QCD Phenomenology and Nucleon Structure

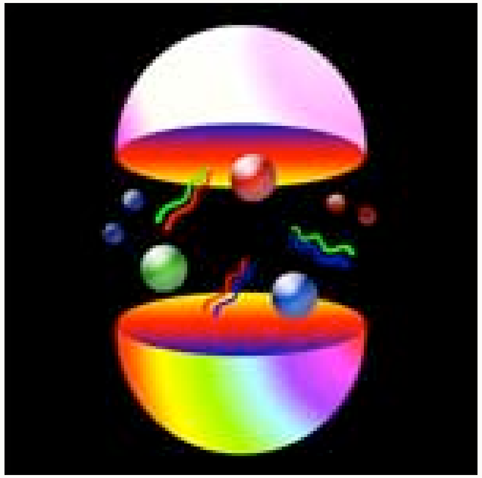
Stan Brodsky, SLAC

Lecture I

National Nuclear Physics Summer School

The World of Quarks and Gluons:

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: Holography



QCD Lagrangian

Generalization of QED

gluon dynamics

quark kinetic energy + quark-gluon dynamics

mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge

field strength tensor

covariant derivative

quark field

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

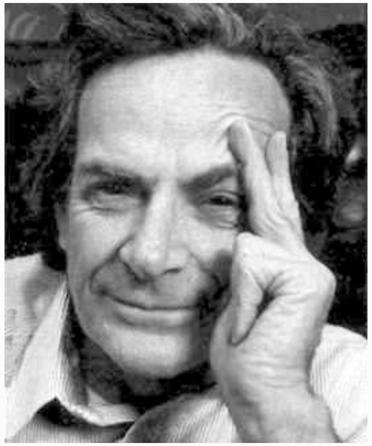
- Novel QCD Phenomena: hidden color, color transparency, intrinsic charm, anomalous heavy quark phenomena, anomalous spin-spin effects, odderon, anomalous Regge behavior ...

- Remarkable Predictions of AdS/CFT

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities.

—Mark Twain

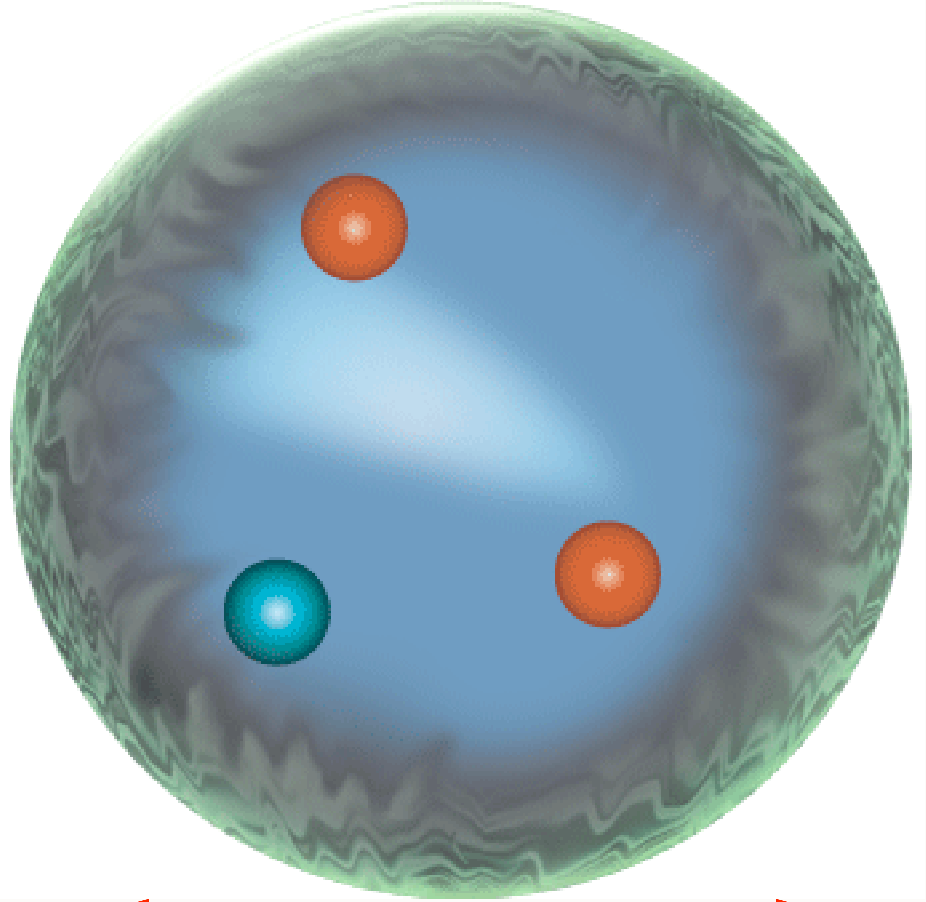
Quarks in the Proton



Feynman: "Parton" model

Bjorken Scaling:
Pointlike Quarks

$p = (u\, u\, d)$

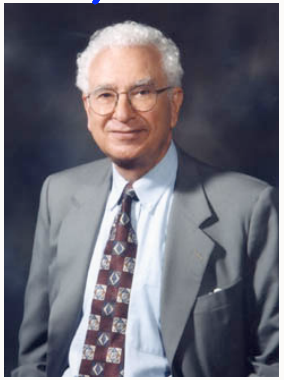


$1\, fm$

$10^{-15} m = 10^{-13} cm$

Ne'eman: $SU(3)_F$

Zweig: "Aces, Duces, Treys"



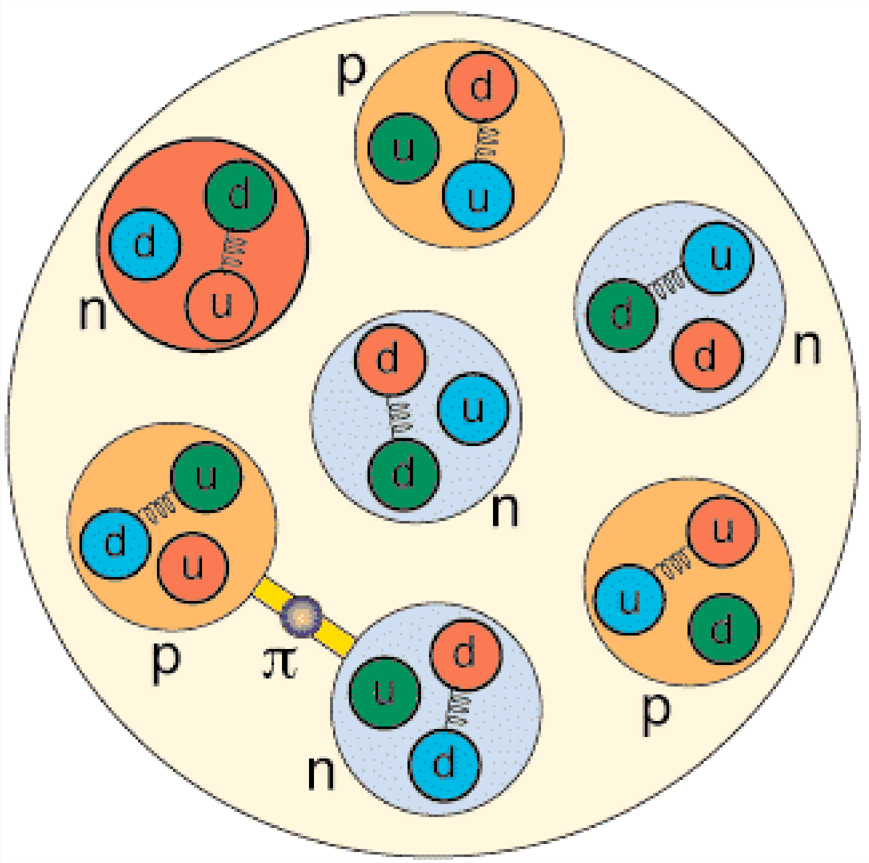
Gell Mann: "Three Quarks for Mr. Mark"

The Quark Structure of the Nucleus

$e_u = +\frac{2}{3} \quad e_d = -\frac{1}{3}$

$p = (uud)$

$n = (ddu)$



$2e_u + e_d = e_p$

$2e_d + e_u = e_n$

$2 \times (+\frac{2}{3}) \; + \; 1 \times (-\frac{1}{3}) = 1$

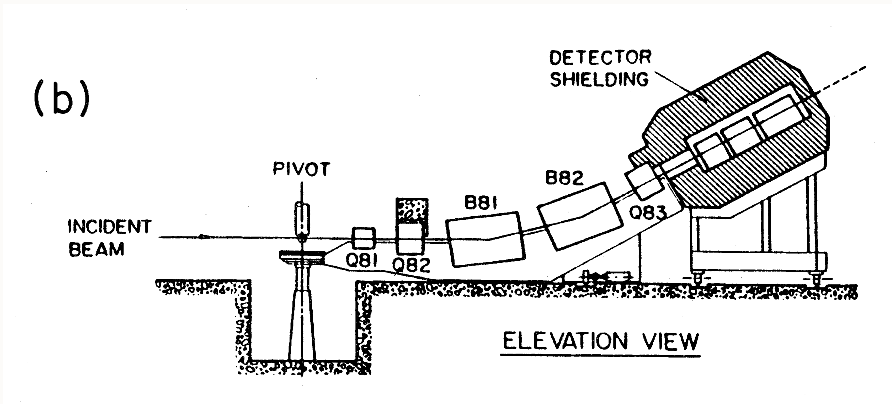
$2 \times (-\frac{1}{3}) \; + \; 1 \times (+\frac{2}{3}) = 0$

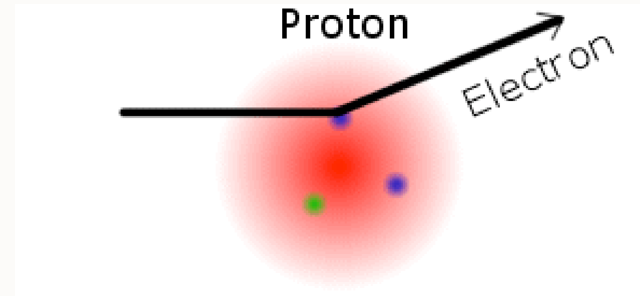
1967 SLAC Experiment:

Scatter Electrons on protons
in a Hydrogen Target

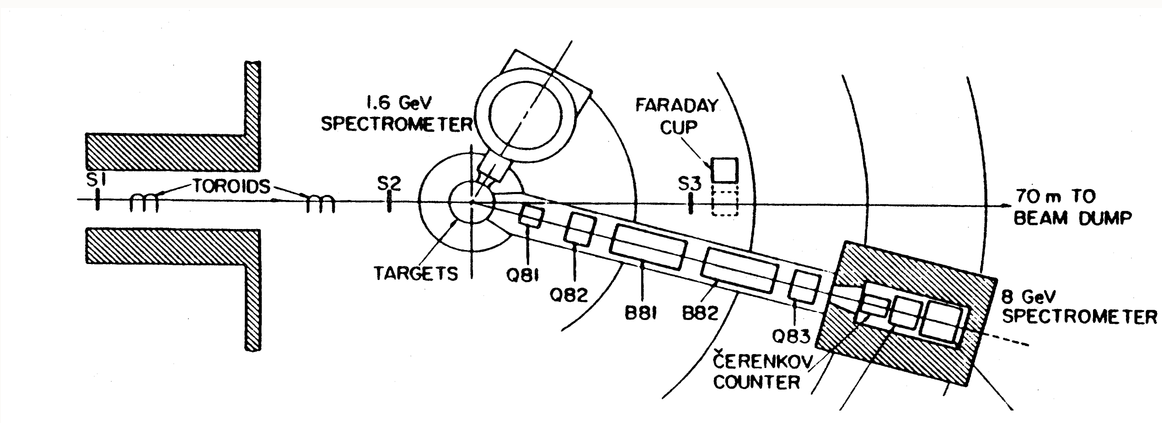
Discovery of the Quark Structure of Matter

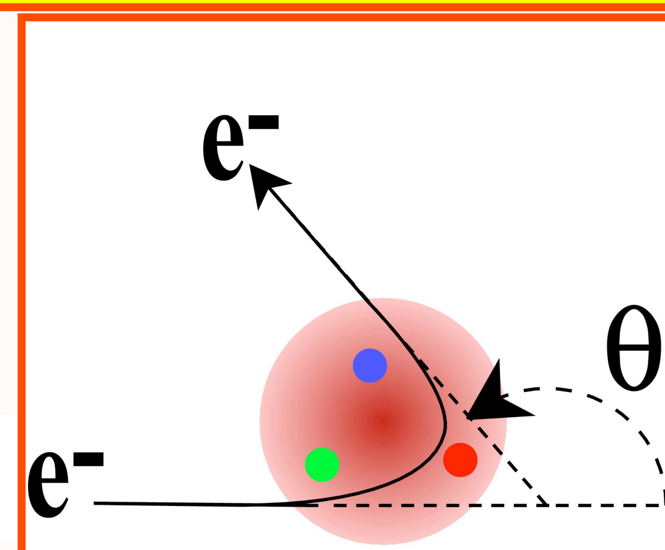
$$ep \longrightarrow e'X$$





Discovery of quarks!

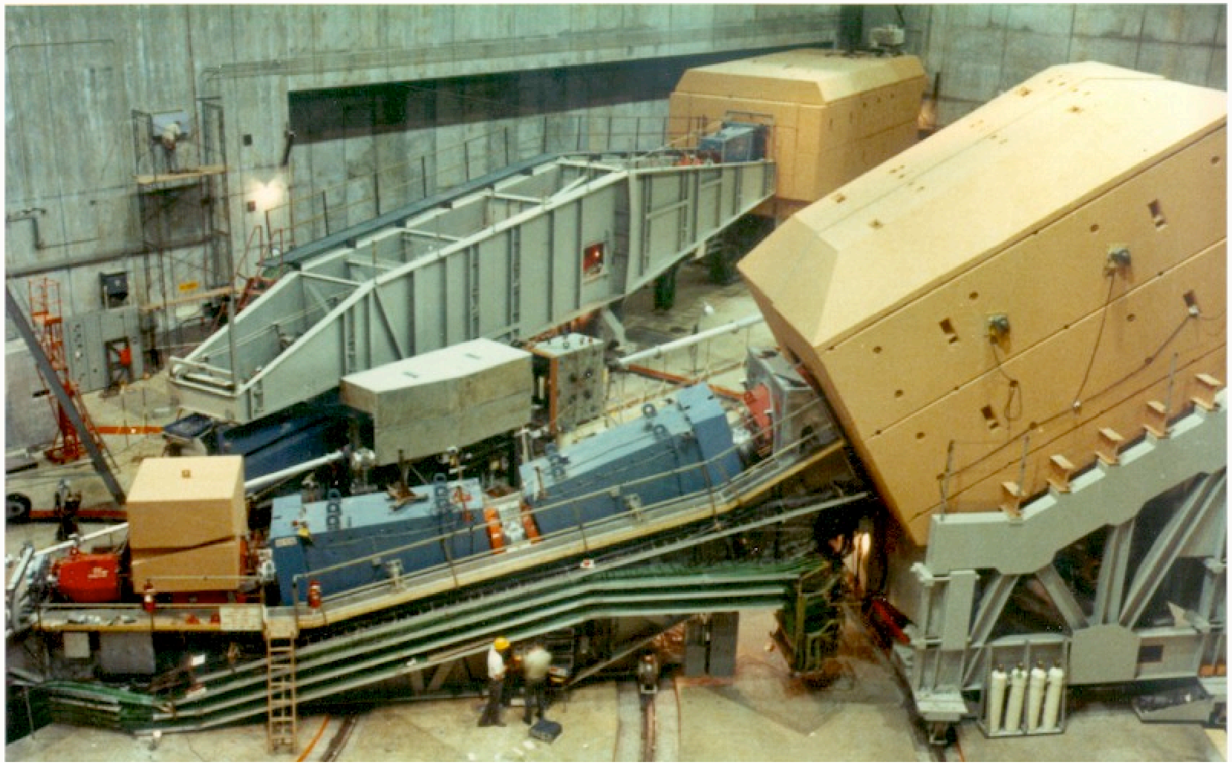


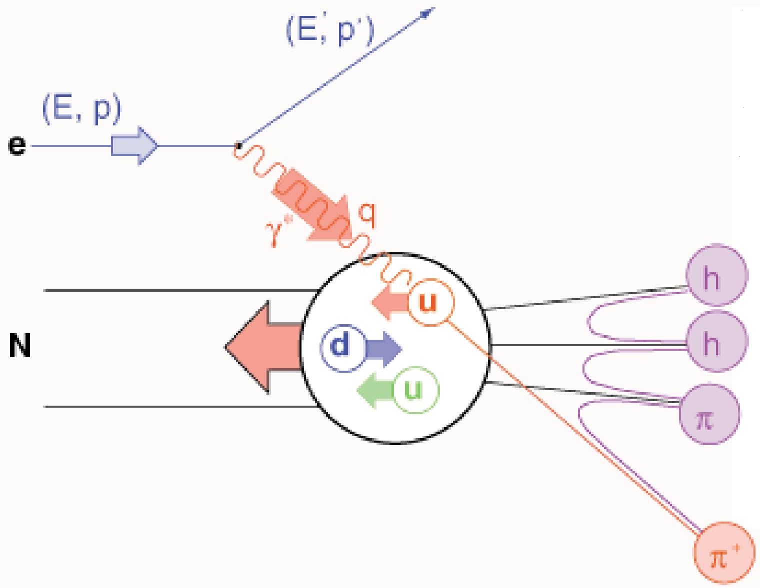


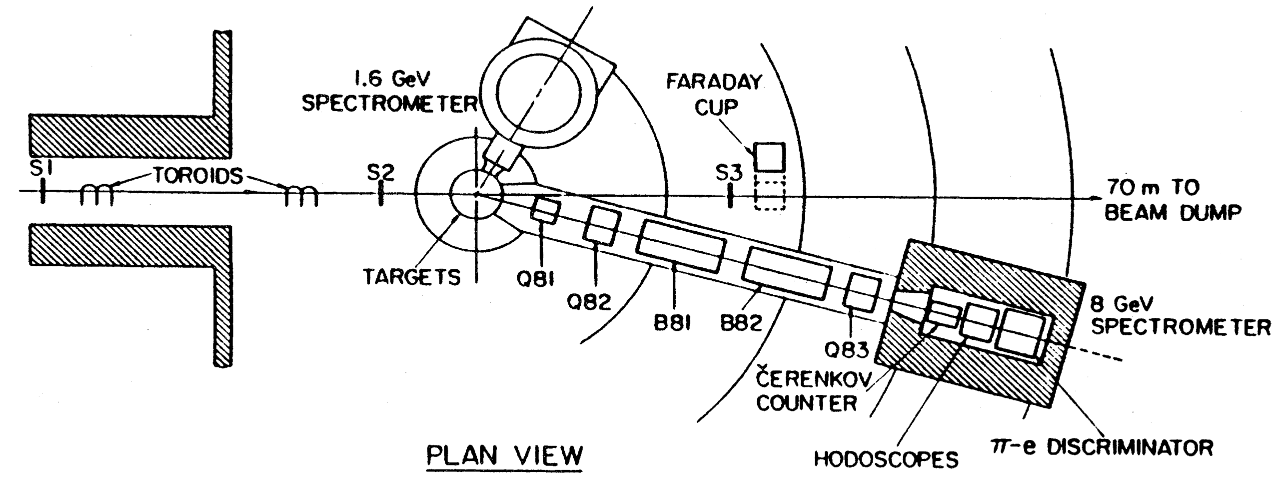
Deep inelastic scattering: Experiments on the proton and the observation of scaling*

