \equiv \frac{\widetilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\widetilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi} \beta_0 \left( L(a,b,c) - \frac{1}{\varepsilon} + \cdots \right)$$
$$\frac{1}{\widetilde{\alpha}(a,b,c)} = \frac{1}{\widetilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi} \beta_0 \left[ L(a,b,c) - L(a_0,b_0,c_0) \right]$$

L(a,b,c) = 3-scale "log-like" function L(a,a,a) = log(a)

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### **3 Scale Effective Scale**

$$L(a,b,c) \equiv \log(Q_{eff}^2(a,b,c)) + i \operatorname{Im} L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\widetilde{\alpha}(a,b,c)} = \frac{1}{\widetilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi} \beta_0 [L(a,b,c) - L(a_0,b_0,c_0)]$$
$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\widetilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

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### **Properties of the Effective Scale**

$$\begin{aligned} Q_{eff}^{2}(a,b,c) &= Q_{eff}^{2}(-a,-b,-c) \\ Q_{eff}^{2}(\lambda a,\lambda b,\lambda c) &= |\lambda| Q_{eff}^{2}(a,b,c) \\ Q_{eff}^{2}(a,a,a) &= |a| \\ Q_{eff}^{2}(a,-a,-a) &\approx 5.54 |a| \\ Q_{eff}^{2}(a,a,c) &\approx 3.08 |c| \quad \text{for } |a| >> |c| \\ Q_{eff}^{2}(a,-a,c) &\approx 22.8 |c| \quad \text{for } |a| >> |c| \\ Q_{eff}^{2}(a,b,c) &\approx 22.8 \frac{|bc|}{|a|} \quad \text{for } |a| >> |b|, |c| \end{aligned}$$

Surprising dependence on Invariants

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## The Effective Scale



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## **Heavy Quark Hadro-production**



- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale

much larger cross section than  $\overline{MS}$  with scale  $\mu_R = M_{Q\overline{Q}}$  or  $M_Q$ 

• Future : repeat analysis using the full massdependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections



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### **Future Directions**

#### Gauge-invariant four gluon vertex



 $L_4(p_1, p_2, p_3, p_4)$ 

 $Q_{4\,eff}^2(p_1, p_2, p_3, p_4)$ 

Hundreds of form factors!



QCD Phenomenology

## **Summary and Future**

 Multi-scale analytic renormalization based on physical, gauge-invariant Green's functions

 Optimal improvement of perturbation theory with no scale-ambiguity since physical kinematic invariants are the arguments of the (multi-scale) couplings



QCD Phenomenology

#### Conventional renormalization scale-setting method:

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- Prediction depends on choice of renormalization scheme
- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms
- Conventional procedure has no scientific basis.
- FAC and PMS give unphysical results.
- Renormalization scale not arbitrary: Analytic constraint from flavor thresholds



QCD Phenomenology

## Use BLM!

- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit (N<sub>C</sub> = 0)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...



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## Factorization scale

 $\mu$ factorization  $\neq \mu$ renormalization

- Arbitrary separation of soft and hard physics
- Dependence on factorization scale not associated with beta function - present even in conformal theory
- Keep factorization scale separate from renormalization scale  $\frac{d\mathcal{O}}{d\mathcal{O}} = 0$
- $d\mu_{factorization}$  Residual dependence when one works in fixed order in perturbation theory.



QCD Phenomenology

## Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme



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### New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support 0 < x < 1.
- Quark Interchange dominant force at short distances



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## Essential to test QCD

- J-PARC
- GSI antiprotons
- 12 GeV Jlab
- BaBar/Belle: ISR, two-gamma, timelike DVCS
- RHIC/LHC Nuclear Collisions; LHCb
- electron-proton, electron-nucleus collisions



QCD Phenomenology

# Novel Tests of QCD at GSI

Polarized antiproton Beam Secondary Beams

- Characteristic momentum scale of QCD: 300 MeV
- Many Tests of AdS/CFT predictions possible
- Exclusive channels: Conformal scaling laws, quark-interchange
- pp scattering: fundamental aspects of nuclear force
- Color transparency: Coherent color effects
- Nuclear Effects, Hidden Color, Anti-Shadowing
- Anomalous heavy quark phenomena
- Spin Effects: A<sub>N</sub>, A<sub>NN</sub>



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## QCD Phenomenology and Nucleon Structure



Thanks to Adam and Steve!



#### National Nuclear Physics Summer School



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