Friedman, Kendall, Taylor: Nobel Prize

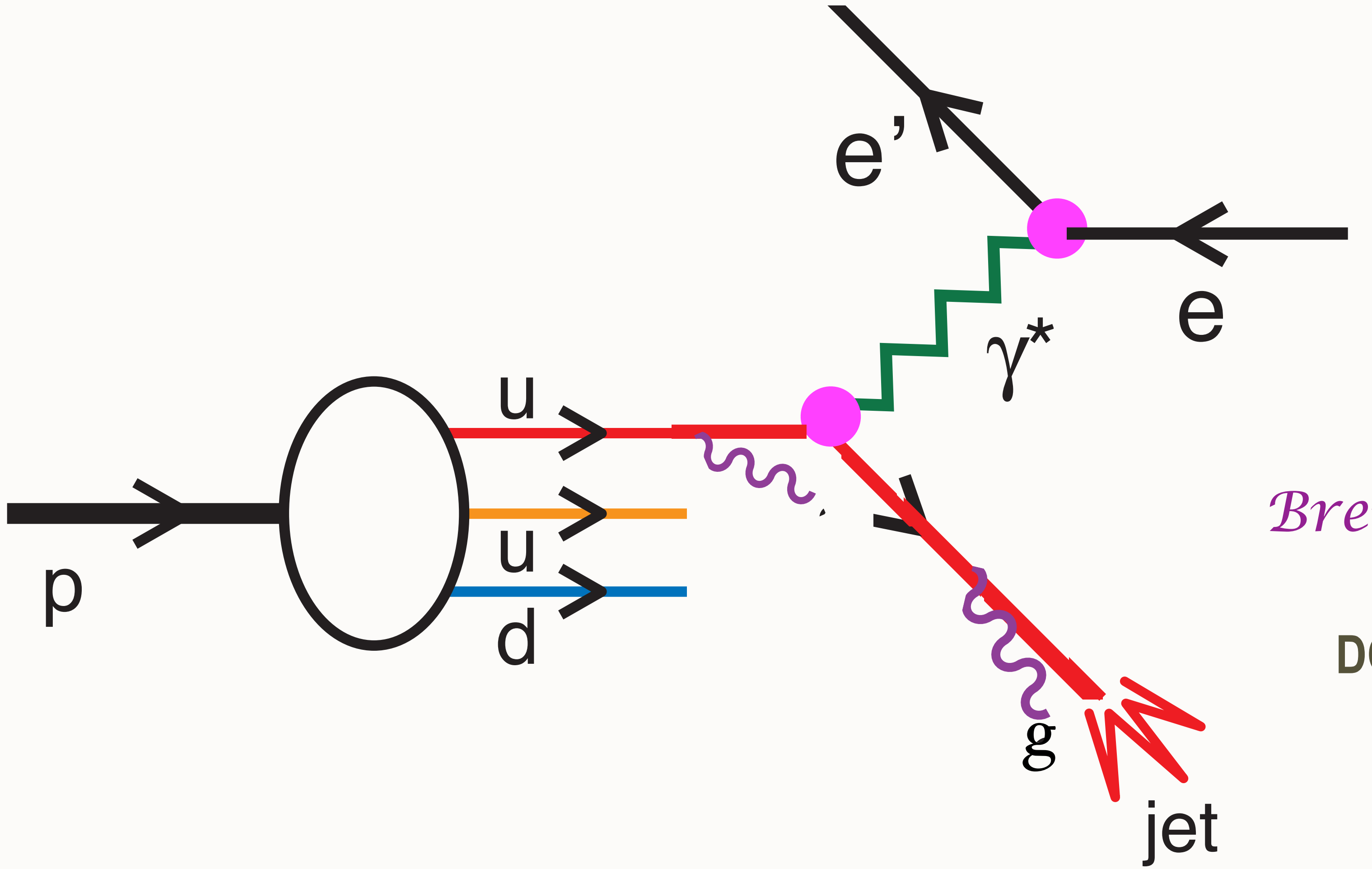
SLAC Two-Mile Linear Accelerator







First Evidence for Quark Structure of Matter

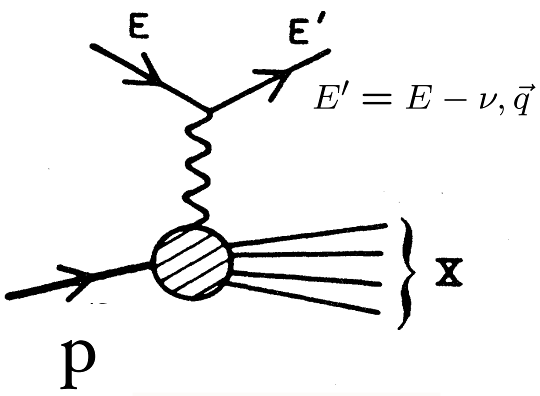


Gluonic Bremmstrahlung

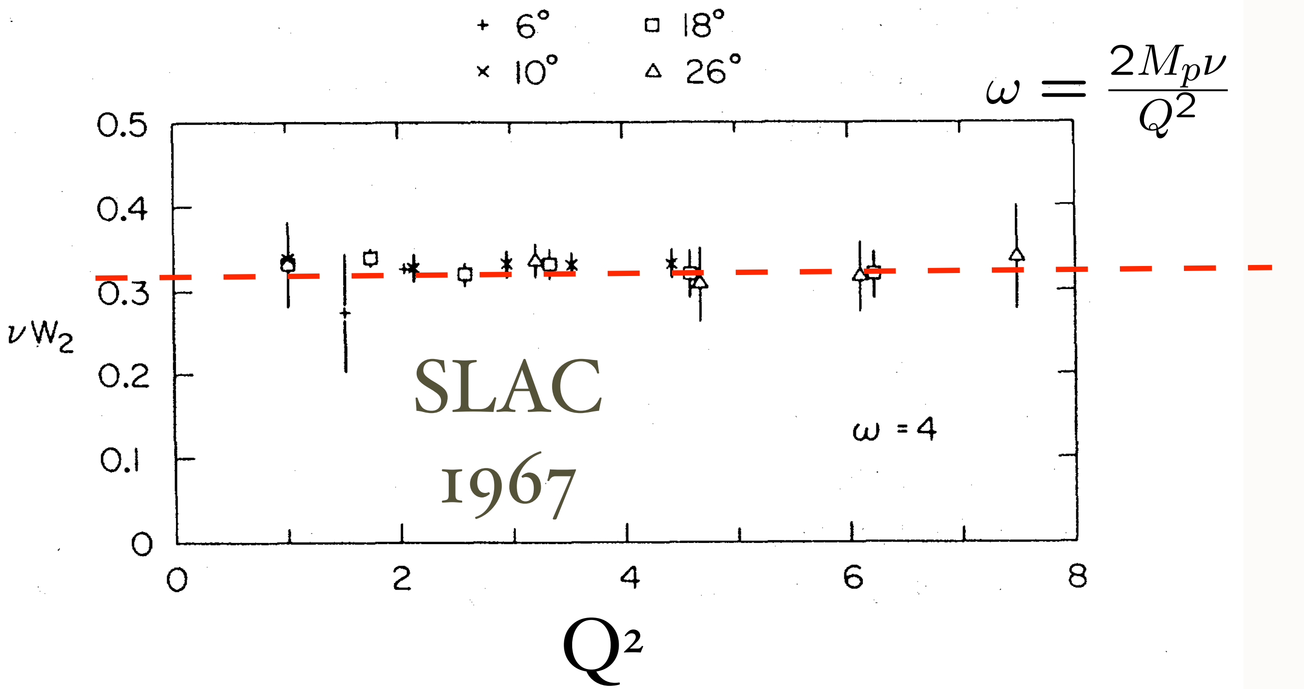
DGLAP Evolution

Deep Inelastic Electron-Proton Scattering

$$ep \to e' X$$



$$Q^2 = \vec{q}^2 - \nu^2$$



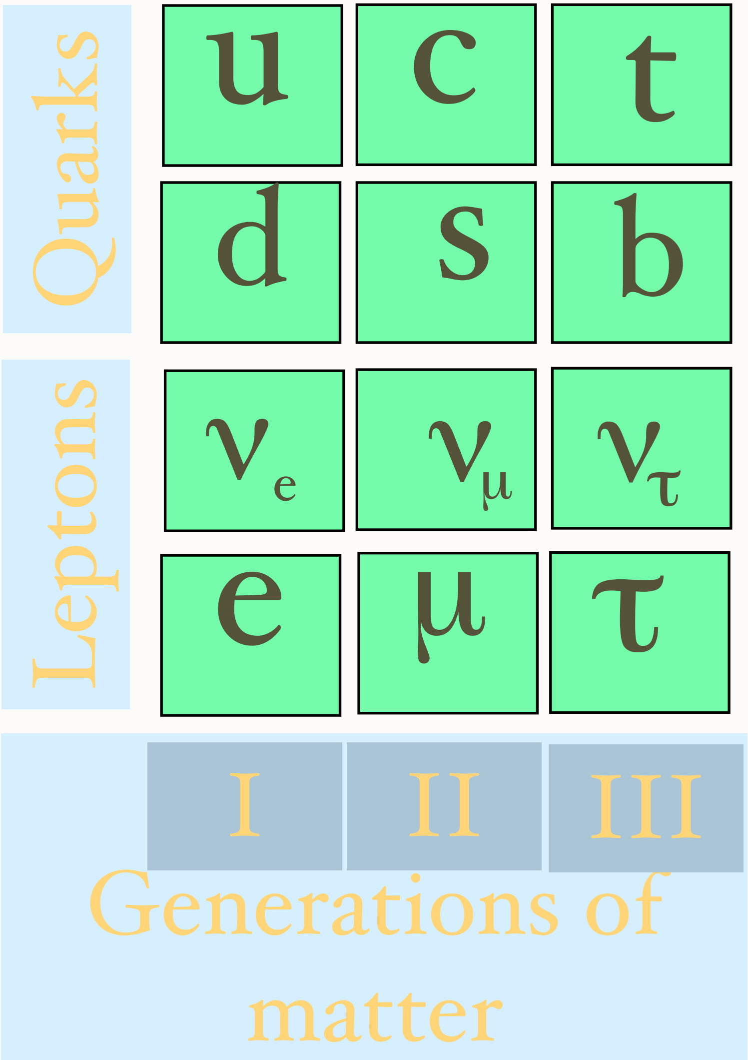
No intrinsic length scale !

Measure rate as a function of energy loss ν and momentum transfer Q

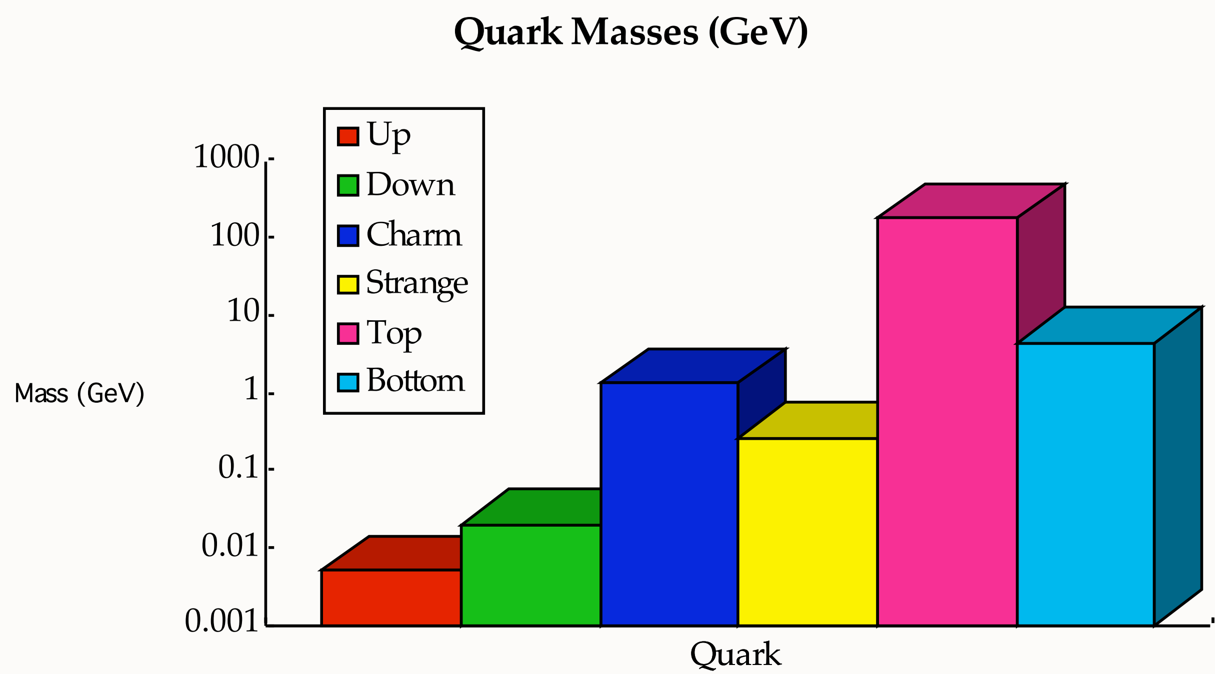
Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!



Quarks
u
c
t
d
s
b
Leptons
ν_e
ν_μ
ν_τ
e
μ
τ
I
II
III
Generations of matter



Quark Masses (GeV)
Up
Down
Charm
Strange
Top
Bottom
1000
100
10
1
0.1
0.01
0.001
Mass (GeV)
Quark

	u	d	s	c	b
$\bar{u}$	π^0, η, η' ρ^0, ω	π^- ρ^-	K^- K^{*-}	D^0 D^{*0}	$\bar{B}^-$ $\bar{B}^{*-}$
$\bar{d}$	π^+ ρ^+	π^0, η, η' ρ^0, ω	$\bar{K}^0$ $\bar{K}^{*0}$	D^+ D^{*+}	$\bar{B}^0$ $\bar{B}^{*0}$
$\bar{s}$	K^+ K^{*+}	K^0 $\bar{K}^{*0}$	η, η' ϕ	D_s D_s^*	$\bar{B}_s$ $\bar{B}_s^*$
$\bar{c}$	$\bar{D}^0$ $\bar{D}^{*0}$	D^- D^{*-}	$\bar{D}_s$ $\bar{D}_s^*$	η_c J/ψ	$\bar{B}_c$ $\bar{B}_c^*$
$\bar{b}$	B^+ B^{*+}	B^0 B^{*0}	B_s B_s^*	B_c B_c^*	η_b Υ

Constructing mesons

$$M = (q\bar{q})$$

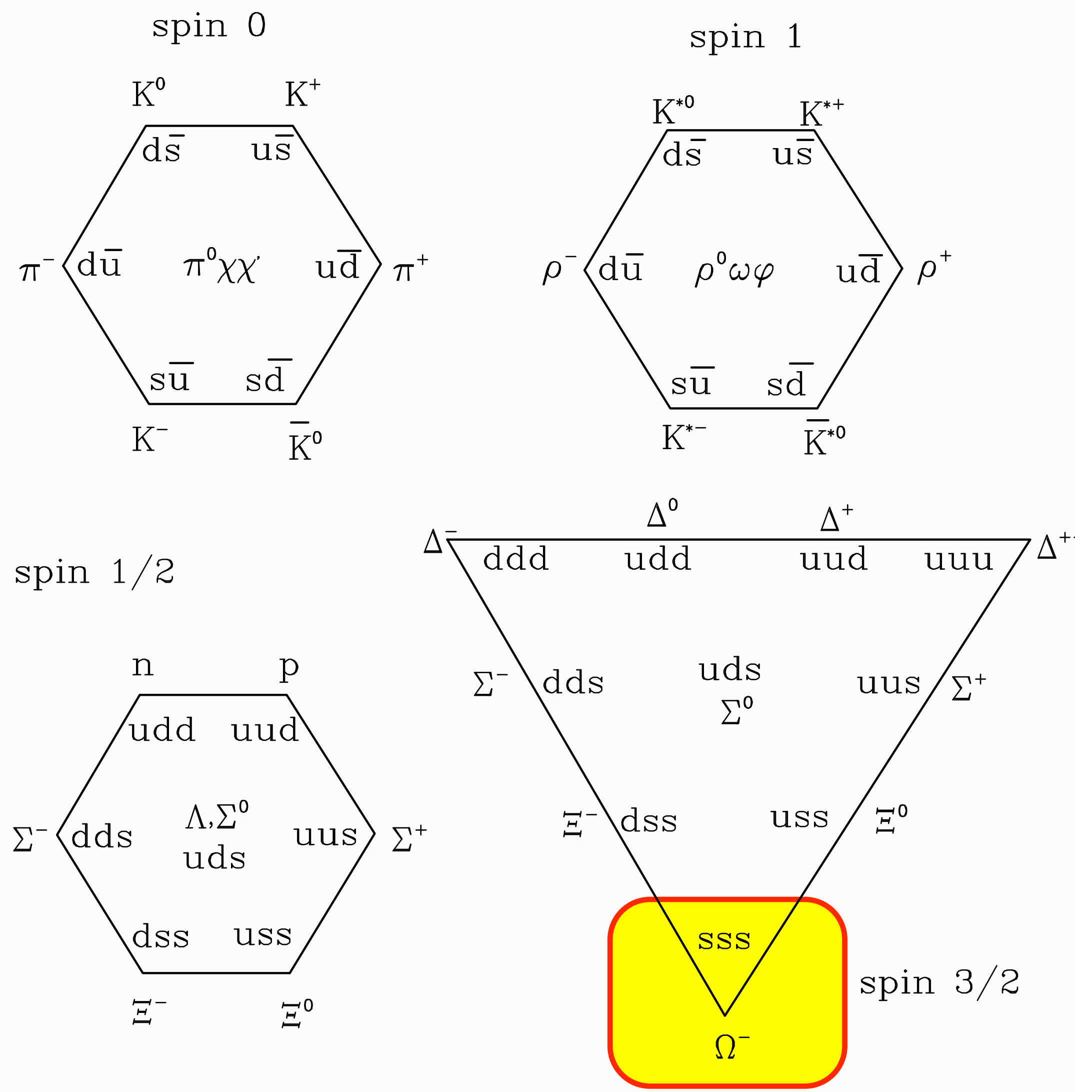
$$\pi^+ = (u\bar{d})$$

Pseudoscalar ($J^P = 0^-$) (upper lines) and vector ($J^P = 0^-$) (lower lines) mesons with different flavour content.

The Hadron Spectrum

$SU(3)_{flavor}$

Ne'eman,
Gell Mann,
Zweig
Y. Eisenberg
Samios

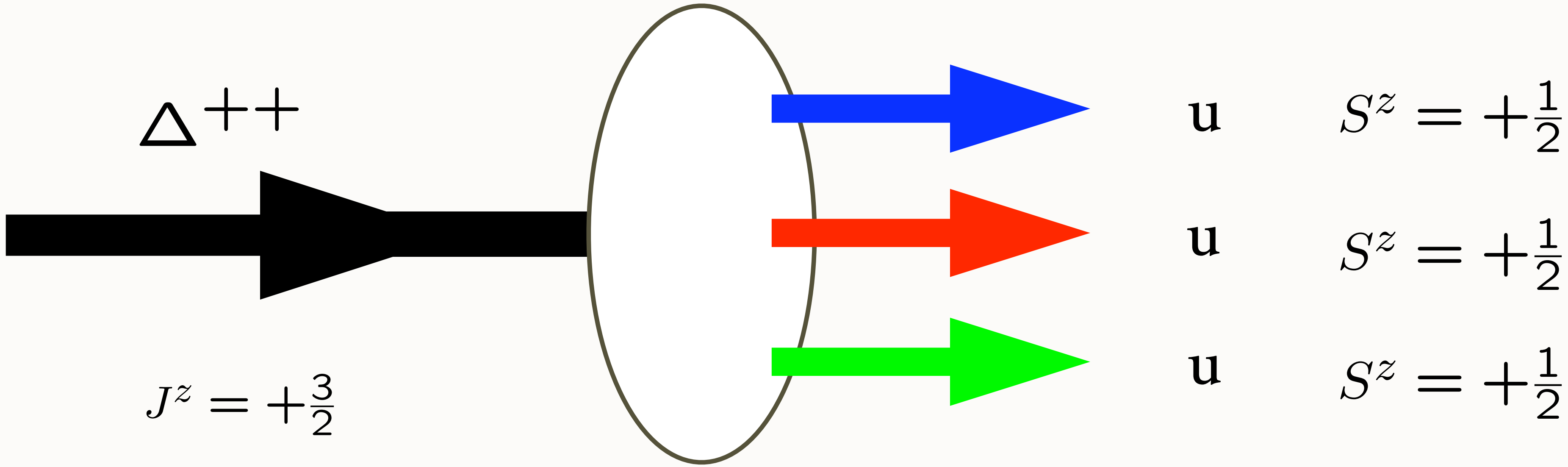


Prediction and Measurement of $\Omega^- = (sss)$

Why are there three colors of quarks?

Pauli Exclusion Principle!

spin-half quarks cannot be in same quantum state !



Three Colors (Parastatistics) Solves Paradox

3 Colors Combine : WHITE

Greenberg: Parastatistics

QCD Lagrangian

Generalization of QED

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge — field strength tensor — covariant derivative — quark field

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

$$\mathcal{L} = -\frac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu}_{\alpha} - \sum_n \bar{\psi}_n \gamma^\mu [\partial_\mu - i g A^\alpha_\mu t_\alpha] \psi_n - \sum_n m_n \bar{\psi}_n \psi_n$$

$$[t_\beta, t_\gamma] = i C^\alpha_{\beta\gamma} t_\alpha$$

where $C^\alpha_{\beta\gamma}$ are the SU(3) algebra structure constants

The gluon field tensors $F^\alpha_{\mu\nu}$ are defined as

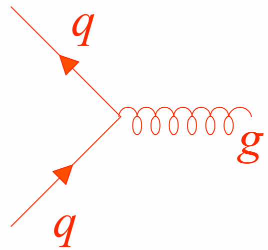
$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + C^\alpha_{\beta\gamma} A^\beta_\mu A^\gamma_\nu .$$

Quarks couple to gluons through the color currents

$$J^\mu_\alpha = -ig \sum_n \bar{\psi}_n \gamma^\mu A^\alpha_\mu t_\alpha \psi_n .$$

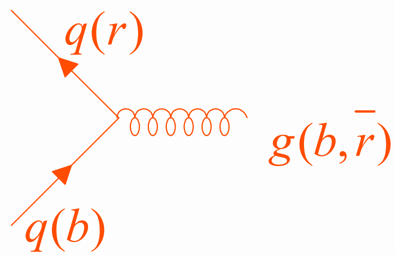
Fundamental Couplings

Only quarks and gluons involve basic vertices: Quark-gluon vertex

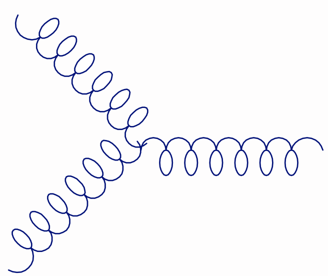


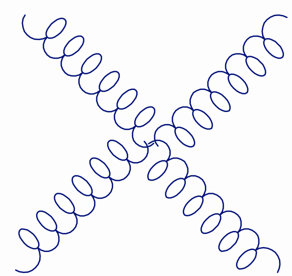
Similar to QED

More exactly

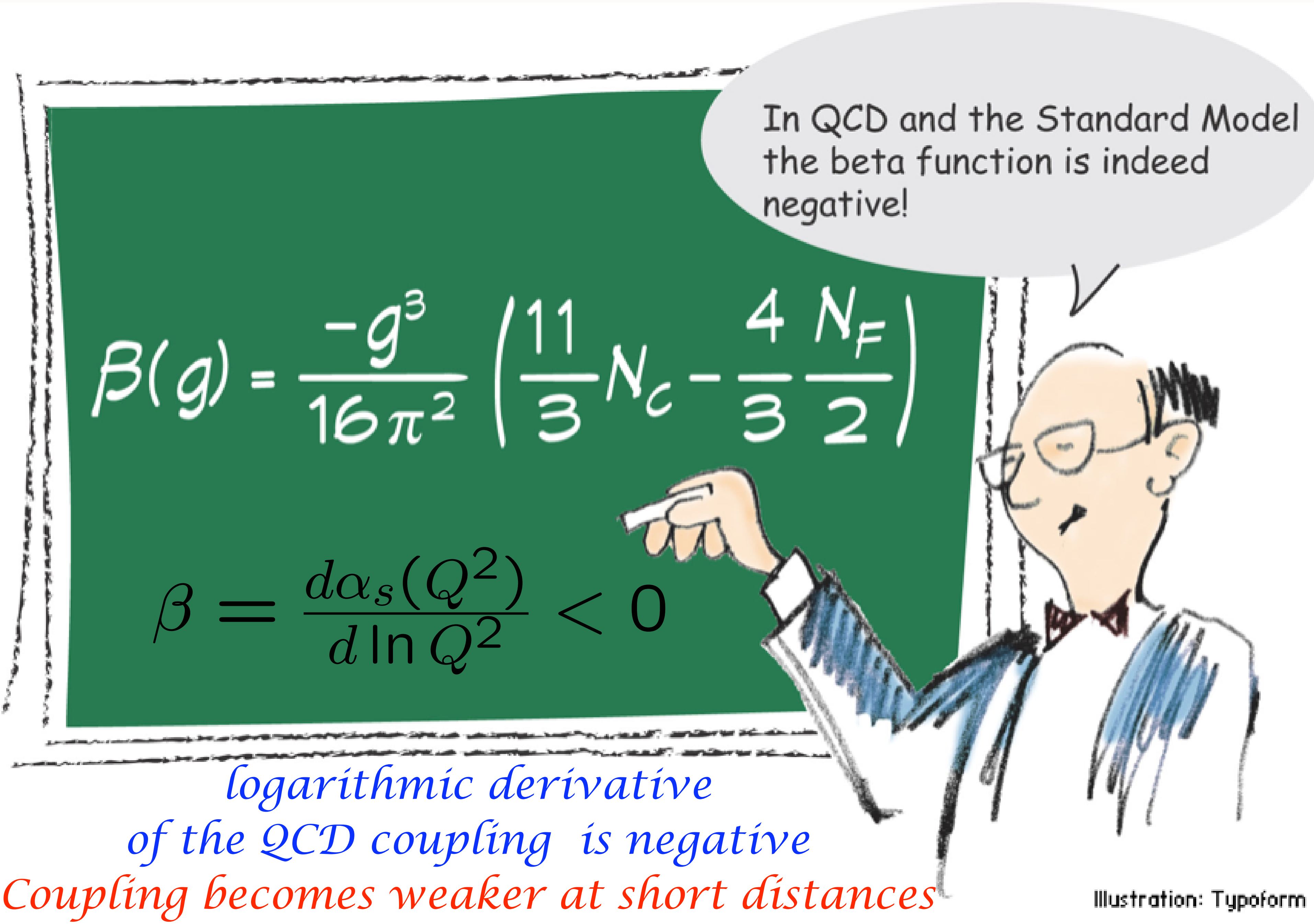


Gluon vertices





colored particles couple to gluons



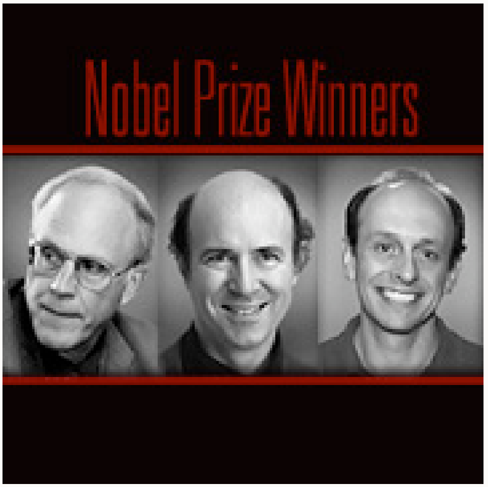
$$\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} < 0$$

logarithmic derivative of the QCD coupling is negative

Coupling becomes weaker at short distances or high momentum transfer

Verification of Asymptotic Freedom

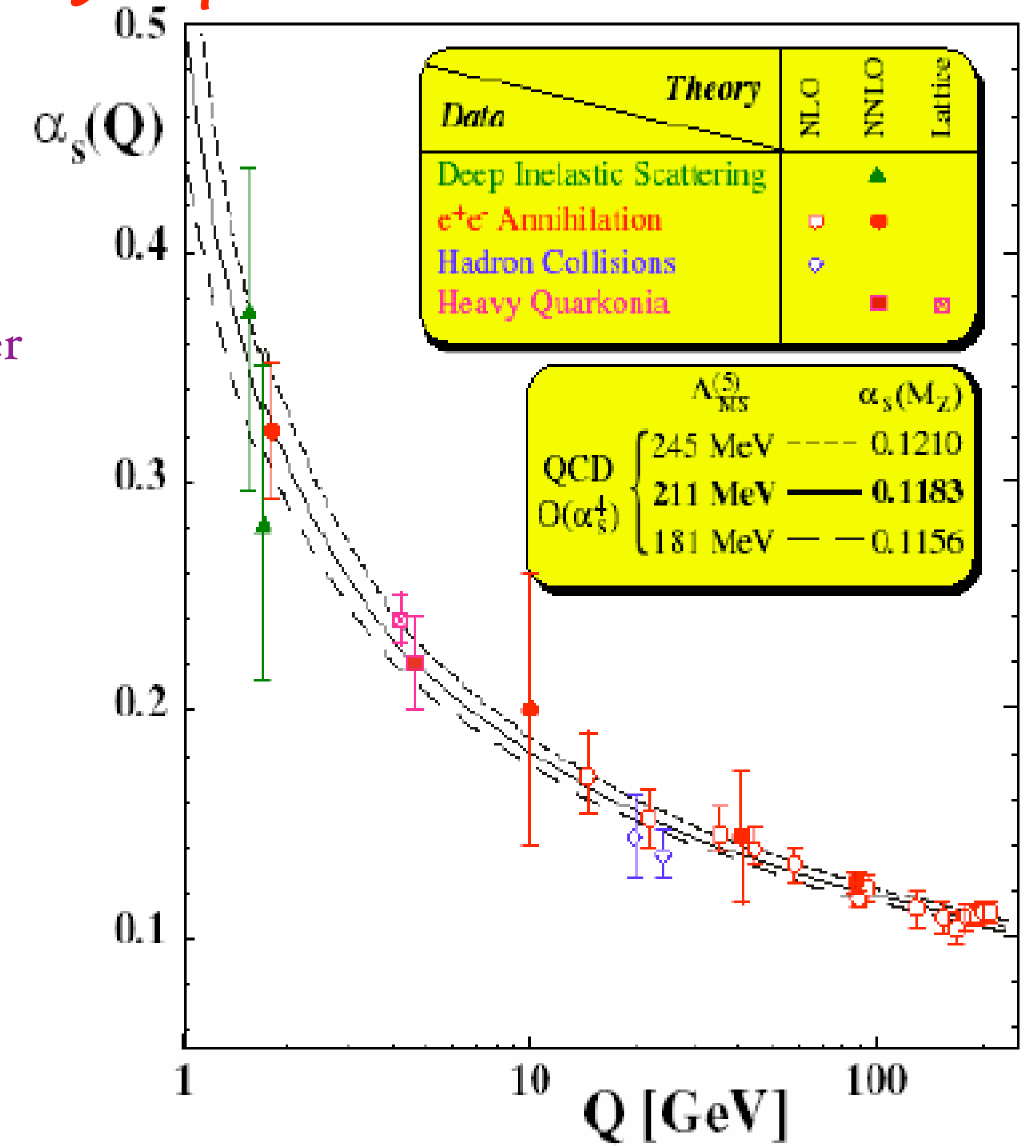
$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$



Gross, Wilzcek, Politzer
Khriplovich, `t Hooft

$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

proportional to $\alpha_s(Q)$



Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$

QCD Lagrangian

gluon dynamics · quark kinetic energy + quark-gluon dynamics · mass term

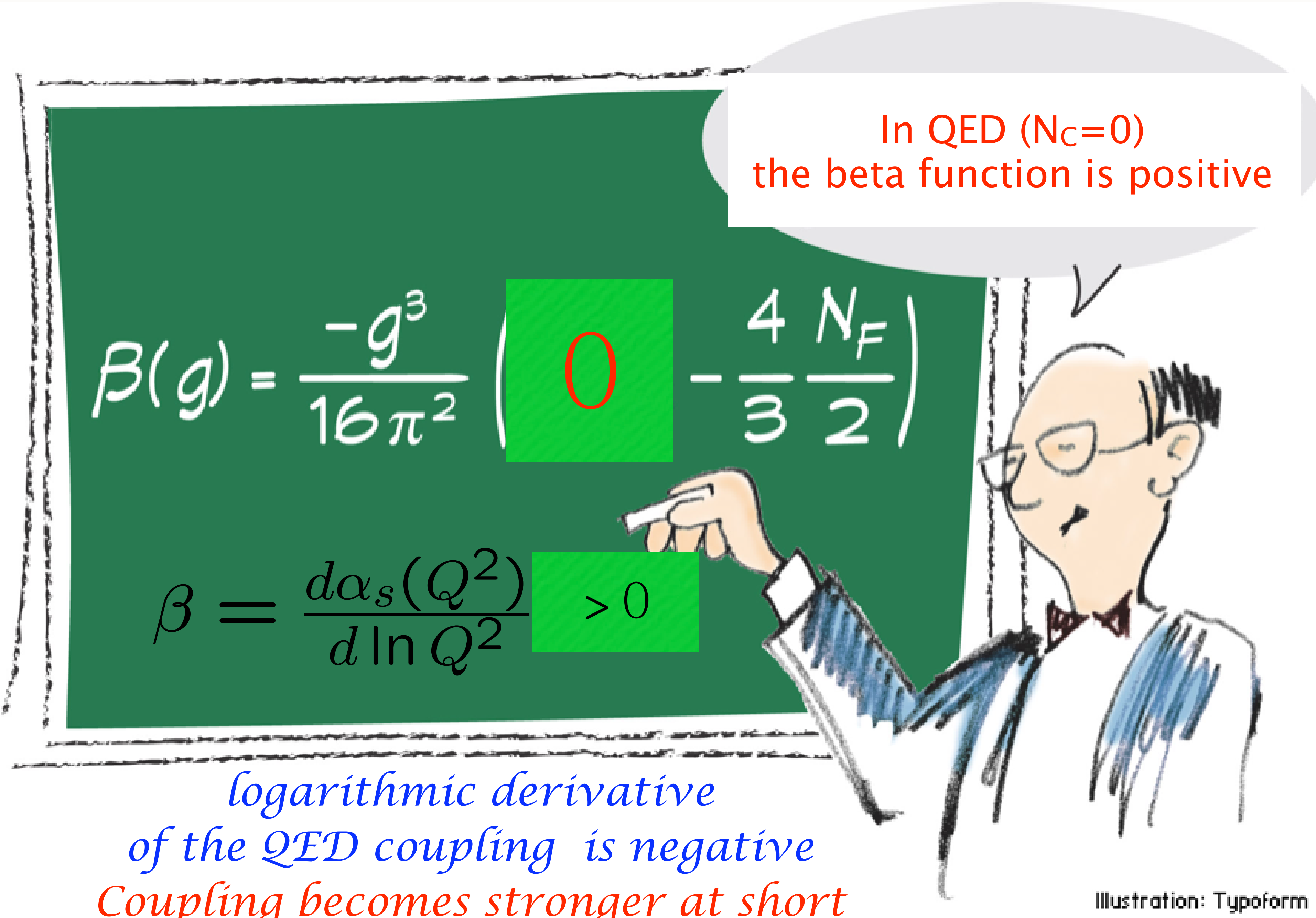
$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge · field strength tensor · covariant derivative · quark field

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F \qquad [C_F = \frac{N_C^2 - 1}{2N_C}]$$

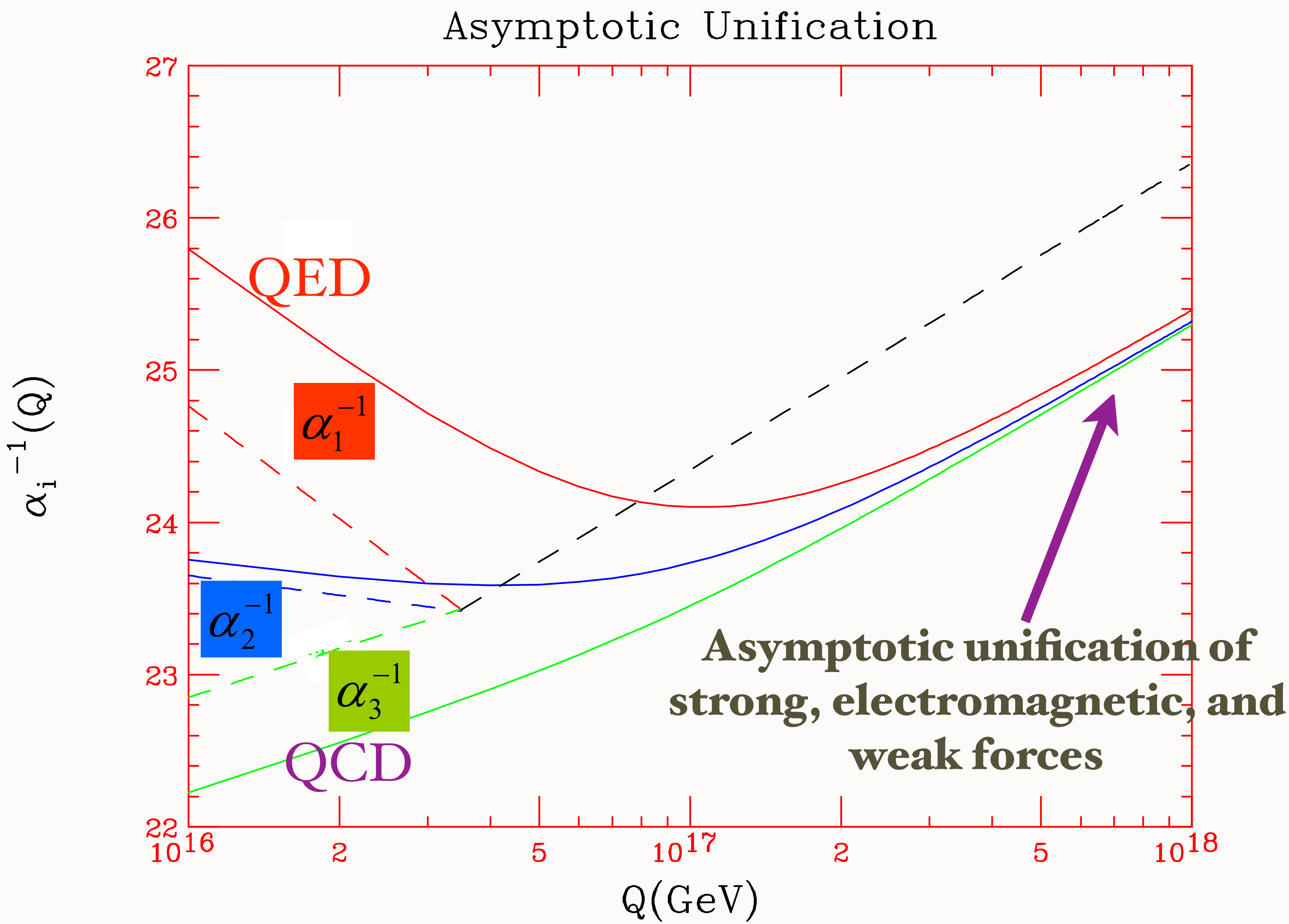
Analytic limit of QCD: Abelian Gauge Theory

P. Huet, sjb

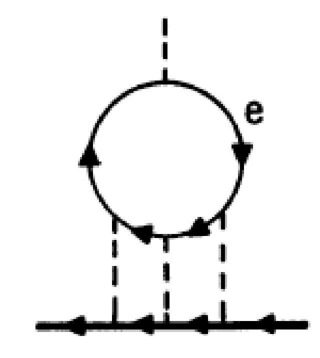


logarithmic derivative
of the QED coupling is negative
Coupling becomes stronger at short distances or high momentum transfer

Binger, sjb


Asymptotic Unification
27
26
25
24
23
22
$\alpha_i^{-1}(Q)$
QED
α_1^{-1}
α_2^{-1}
α_3^{-1}
QCD
10^{16}
2
5
10^{17}
2
5
10^{18}
Q(GeV)
Asymptotic unification of strong, electromagnetic, and weak forces

Given the elementary gauge theory interactions, all fundamental processes described in principle!



Example from QED:

$N_C=0$

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

$\frac{1}{2}g_e = 1.001\ 159\ 652\ 201(30)$ QED prediction (Kinoshita, et al.)

$\frac{1}{2}g_e = 1.001\ 159\ 652\ 193(10)$ Measurement (Dehmelt, et al.)

g_e accurate to 11 figures!

Dirac: $g_e \equiv 2$

Radiative Corrections of Eighth- and Tenth-Orders to Lepton g-2

Toichiro Kinoshita [a]

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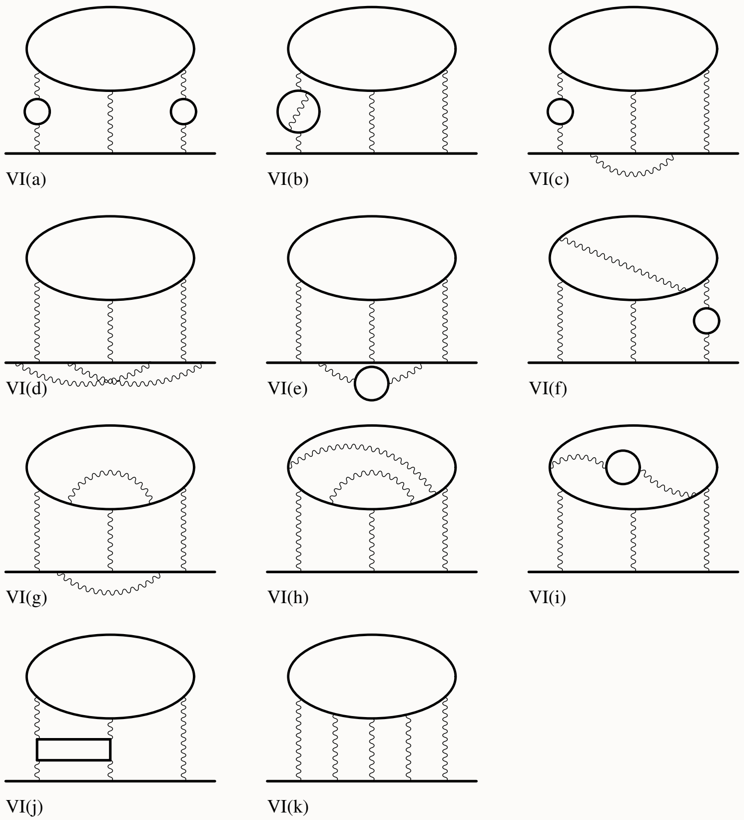
Cornell University
Ithaca, NY 14853, USA
E-mail: tk@hepth.cornell.edu

Nuclear Physics B (Proc. Suppl.) 157 (2006) 101–105



Phys. Rev. Lett. 23, 441–443 (1969)

PHOTON-PHOTON SCATTERING CONTRIBUTION TO THE SIXTH-ORDER MAGNETIC MOMENT OF THE MUON*

Janis Aldins† and Toichiro Kinoshita
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

and

Stanley J. Brodsky and Andrew J. Dufner
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 25 July 1969)

We report a calculation of the three-photon–exchange (electron-loop) contribution to the sixth-order anomalous magnetic moment of the muon. Our result, which contains a logarithmic dependence on the muon-to-electron mass ratio, brings the theoretical prediction into agreement with the CERN measurements, within the 1–standard-deviation experimental accuracy.

$$\Delta a_{\text{ph-ph}} = [(6.4 \pm 0.1)\ln(m_\mu/m_e) + \text{const}] \times (\alpha/\pi)^3.$$

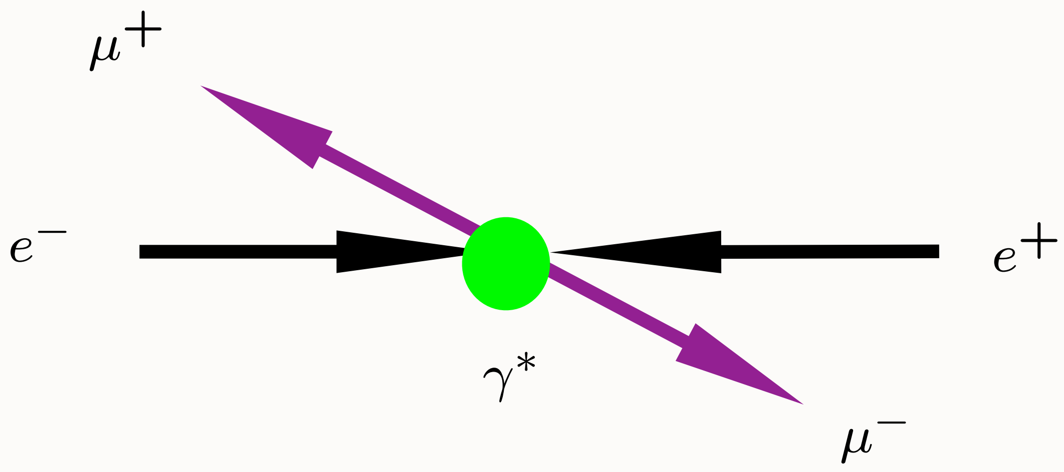
High-Precision Atomic Physics Tests of QED

All Accurate to ppm

- Lamb Shift in Hydrogen
- Hyperfine splitting of muonium and hydrogen
- Muonic Atom spectroscsopy
- Positronium Lifetime

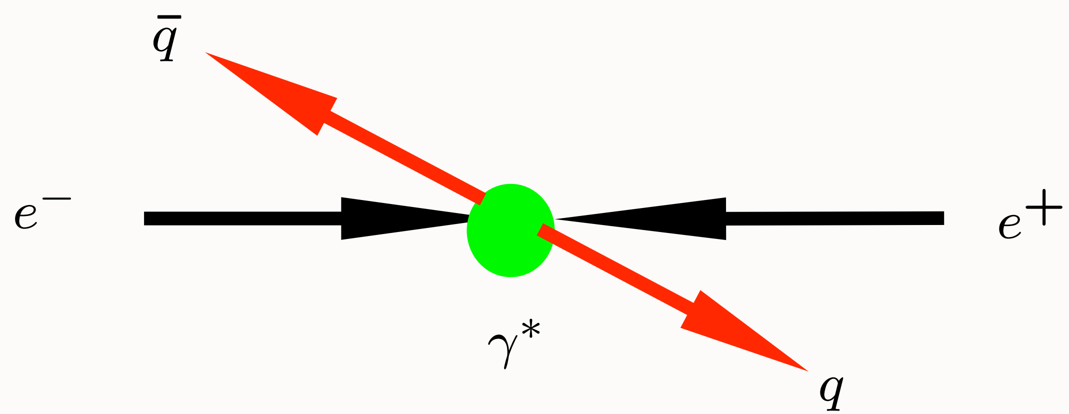
Crucial tool of atomic physics: Wavefunctions

Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to \mu^+\mu^-$$

Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to q\bar{q}$$

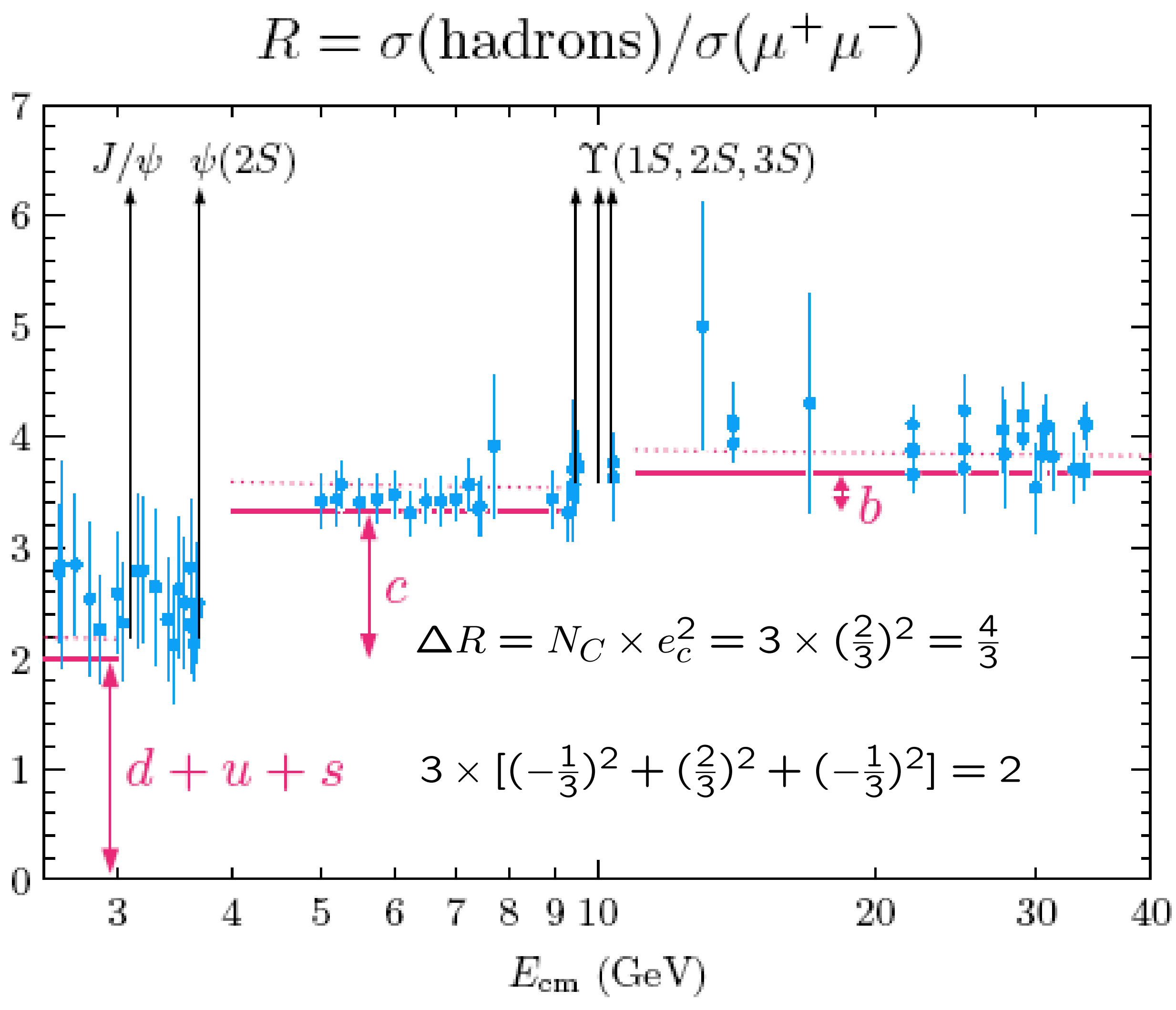
Rate proportional to quark charge squared and number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

How to Count Quarks

$J/\psi = (c\bar{c})_{1S}$

$\Upsilon = (b\bar{b})_{1S}$



$3 \times (-\frac{1}{3})^2 = \frac{1}{3}$

$N_C = 3$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

Hadron Dynamics at the Amplitude Level

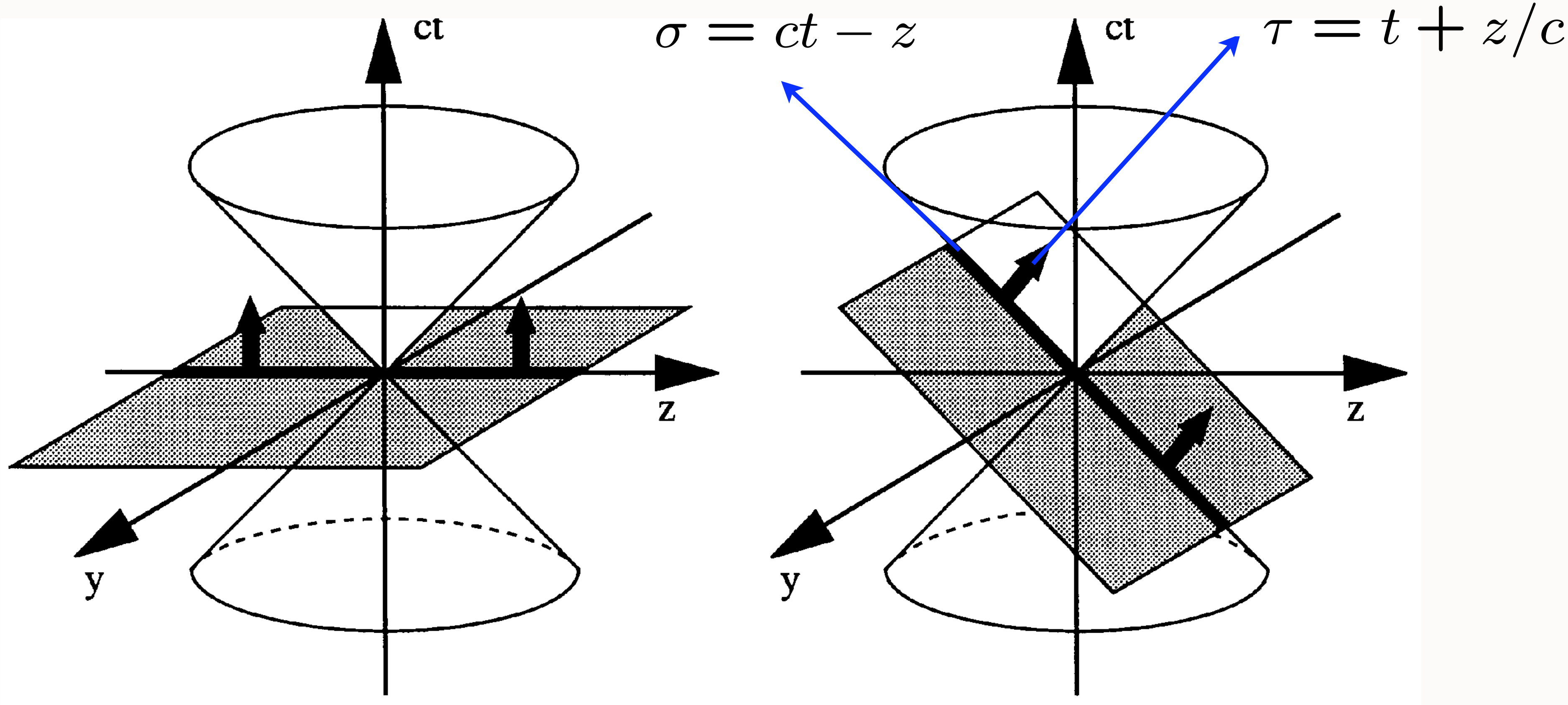
- DIS studies have primarily focussed on probability distributions: integrated and unintegrated.
- Impact of ISI and FSI: Single Spin Asymmetries, Diffractive Deep Inelastic Scattering, Shadowing, Anti-shadowing
- Test QCD at the amplitude level: Phases, multi-parton correlations, spin, angular momentum, exclusive processes
- Wavefunctions: Fundamental QCD Dynamics

Wavefunctions: Fundamental description of composite systems

- Basic quantum mechanical quantities in atomic and nuclear physics
- Physics at the amplitude level
- **Schrödinger** wavefunction in nonrelativistic theory
- Relativistic formulation: Bethe Salpeter amplitudes evaluated at fixed time t
- Problem: "Instant" form: Frame-dependent

Dirac's Amazing Idea: The "Front Form"

Evolve in light-cone time!



Instant Form

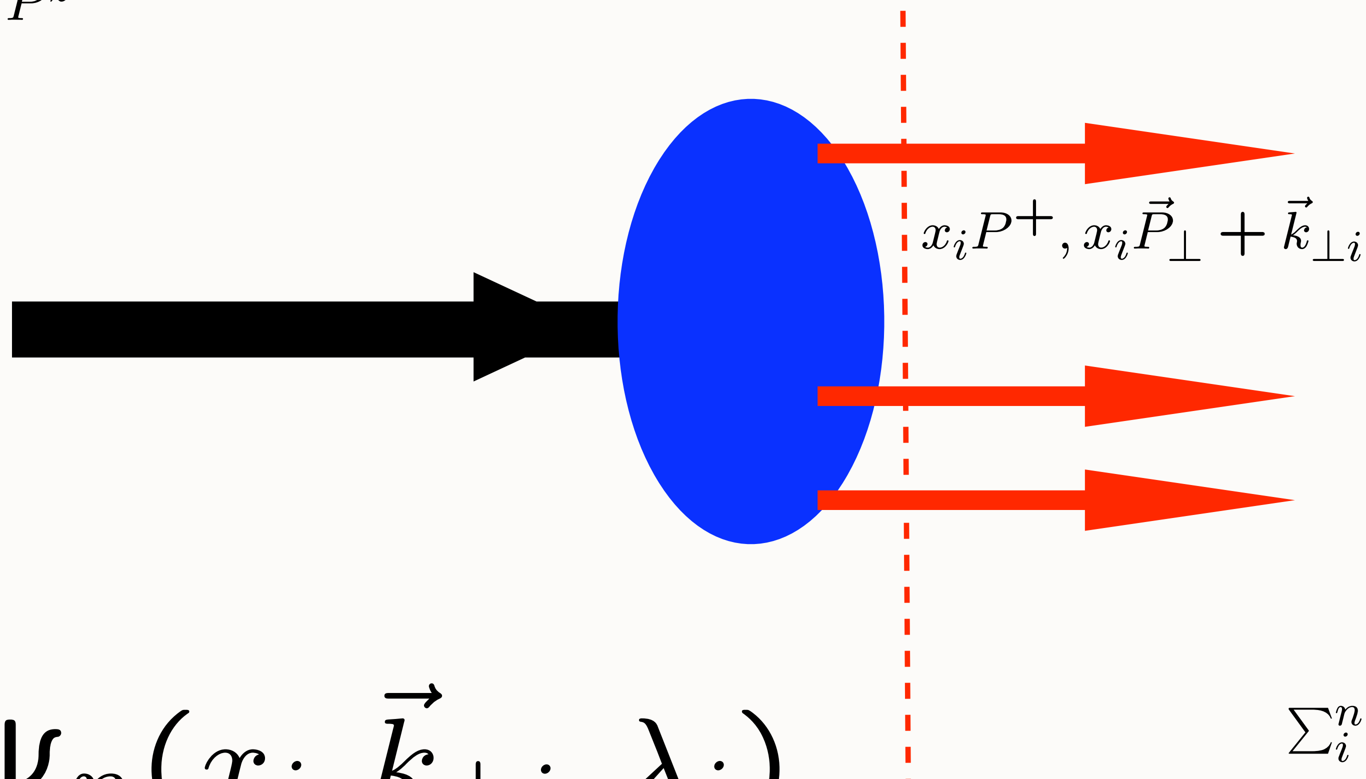
Front Form

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$

$P^+, \vec{P}_\perp$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \; |\Psi_h\rangle$$

Compute LFWFS from first principles

$$H_{LC}^{QCD} = P_\mu P^\mu = \quad P^- P^+ - \vec{P}_\perp^2$$

The hadron state $|\Psi_h\rangle$ is expanded in a Fock-state complete basis of non-interacting n-particle states $|n\rangle$ with an infinite number of components

$$\left|\Psi_h(P^+, \vec{P}_\perp)\right\rangle =$$

$$\sum_{n,\lambda_i} \int \; [dx_i \; d^2\vec{k}_{\perp i}] \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

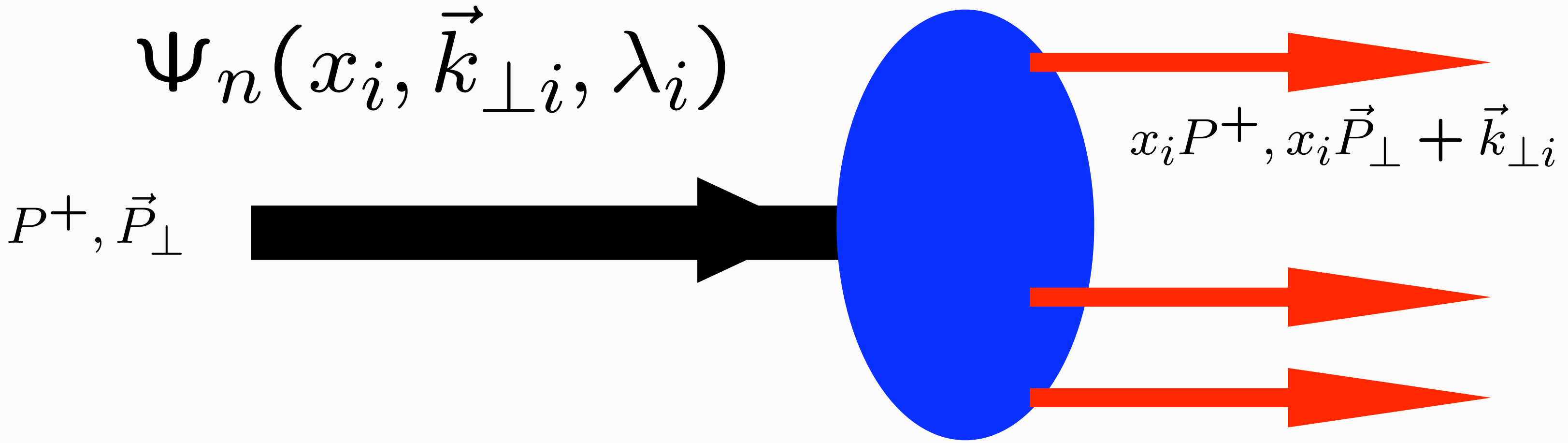
$$\times \; |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_n \int [dx_i \; d^2\vec{k}_{\perp i}] \; |\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 = 1$$

$\sum_i^n x_i = 1$ $\qquad$ $\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_{i=1}^n k_i^+ = \sum_{i=1}^n x_i \vec{P}^+ = \vec{P}^+$ $\qquad$ $\sum_{i=1}^n (x_i \vec{P}_\perp + \vec{k}_{\perp i}) = \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$$\vec{\ell}_j \equiv (\vec{k}_\perp \times \vec{b}_\perp)_j = (\vec{k}_\perp \times \frac{i\partial}{\partial \vec{k}_\perp})_j$$

n-1 Intrinsic Orbital Angular Momenta

Frame Independent $\qquad j = 1, 2, \cdots (n-1)$

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

LFWFs of Electron (n=2)

$J_z = +\frac{1}{2}$

Gives Schwinger Anomalous Moment $\frac{\alpha}{2\pi}$

$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(-k^1+ik^2)}{x(1-x)}\,\varphi\ , & L_z = -1 \\ \psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(+k^1+ik^2)}{1-x}\,\varphi\ , & L_z = 1 \\ \psi^{\uparrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}(M-\frac{m}{x})\,\varphi\ , & L_z = 0 \\ \psi^{\uparrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = 0\ , & \end{cases}$$

where

$$\varphi = \varphi(x,\vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}_\perp^2+m^2)/x - (\vec{k}_\perp^2+\lambda^2)/(1-x)}\ .$$

M→m + λ

Spin-1 mass λ

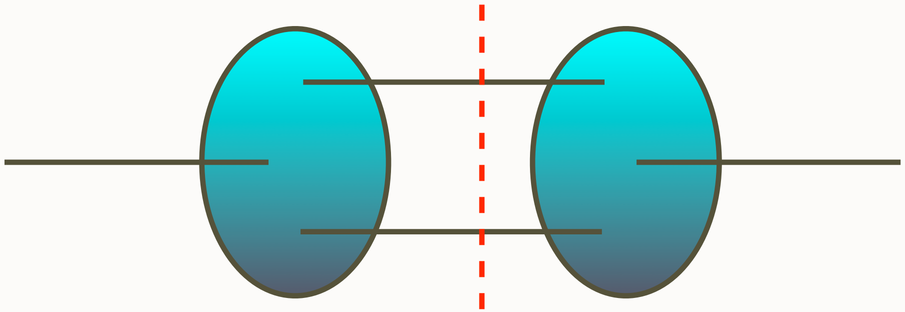
Spin-1/2 mass m

$$\begin{cases} \psi^{\downarrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = 0\ , \\ \psi^{\downarrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}(M-\frac{m}{x})\,\varphi\ , \\ \psi^{\downarrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(-k^1+ik^2)}{1-x}\,\varphi\ , \\ \psi^{\downarrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(+k^1+ik^2)}{x(1-x)}\,\varphi\ . \end{cases}$$

Drell, sjb

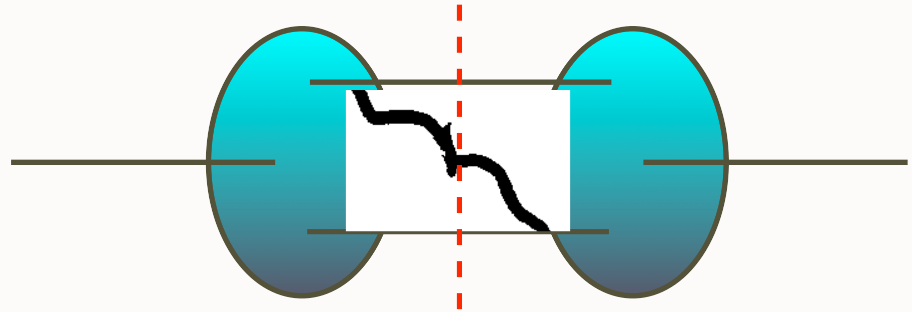
Hwang, Schmidt, sjb

Relativistic Quantum Field Theory: Uncertainty in particle number n



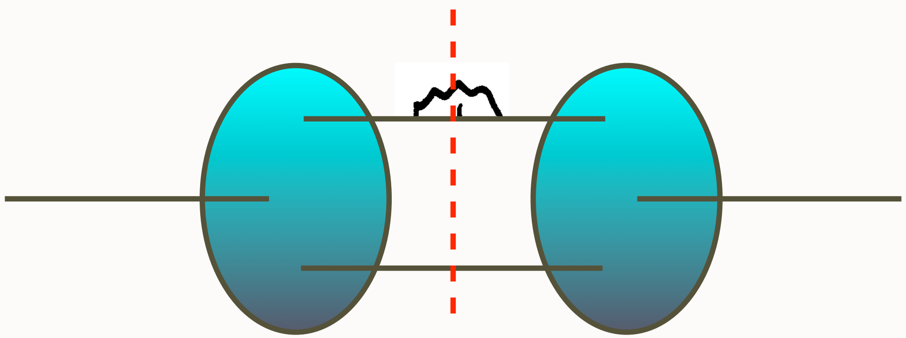
Positronium n=2

e^+e^-



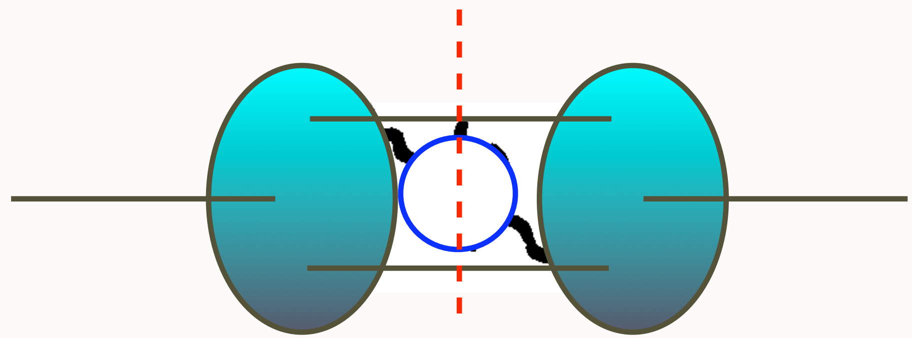
Hyperfine splitting n=3

$e^+e^-\gamma$



Lamb Shift n=3

$e^+e^-\gamma$



Vacuum Polarization n=4

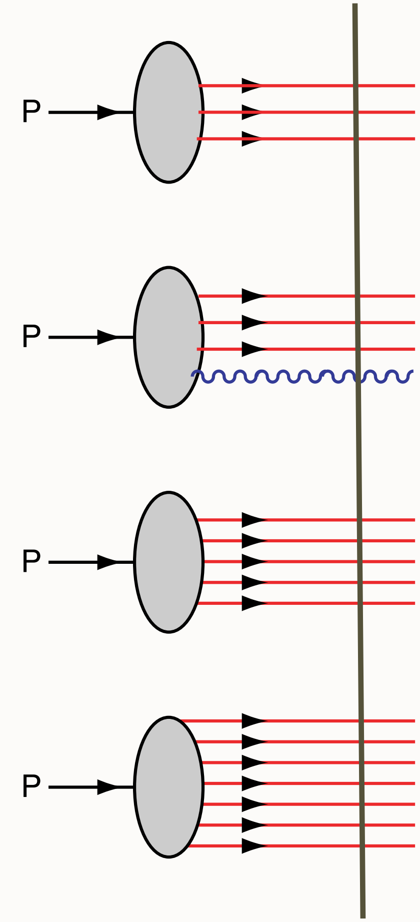
$e^+e^-e^+e^-$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$



$$H_{LF}^{QCD}|\psi\rangle = M^2|\psi\rangle$$

Invariant under boosts. Independent of P^μ

Central Property of Quantum Field Theory

Quantum Fluctuations

Fluctuations in

* particle number $n = 2, 3, \ldots$

Einstein

* off-shellness $E \neq \sum E_i$

$$\mathcal{M}_n^2 \neq M_p^2$$

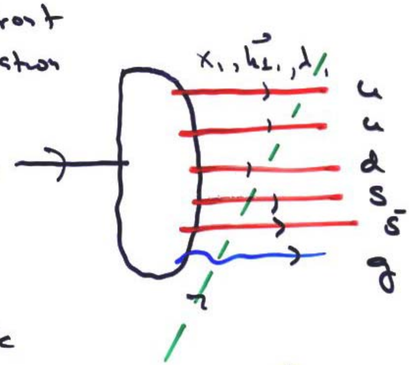
* size, momenta, space coordinates

* orbital angular momentum

$$J_z = \sum_{i=1}^{n} S_z^i + \sum_{i=1}^{n-1} L_z^i$$

Dirac:

Light-Front Quantization



$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{p^0 + p^z}$$

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \left(\frac{k_\perp^2 + m^2}{x} \right)_i$$

Fixed $\tau = t + z/c$

$$|\Psi_p\rangle = \sum_{n=2}^{\infty} |n\rangle\langle n|\Psi_p\rangle$$

$\uparrow$ $\Psi_{n/p}(x_i, \vec{k}_{\perp i}, \lambda_i)$

Hadrons Fluctuate in Particle Number

- Proton Fock States

$$|uud>, |uudg>, |uuds\bar{s}>, |uudc\bar{c}>, |uudb\bar{b}> \cdots$$

- Strange and Anti-Strange Quarks not Symmetric

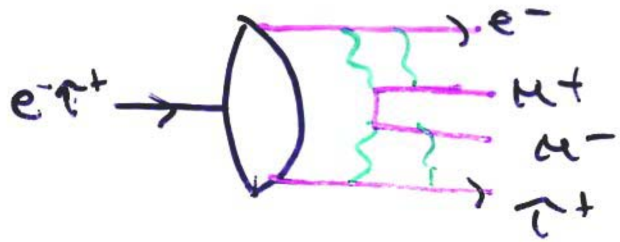
$$s(x) \neq \bar{s}(x)$$

- "Intrinsic Charm": High momentum heavy quarks
- "Hidden Color": Deuteron not always p + n
- Orbital Angular Momentum Fluctuations - Anomalous Magnetic Moment

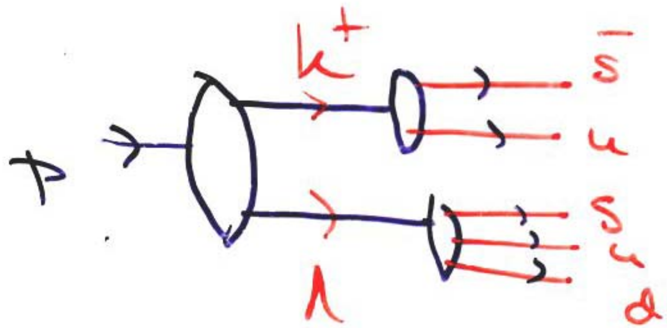
Properties of Intrinsic sea

$$Q(x) \neq \bar{Q}(x)$$

QED analog



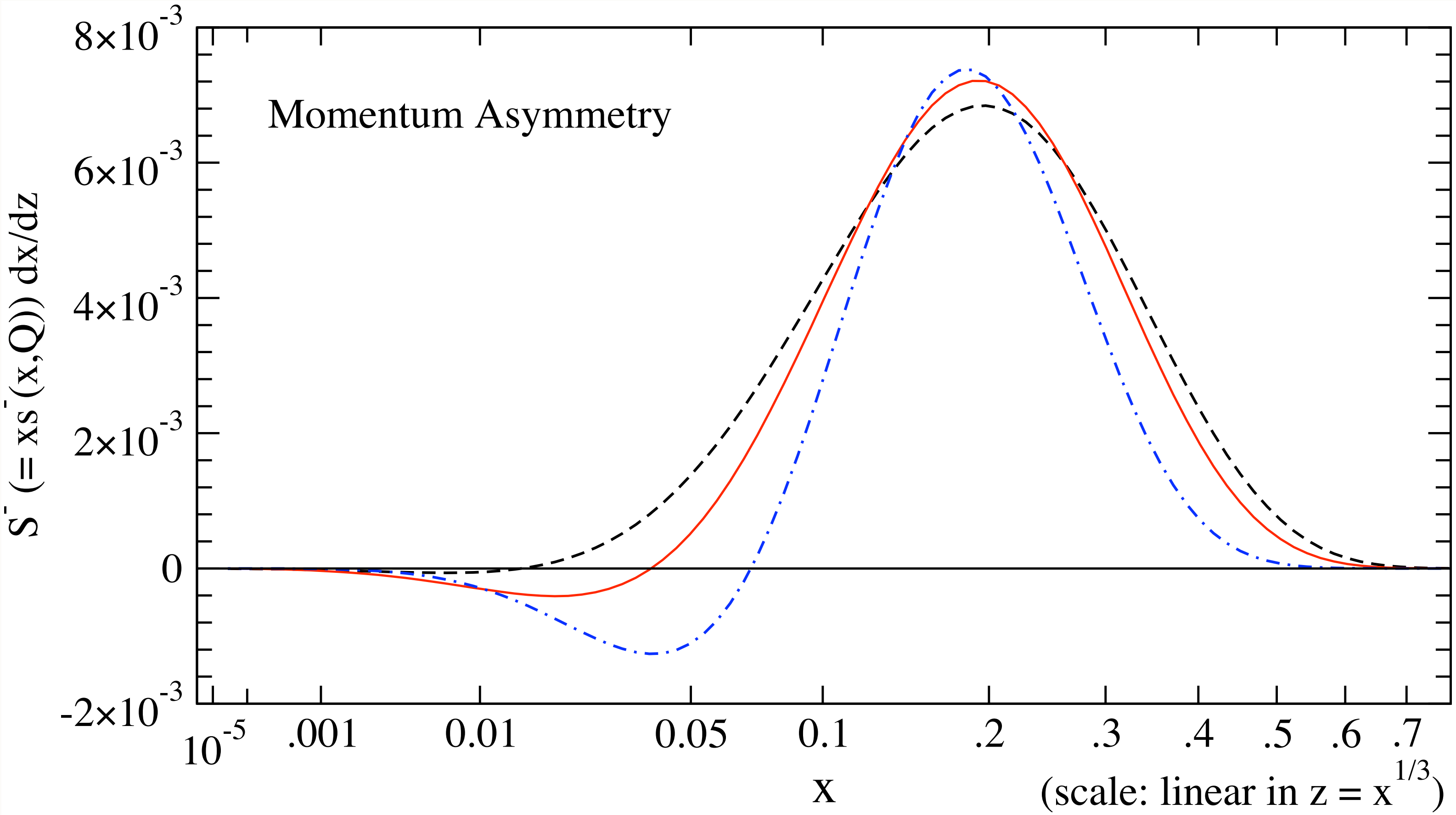
Coulomb interactions break μ± symmetry



$$\langle x_{\bar{s}} \rangle < \langle x_s \rangle$$

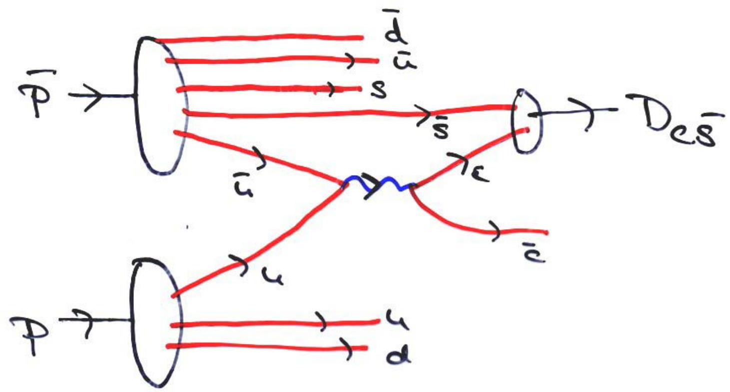
$$\lambda_s \sim -\lambda_p$$

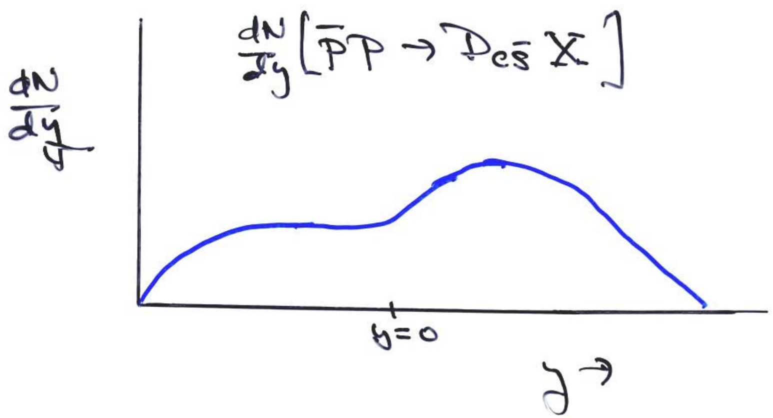
$$S^{-}(x) = x[s(x) - \bar{s}(x)]$$



S. Kretzer; B.Q. Ma and sjb

Test of $s(x) \neq \bar{s}(x)$ in $\bar{p}p$ reactions





Asymmetric distribution! $D_{c\bar{s}}$ harder than $D_{\bar{c}s}$

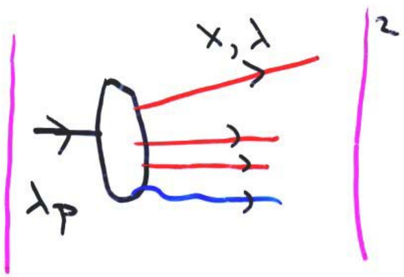
reflects $\bar{s}(x)$ harder than $s(x)$ in $\bar{p}$!

Conventional wisdom: $D_{c\bar{s}}$ and $D_{\bar{s}c}$ identical

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$$q_{\lambda/\lambda_p} = \sum$$



$$q_{\lambda/\lambda_p}(x, \Lambda)$$

- transversity: density matrix
- light-cone helicity

$$= \sum_{n,q} \int \left| \psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1}^{n} dx_j \prod_{j=1}^{n} d^2 k_{\perp j}$$

$$\delta\left(\sum_i^n x_i - 1\right) \delta\left(\sum_i^n \vec{k}_{\perp i}\right)$$

$$\delta(x - x_q)\, \delta_{\lambda, \lambda_q}$$

$$\Theta(\Lambda^2 - \mathcal{M}_n^2)$$

$x \to x_{Bj}$

DGLAP, Factorization

Light-Cone Scheme

Exact Representation of Form Factors using LFWFs

Hadron form factors can be expressed as a sum of overlap integrals of light-front wave functions:

Drell Yan, West, Drell, SJB

$$F(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\vec{k}_{\perp i}\right] \sum_j e_j \psi_n^*(x_i, \vec{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i), \qquad (1)$$

where the variables of the light-cone Fock components in the final-state are given by

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\, \vec{q}_\perp, \qquad (2)$$

for a struck constituent quark and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\, \vec{q}_\perp, \qquad (3)$$

for each spectator. The momentum transfer is $q^2 = -\vec{q}_\perp^{\,2} = -2P \cdot q = -Q^2$. The measure of the phase-space integration is

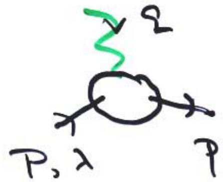
$$\left[dx_i\right] = \prod_{i=1}^{n} \frac{dx_i}{\sqrt{x_i}}\, \delta\left(1 - \sum_{j=1}^{n} x_j\right), \qquad (4)$$

$$\left[d^2\vec{k}_{\perp i}\right] = (16\pi^3) \prod_{i=1}^{n} \frac{d^2\vec{k}_{\perp i}}{16\pi^3} \delta^{(2)}\left(\sum_{\ell=1}^{n} \vec{k}_{\perp \ell}\right). \qquad (5)$$

Light-Front Wavefunctions

* Space-like form factors computed from diagonal $n = n'$ overlap

$$q^+ = 0, \quad q^2 = -q_\perp^2 = -Q^2$$



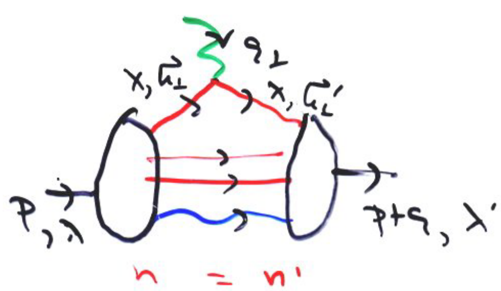
$$= \langle p+q | \frac{J^+(0)}{p^+} | p \rangle$$

Drell, Yan, West
Drell, SJB

$$F_{\lambda\lambda'}(q^2) = \sum_q e_q \sum_{n=n'} \int [d^2k_\perp] \int [dx]$$

$$\psi_{n'}(x, \vec{k}_\perp', \lambda')\, \psi_{n,\lambda}(x, \vec{k}_\perp, \lambda)$$

$$\vec{k}_\perp' = \begin{cases} \vec{k}_\perp + (1-x)\vec{q}_\perp & \text{struck parton} \\ \vec{k}_\perp - x\,\vec{q}_\perp & \text{spectators} \end{cases}$$



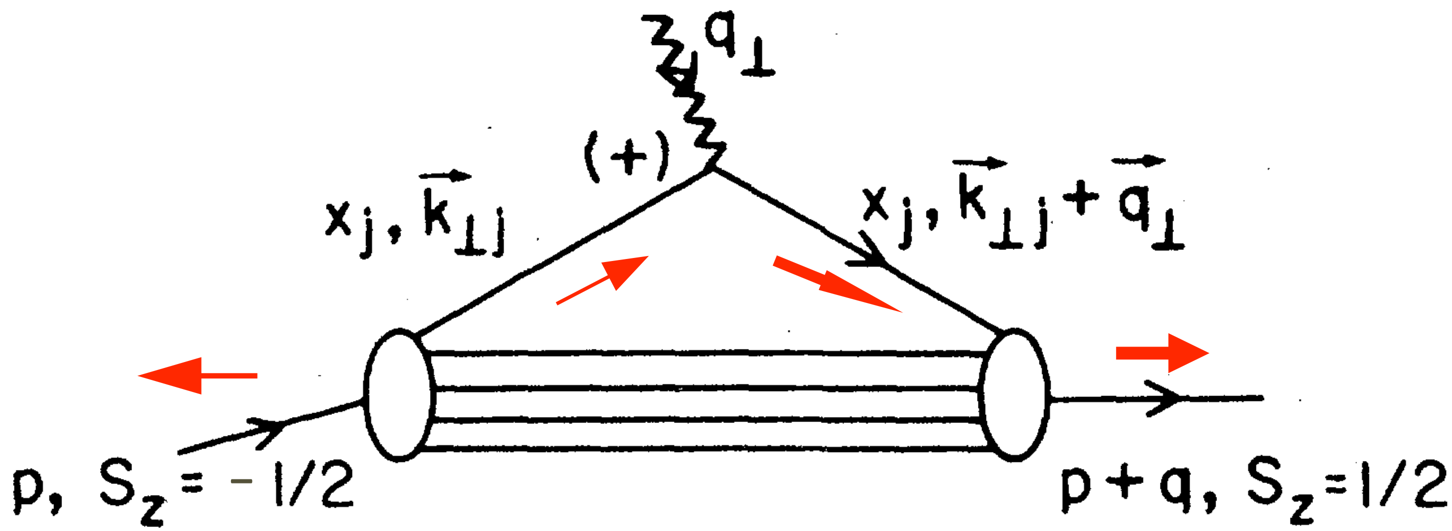
- no ghosts (l.c.g.)
- no vacuum graphs
- no infinite sum of irreducible kernels

Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

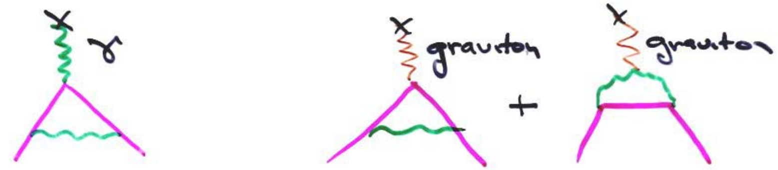
$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous Gravitomagnetic Moment B(0)

QED



$F_2(0) = \frac{\alpha}{2\pi}$

$B(0) = \frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0.$

$B(q^2) \sim \frac{\alpha}{\pi}\sqrt{q^2/m^2}$

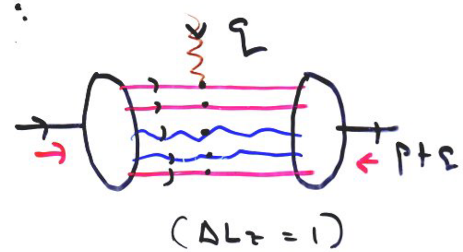
Equivalence Principle : $B(0) = 0$

any spin 1/2 system

Okun + Kobzarev ('62)
X. Ji , Teryaev

L.F. :

$\lim_{q^2 \to 0} \sum_{i=1}^{n}$



$\sum_{i=1}^{n} B_i(0) = 0$

Fock state by Fock state.

Result of Lorentz prop of LF wavefunction.

Hwang, Ma, Schmidt, SJB

key Question for LGTh

$B_p(0) = 0$?

Important indicator of lattice errors.

Gardner, Hwang, sjb,

Light-Cone Wavefunction Representations of Anomalous Magnetic Moment and Electric Dipole Moment

In the case of a spin-$\frac{1}{2}$ composite system, the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$, electric dipole moment form factor $F_3(q^2)$ are defined by

$$\langle P'|J^\mu(0)|P\rangle = \overline{U}(P')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha\right]U(P)\,, \quad (47)$$

Compute matrix elements of good current J+

$$F_1(q^2) = \left\langle P+q,\uparrow\left|\frac{J^+(0)}{2P^+}\right|P,\uparrow\right\rangle = \left\langle P+q,\downarrow\left|\frac{J^+(0)}{2P^+}\right|P,\downarrow\right\rangle\,, \quad (48)$$

$$\frac{F_2(q^2)}{2M} = \frac{1}{2}\left[+\frac{1}{-q^1+\mathrm{i}q^2}\left\langle P+q,\uparrow\left|\frac{J^+(0)}{2P^+}\right|P,\downarrow\right\rangle + \frac{1}{q^1+\mathrm{i}q^2}\left\langle P+q,\downarrow\left|\frac{J^+(0)}{2P^+}\right|P,\uparrow\right\rangle\right], \quad (49)$$

$$\frac{F_3(q^2)}{2M} = \frac{i}{2}\left[+\frac{1}{-q^1+\mathrm{i}q^2}\left\langle P+q,\uparrow\left|\frac{J^+(0)}{2P^+}\right|P,\downarrow\right\rangle - \frac{1}{q^1+\mathrm{i}q^2}\left\langle P+q,\downarrow\left|\frac{J^+(0)}{2P^+}\right|P,\uparrow\right\rangle\right]. \quad (50)$$

Relation between edm and anomalous magnetic moment

$$\frac{F_2(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3} \sum_j e_j \, \frac{1}{2} \; \times$$

$$\left[+ \frac{1}{-q^1 + \mathrm{i}q^2} \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i) + \frac{1}{q^1 + \mathrm{i}q^2} \psi_a^{\downarrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \vec{k}_{\perp i}, \lambda_i) \right]$$

Drell, sjb,

$$\frac{F_3(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp \mathrm{d}x}{16\pi^3} \sum_j e_j \, \frac{i}{2} \; \times$$

$$\left[+ \frac{1}{-q^1 + \mathrm{i}q^2} \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i) - \frac{1}{q^1 + \mathrm{i}q^2} \psi_a^{\downarrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \vec{k}_{\perp i}, \lambda_i) \right],$$

Gardner, Hwang, sjb,

$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$ struck quark $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$ spectator

CP-violating phase of LFWF

$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

New relation between d_n and d_p

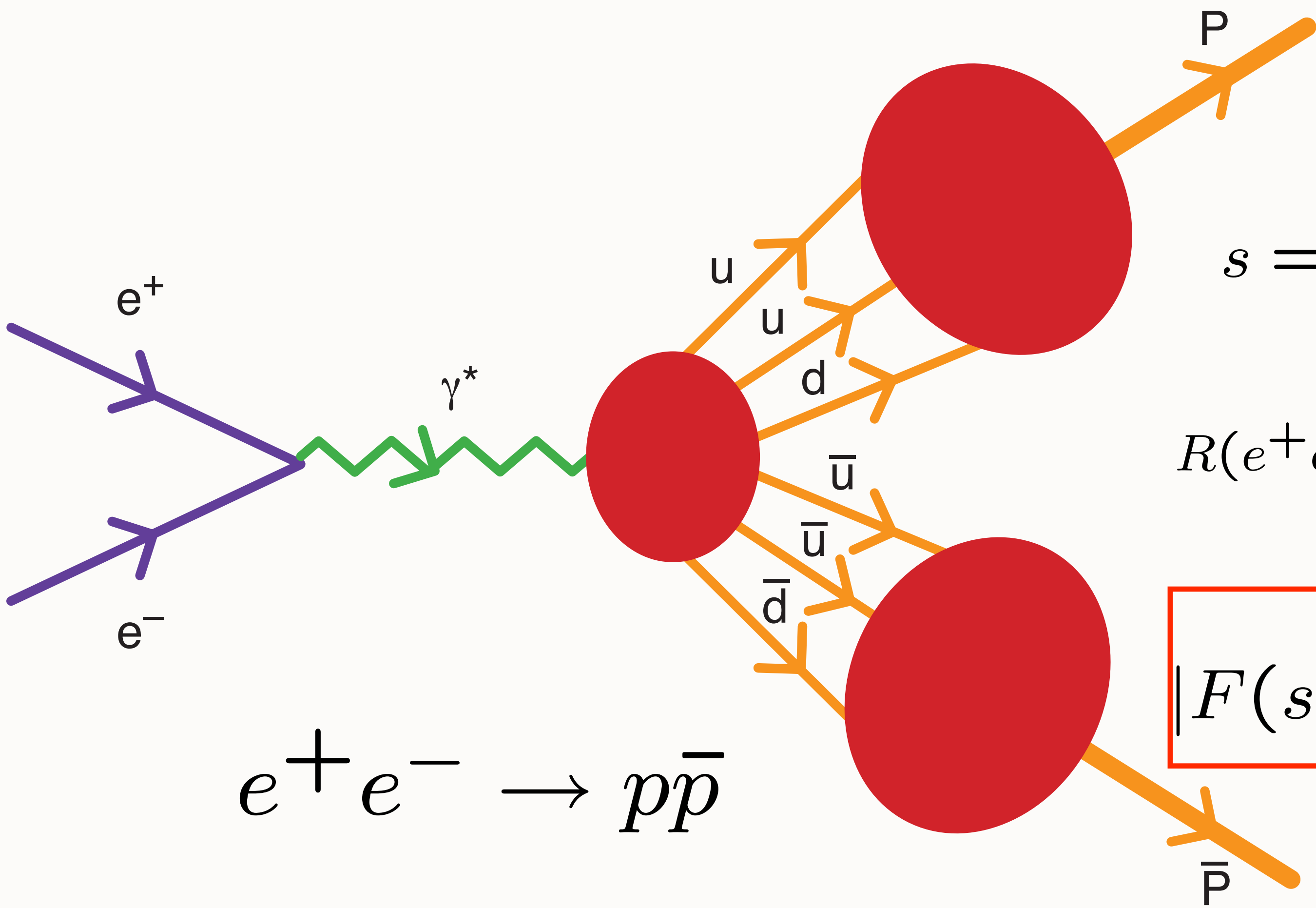
Nuclear Chromodynamics:

Novel Effects of QCD in Nuclear Systems

- QCD Color Transparency and Opaqueness
- Hidden Color
- Exclusive Nuclear Reactions, x > 1
- Nuclear shadowing and antishadowing
- Diffractive Phenomena

Exclusive Processes

What if we ask for a specific final state?



$$e^+e^- \to p\bar{p}$$

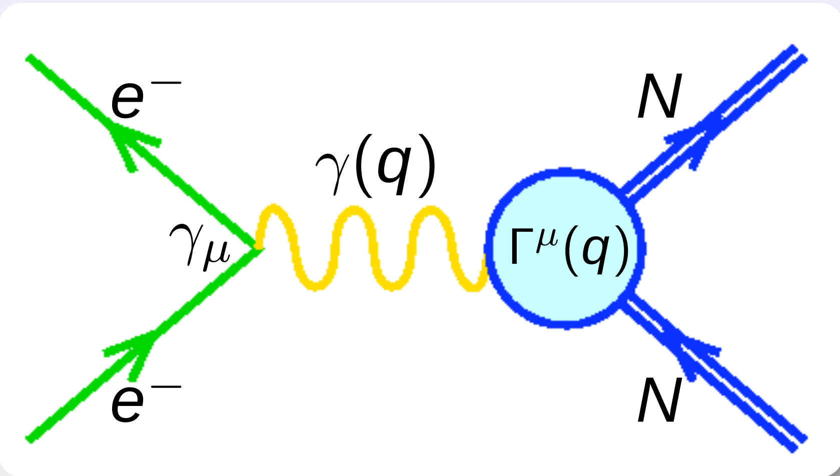
$$s = (E_{e^+} + E_{e^-})^2$$

$$R(e^+e^- \to H\bar{H}) \propto |F(s)|^2$$

$$|F(s)| \propto [\frac{1}{s}]^{n_q - 1}$$

Probability decreases with number of constituents!

Nucleon Form Factors

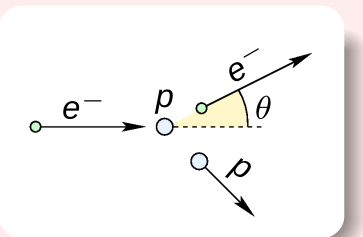


Nucleon current operator (Dirac & Pauli)

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_N}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

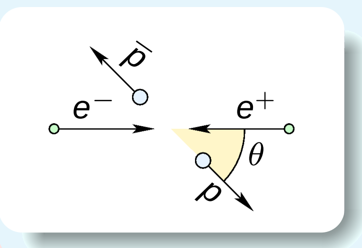
Electric and Magnetic Form Factors

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$
$$\tau = \frac{q^2}{4M_N^2}$$



Elastic scattering $\quad ep \longrightarrow ep$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}}\left[G_E^2 + \tau\left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1+\tau}$$



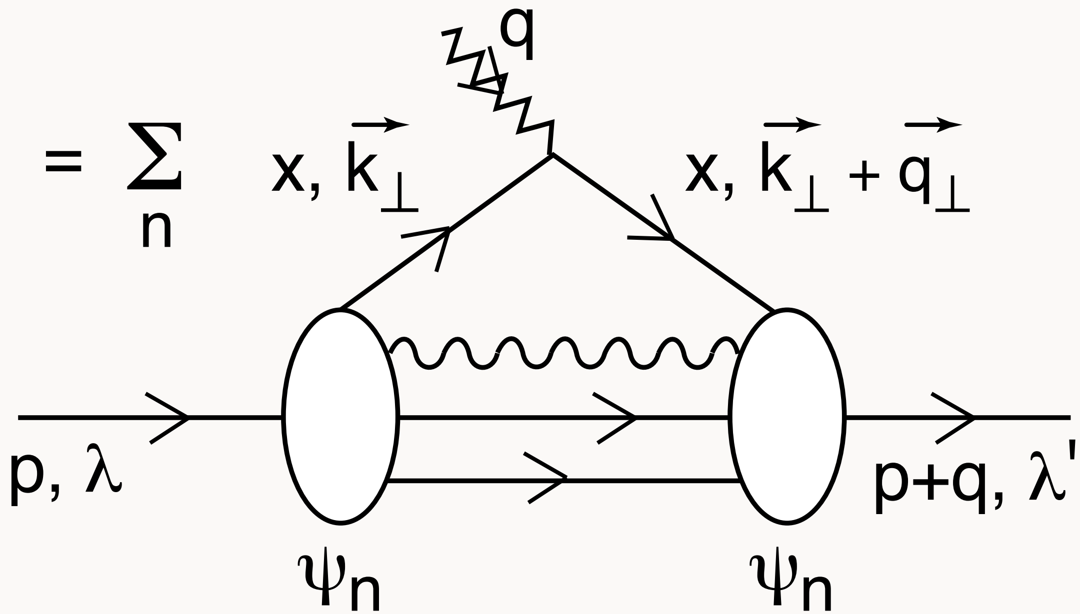
Annihilation $\quad e^+e^- \longrightarrow p\bar{p}$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\sqrt{1-1/\tau}}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

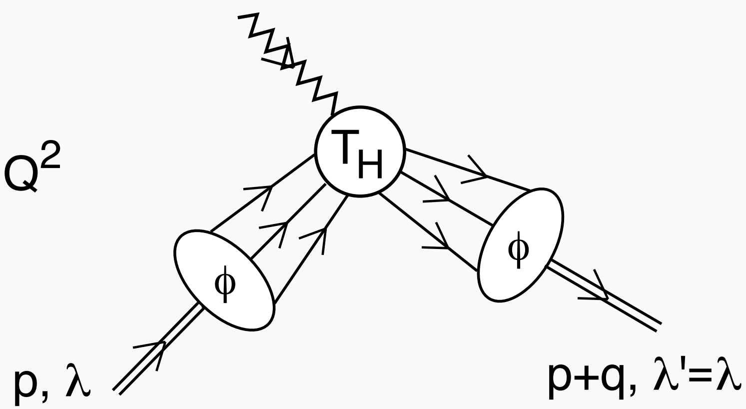
Simone Pacetti | Ratio $|G_E^p(q^2)/G_M^p(q^2)|$ and dispersion relations

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+(0) | p \lambda \rangle$

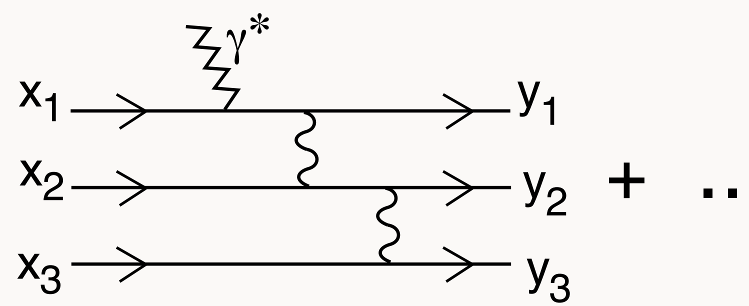
$$F_{\lambda\lambda'}(Q^2) = \sum_n$$



⇨ Large Q^2



$T_H = \sum$



Scaling from PQCD or AdS/CFT

$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i)$$

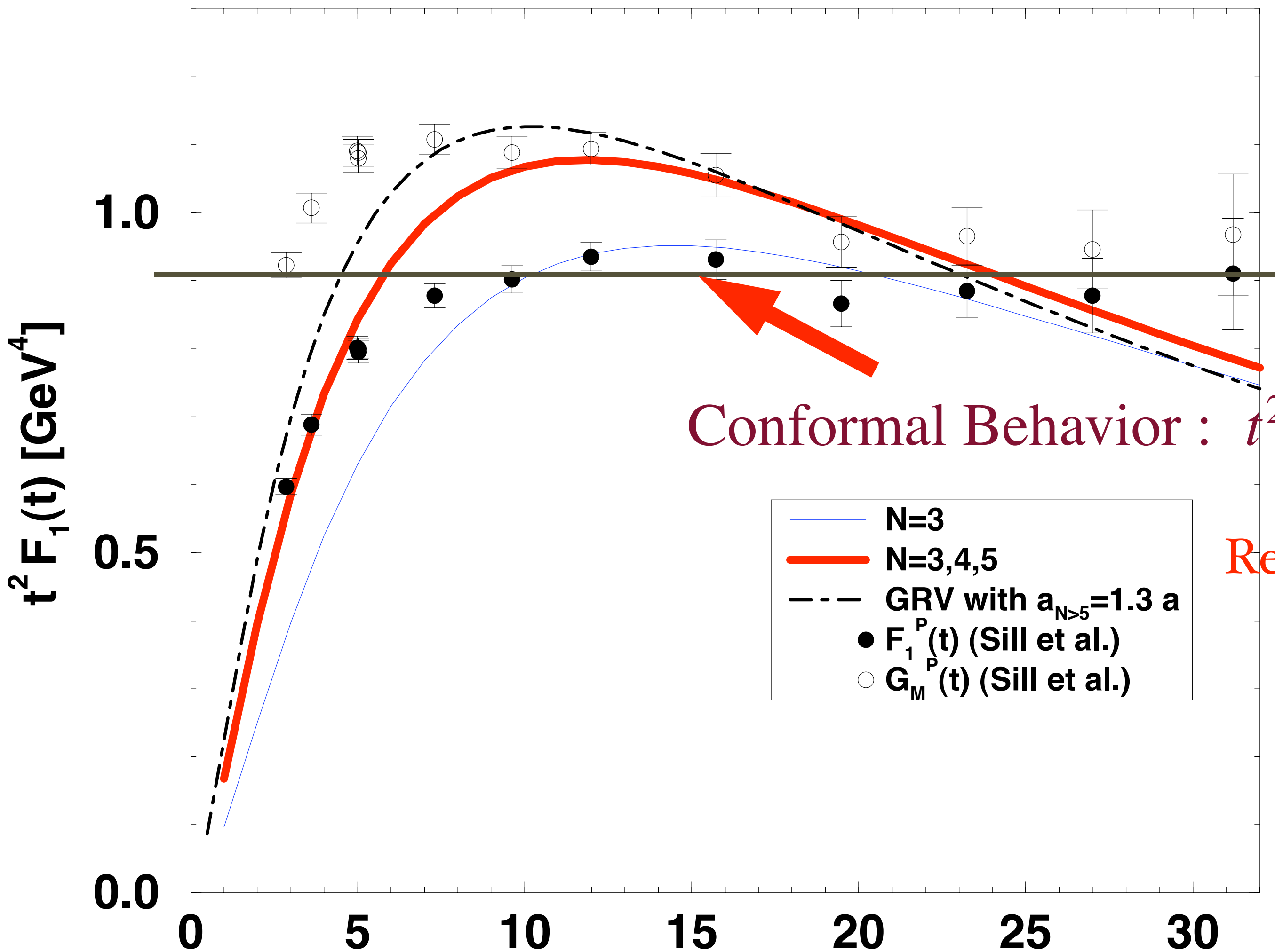
Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \ \psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage; SJB
Efremov, Radyuskin

Proton Form Factor



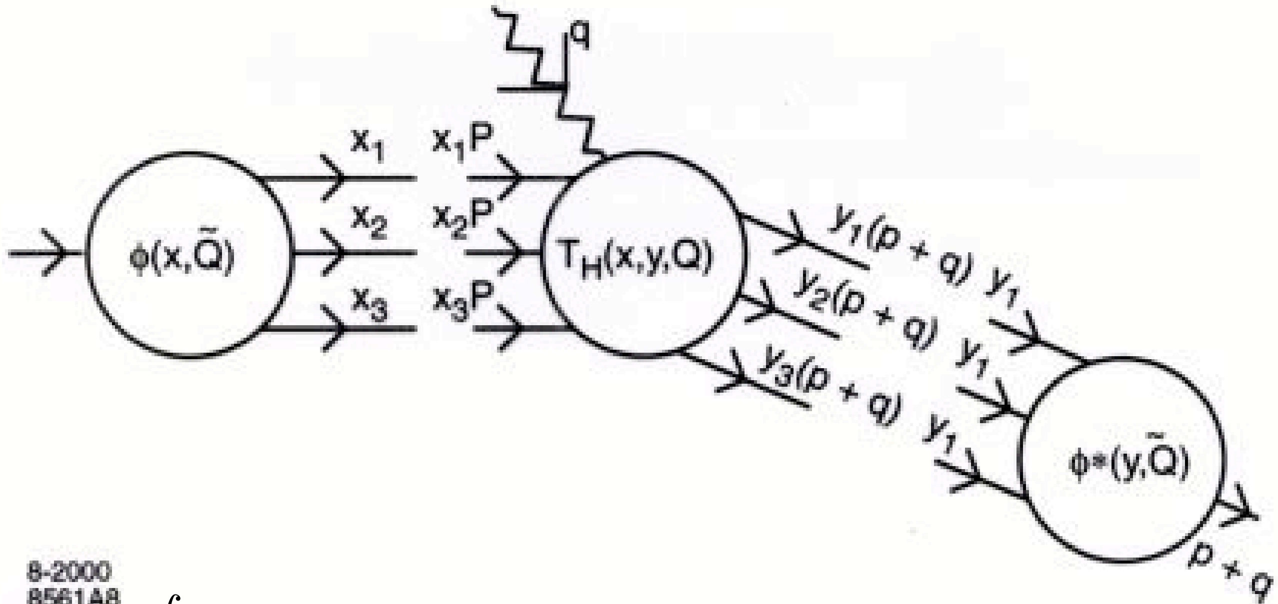
-t (GeV^2)

Conformal Behavior : $t^2 F_1(t) = \text{const}$

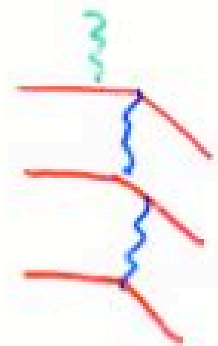
Remarkable scaling behavior

Non-perturbative model: Diehl, Kroll

Primary Test of QCD Factorization, Scaling



$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

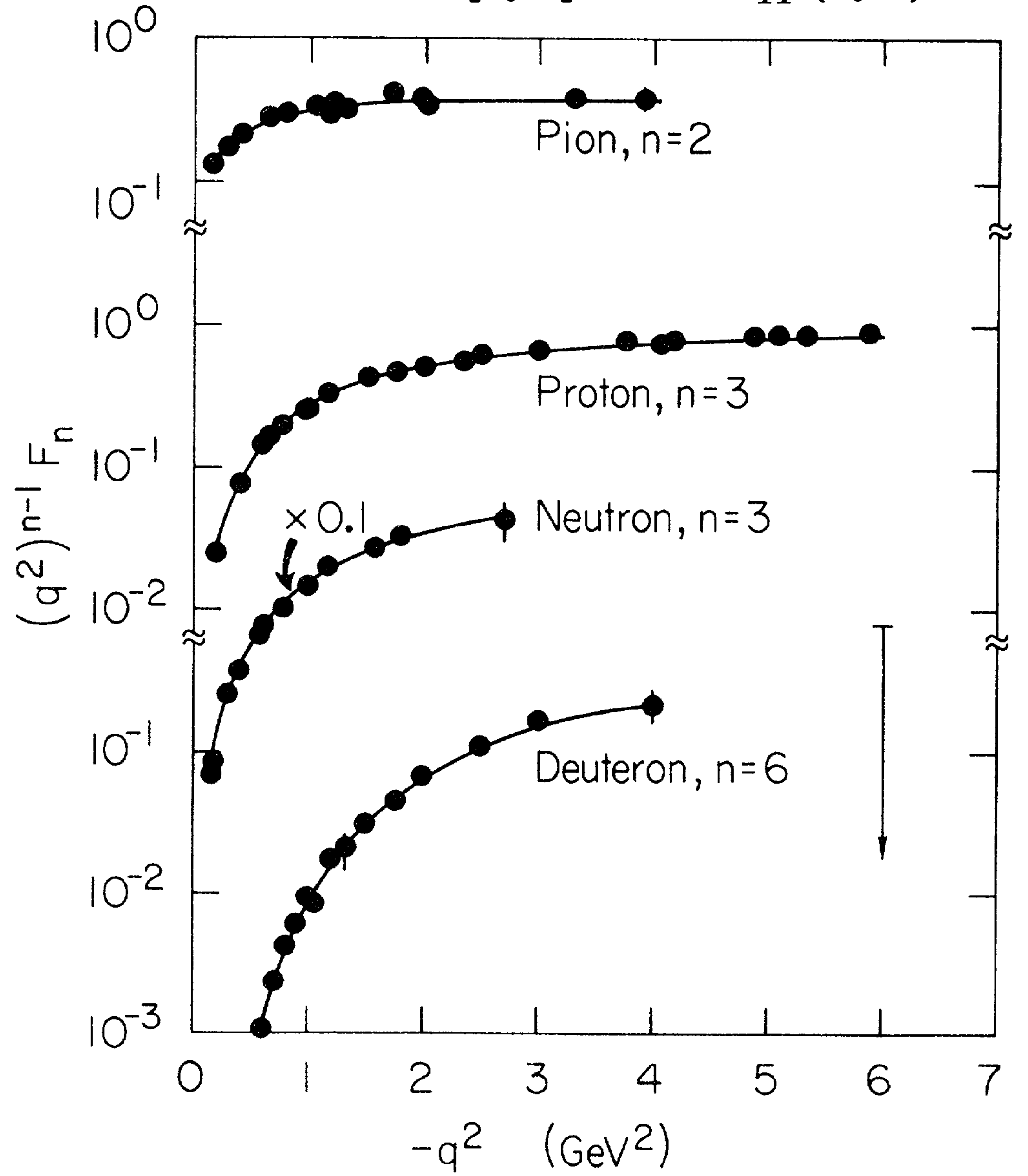


Conformal Behavior : $t^2 F_1(t) = \text{const}$

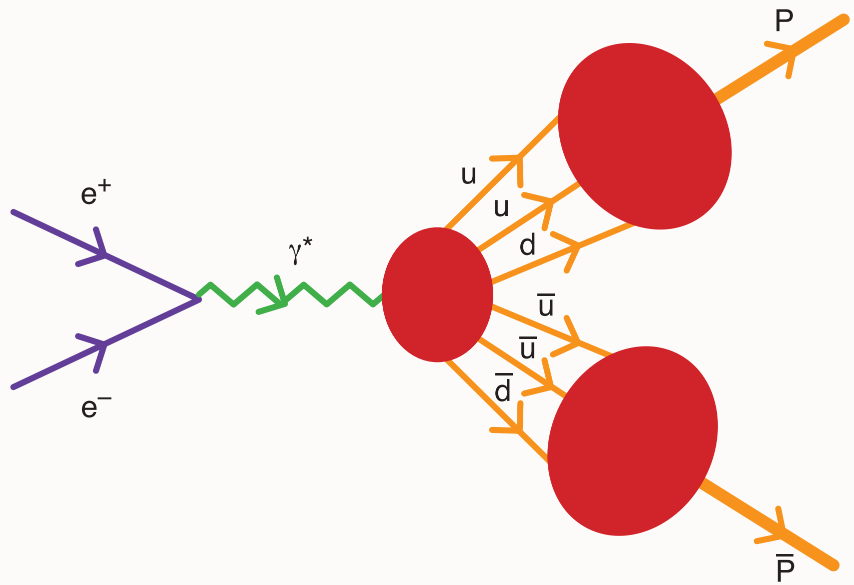
Quark Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
- $F_H(Q) \propto \frac{1}{(Q^2)^{n-1}}$ n = # elementary constituents

Quark counting rules predict: $[Q^2]^{n_H - 1} F_H(Q^2) \rightarrow$ **constant**



Timelike proton form factor in PQCD



$$G_M(Q^2) \rightarrow \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left(\log \frac{Q^2}{\Lambda^2}\right)^{\gamma_n^B + \gamma_n^B}$$

$$\times \left[1 + \mathcal{O}\left(\alpha_s(Q^2), \frac{m^2}{Q^2}\right)\right] .$$

Lepage and Sjb

PQCD and Exclusive Processes

Lepage; SJB
Efremov, Radyuskin

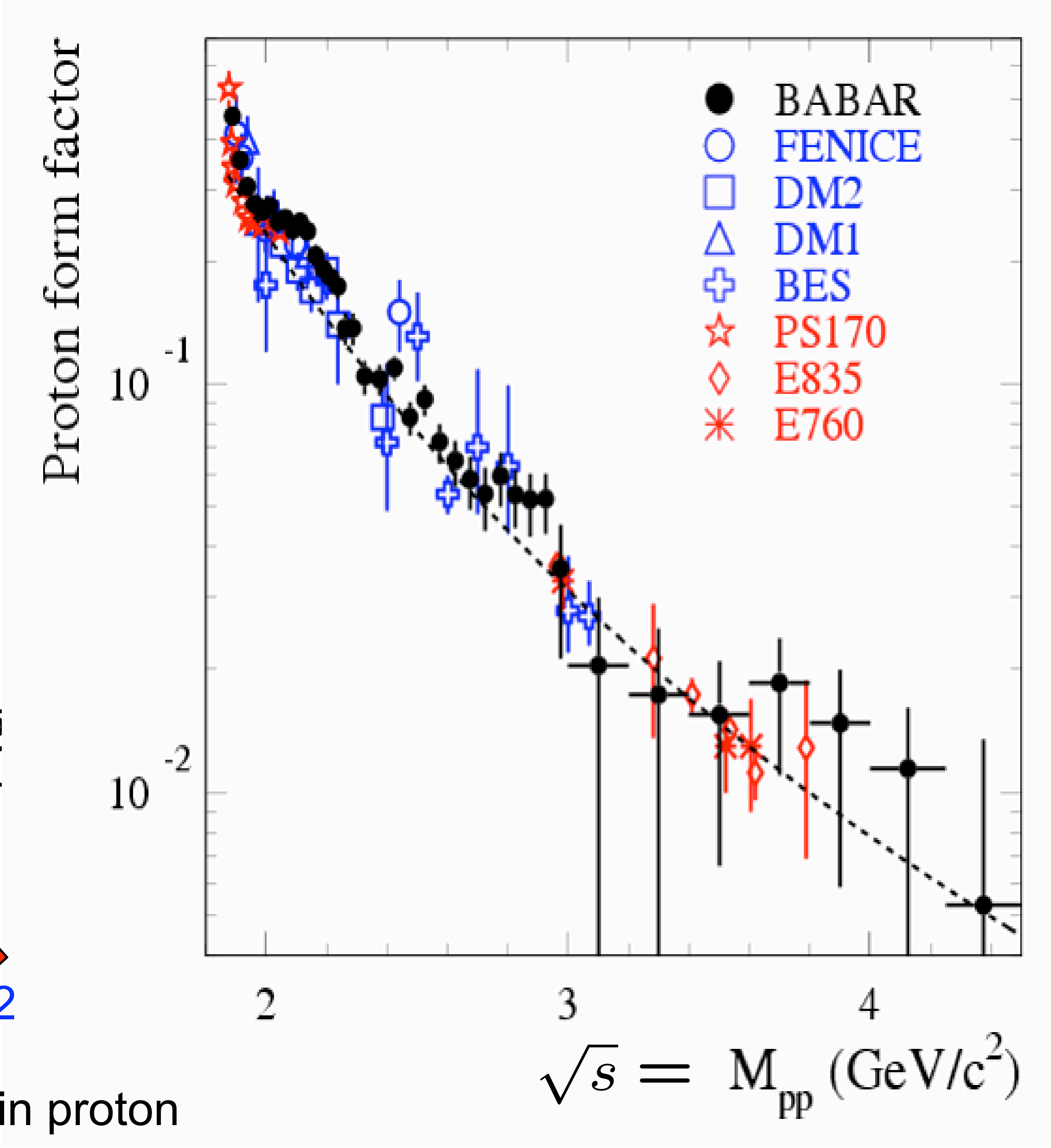
$$M = \int \prod dx_i dy_i \phi_F(x, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \phi_I(y_i, Q)$$

- Iterate kernel of LFWFs when at high virtuality; distribution amplitude contains all physics below factorization scale
- Rigorous Factorization Formulae: Leading twist
- Underly Exclusive B-decay analyses
- Distribution amplitude: gauge invariant, OPE, evolution equations, conformal expansions
- BLM scale setting: sum nonconformal contributions in scale of running coupling
- Derive Dimensional Counting Rules/ Conformal Scaling

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2,$$

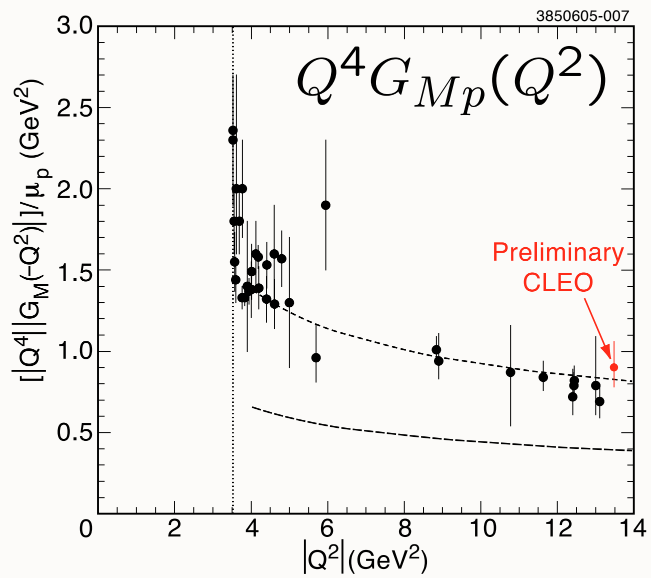
$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$

$n_q - 1 = 3 - 1 = 2$

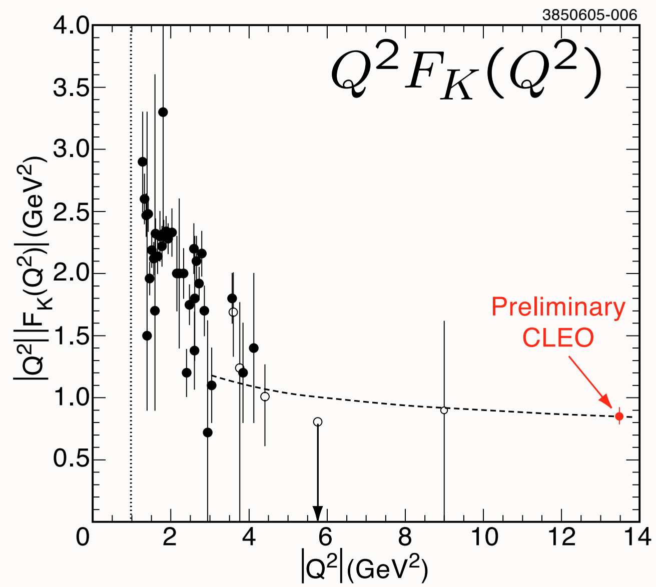


Quark counting for 3 quarks in proton

Test of quark counting rule: timelike form factors



Proton timelike form factor.



Kaon timelike form factor.

$$s = E^2_{\rm cm} = Q^2$$

$$\boldsymbol{Q^2|F_K(13.48\ \mathrm{GeV}^2)| \ = \ 0.85 \pm 0.05(\mathrm{stat}) \pm 0.02(\mathrm{syst})\ \mathrm{GeV}^2}$$

$$\boldsymbol{Q^4|G^p_M(13.48\ \mathrm{GeV}^2)|/\mu_p \ = \ 0.91 \pm 0.13(\mathrm{stat}) \pm 0.06(\mathrm{syst})\ \mathrm{GeV}^4}$$

The proton magnetic form factor result agrees with that measured in the reverse reaction $p\bar{p} \to e^+e^-$ at Fermilab. **The kaon form factor measurement is the first ever direct measurement at $|Q^2| > 4$ GeV^2.**

The pion form factor is being measured.

Seth

Conformal Behavior of LFWFs Predicted by AdS/CFT Leads to PQCD Scaling Laws

- Bjorken Scaling of DIS
- Counting Rules of Structure Functions at large x
- Dimensional Counting Rules for Exclusive Processes and Form Factors